Temperature-Dependent Decay of Quasi-Two-Dimensional Vortices across the BCS-BEC Crossover

Xiang-Pei Liu,[1,2,3,*] Xing-Can Yao,[1,2,3,*] Xiaopeng Li,[4,5,*] Yu-Xuan Wang,[1,2,3]
Chun-Jiong Huang,[6] Youjin Deng,[1,2,3,7] Yu-Ao Chen,[1,2,3] and Jian-Wei Pan[1,2,3]

[1]*Hefei National Research Center for Physical Sciences at the Microscale and School of Physical Sciences, University of Science and Technology of China, Hefei 230026, China*
[2]*Shanghai Research Center for Quantum Science and CAS Center for Excellence in Quantum Information and Quantum Physics, University of Science and Technology of China, Shanghai 201315, China*
[3]*Hefei National Laboratory, University of Science and Technology of China, Hefei 230088, China*
[4]*State Key Laboratory of Surface Physics, Institute of Nanoelectronics and Quantum Computing, and Department of Physics, Fudan University, Shanghai 200433, China*
[5]*Shanghai Qi Zhi Institute, AI Tower, Xuhui District, Shanghai 200232, China*
[6]*Department of Physics and HKU-UCAS Joint Institute for Theoretical and Computational Physics at Hong Kong, The University of Hong Kong, Hong Kong, China*
[7]*MinJiang Collaborative Center for Theoretical Physics, College of Physics and Electronic Information Engineering, Minjiang University, Fuzhou 350108, China*

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We systematically study the decay of quasi-two-dimensional vortices in an oblate strongly interacting Fermi gas over a wide interaction range and observe that, as the system temperature is lowered, the vortex lifetime increases in the Bose-Einstein condensate (BEC) regime but decreases at unitarity and in the Bardeen-Cooper-Schrieffer (BCS) regime. The observations can be qualitatively captured by a phenomenological model simply involving diffusion and two-body collisional loss, in which the vortex lifetime is mostly determined by the slower process of the two. In particular, the counterintuitive vortex decay in the BCS regime can be interpreted by considering the competition between the temperature dependence of the vortex annihilation rate and that of unpaired fermions. Our results suggest a competing mechanism for the complex vortex decay dynamics in the BCS-BEC crossover for the fermionic superfluids.

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The dynamics of vortices and antivortices in a superfluid play a pivotal role in various fascinating quantum phenomena, such as quantum turbulence [1,2], the Berezinskii-Kosterlitz-Thouless phase transition [3–5], and large-scale vortex clustering [6–8]. Of particular interest is the decay dynamics of vortices. Different from the low-energy phonon excitations, which are usually quickly thermalized, topological high-energy vortices are quasistable and can survive for a significantly longer time before dying out, during which the system approaches equilibrium and the global phase coherence of superfluidity builds up. At nonzero temperature T, vortex decay is mainly driven by two energy dissipation mechanisms, i.e., diffusion of vortices and annihilation of vortex and antivortex pairs. The diffusive spread of vortices is due to their frictional interactions with the normal component of a superfluid [9–11], and the diffusion constant D is proportional to T. When a vortex and antivortex pair gets close to each other, the effective transverse superfluid Magnus force [12], which is proportional to the superfluid density, further attracts them, yielding a collective annihilation accompanied by the emission of sound waves [13]. Despite decades of efforts, how the decay time (or lifetime) of vortices evolves with the system temperature and interaction strength remains an open question.

A platform for addressing such a question can be provided by atomic fermionic superfluids. By virtue of the Feshbach resonance, the system can be tuned from a Bardeen-Cooper-Schrieffer (BCS) superfluid of Cooper pairs to a Bose-Einstein condensate (BEC) of molecular dimers [14,15]. Based on the Kibble-Zurek mechanism [16–18], dozens of spontaneous vortices have already been generated in fermionic superfluids across the BCS-BEC crossover [19–21]. This allows us to experimentally investigate vortex decay dynamics with drastically different microscopic details, such as interaction strength, atomic density, and even the statistics of constituent particles. Very recently, universal power-law scaling decay behaviors of quasi-2D vortices have been observed [21].

In this Letter, we report the measurements of the T-dependent decay dynamics of quasi-2D vortices [20,21] across the BCS-BEC crossover. The large number of randomly distributed vortices are spontaneously generated via thermally quenching the Fermi gases across the

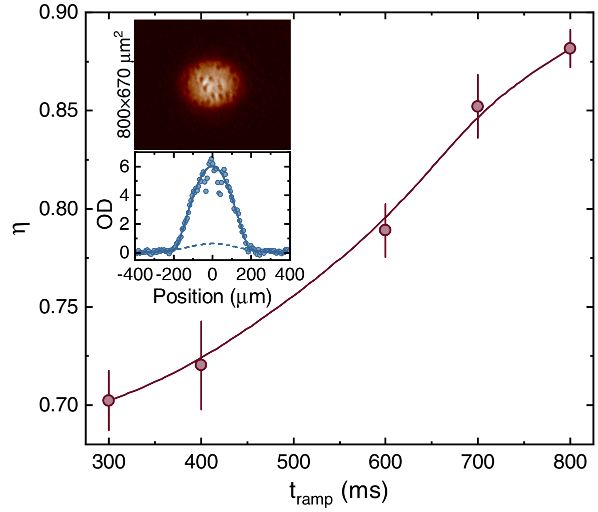

FIG. 1. Quasicondensate fraction η for different trap ramping time t_{ramp} at unitarity, as obtained from a Gaussian plus Thomas-Fermi distribution fitting to the column density profile [taken after 10 ms time-of-flight (TOF)]. The error bars represent standard statistical errors, calculated from three individual measurements. The solid line is for guiding eyes. The inset displays a TOF image and the corresponding fitting.

superfluid transition [20,21]. For a given magnetic field or interaction strength, we extract a series of vortex lifetimes (decay times) by measuring the total vortex density as a function of holding time t for different quench rates. The fraction η of quasicondensate (i.e., consisting of bosonic pairs in the vicinity of zero momentum) is also measured at time $t = 0$ to qualitatively characterize the system temperature T. In the BEC regime, as expected for a weakly interacting Bose gas, the vortices decay more slowly in a lower-T superfluid. In contrast, in the unitarity and BCS regimes, the lifetime of vortices is longer for a hotter superfluid. As an attempt to understand the observations, we present a simple diffusion-annihilation model, in which vortices randomly move and, when a pair of vortex and antivortex are sufficiently close to each other, they might disappear with an annihilation rate K. In this model, it is shown that the vortex decay time τ is mainly from the slower one of the diffusion and annihilation processes. Considering the strong friction and diffusion of vortex motion in the unitarity and BCS regimes [13], we argue that the counterintuitive vortex decay behavior is qualitatively explained by the diffusion-annihilation model.

The experimental method for generating randomly distributed quasi-2D vortices in a ^{6}Li fermionic superfluid has been described in our previous work [20–22]. About 1×10^7 ^{6}Li atoms with balanced populations of the two lowest hyperfine spin states are first confined in an oblate optical trap [$1/e^2$ radii are 200 and 48 μm (gravity direction)]. Then, the trap potential is ramped down in a short time interval (compared with a sufficient evaporative cooling time of about 2 s) to effectively quench the system across the superfluid transition temperature T_c near the broad Feshbach resonance at 832 G. The number of quasicondensate and spontaneous vortices are measured in real time by taking an absorption image along the tight direction of the trap after 10 ms time of flight [20,21]. For the following study, the initial time $t = 0$ is defined as the saturation time point of the quasicondensate fraction, when the vortex cores become clearly visible. The ability to prepare Fermi gas with a large atom number at a temperature slightly above T_c, enables us to generate many spontaneous vortices over a wide range of trap ramping time t_{ramp}. Furthermore, due to the better thermal equilibrium during evaporative cooling, a lower final system temperature is expected for a longer t_{ramp}, implying that the final system temperature can be effectively tuned by changing the t_{ramp}. This is confirmed by the measured quasicondensate fraction η at $t = 0$ for a series of t_{ramp} ranging from 300 to 800 ms, where η monotonically increases with the increase of t_{ramp} (see Fig. 1).

We first investigate the temperature dependence of the vortex decay time at unitarity. To this end, we perform a series of measurements on the temporal evolution of total vortex density ρ_v for $t_{\mathrm{ramp}} = 300$, 400, 600, 700, and 800 ms. Based on the universal power-law decay behavior of the quasi-2D vortices [21], we fit the experimental data by $1/\rho_v(t) = a + \lambda t$, where λ is the decay rate. Unlike the conventional exponential decay dynamics, the vortex lifetime is characterized by $\tau \equiv 1/\lambda$. An example of the fits is given by the inset of Fig. 2(c). Note that data points with less than two vortices are excluded in the fit, as the trivial one-body process dominates the following decay dynamics (i.e., $t > 1$ s). The results of $\tau = 1/\lambda$ versus $1 - \eta$ are presented in Fig. 2(c). It is remarkable that τ monotonically increases as η decreases (i.e., the increase in temperature). This observation is counterintuitive in that the vortices are expected to decay faster at a higher temperature because of more frequent collisions with thermal atoms, as is the case for a weakly interacting bosonic gas [23].

To explore atom-atom interaction effects, we carry out a systematic study of the vortex lifetime in the BEC (783 and 809 G) and BCS (847 and 861 G) regimes. Unfortunately, the system temperature cannot be quantitatively determined, due to the highly nonequilibrium feature of the cloud (i.e., the presence of many vortices) and the lack of reliable knowledge of the equation of state [24,25]. Alternatively, the quasicondensate fraction η is probed at $t = 0$ for each measurement to qualitatively characterize the system temperature, i.e., $1 - \eta$ monotonically increases with T. The experimental results of $\tau = 1/\lambda$ are shown in Fig. 2 as a function $1 - \eta$. In the BEC regime [Fig. 2(a)], the vortex lifetime τ becomes longer when temperature T is lowered, which is within expectation. Nevertheless, as unitarity is approached, this tendency is significantly weakened [Fig. 2(b)]. At unitarity and in the BCS regime [Figs. 2(c)–2(e)], the lifetime τ is even shortened as T decreases. Despite

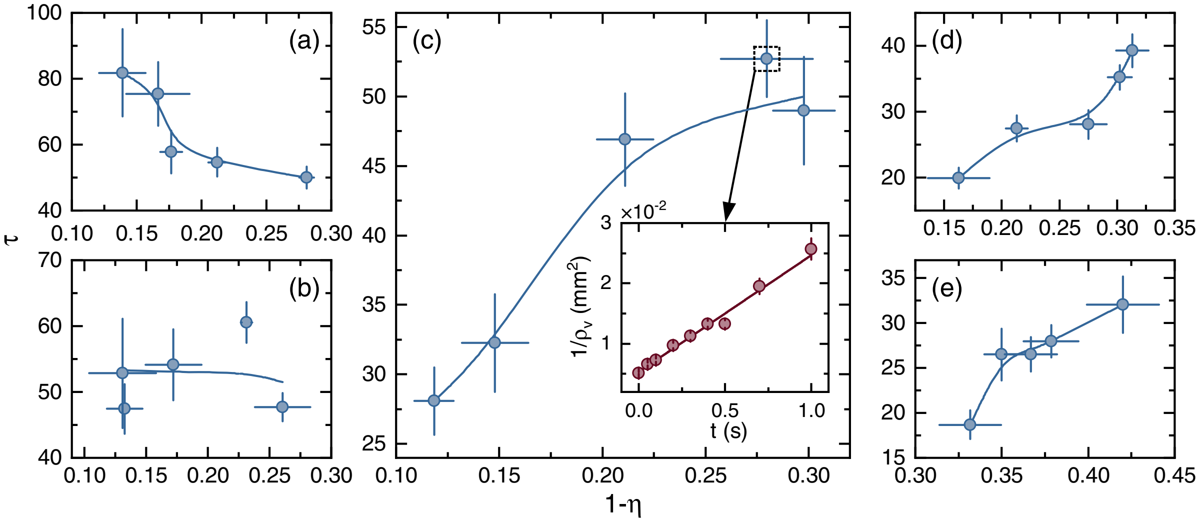

FIG. 2. Temperature dependence of vortex lifetime τ in the BCS-BEC crossover, where larger values of $1-\eta$ correspond to higher temperature. (a)–(e) Results at 783, 809, 832, 847, and 861 G, respectively, with the curves for guiding eyes. An example of the power-law fit $1/\rho_v = a + \lambda t$ is given in the inset of (c), and the solid line represents the linear fit. The vortex lifetime is defined as $\tau = 1/\lambda$, and the temperature is effectively represented by $1-\eta$, with quasicondensate fraction η measured at $t = 0$.

the experimental complexity of controlling the temperature of the strongly interacting fermionic superfluid, the T-dependent tendencies of τ in Fig. 2 is expected to be reliable, as the quasicondensate fraction η, in general, decreases with the temperature.

For an understanding of the vortex decaying behavior across the BEC-BCS regime, first-principle calculations were desired but would be extremely challenging, particularly for nonequilibrium dynamics of such strongly interacting fermionic systems. Nevertheless, in the deep BEC regime and for qualitative temperature dependence, one can assume a universality class and seek for the Ginzburg-Landau field theoretical description to avoid experimental complexity and microscopic details.

As in Ref. [21], we consider a single-site Glauber quench dynamics by simulating the classical XY model on the square lattice. This simplified lattice model, while ignoring the quantum origin of superfluidity and all microscopic details, still exhibits a superfluid phase transition, which is further in the same universality class as that for experimental systems. A random spin configuration is prepared and instantly quenched to a temperature T well below the Berezinskii-Kosterlitz-Thouless (BKT) transition $T_{\mathrm{BKT}} = 0.89$ (see Supplemental Material [26]). The Metropolis simulation is applied by sequentially picking up a spin and rotating it to a random phase with proper acceptance probability. The number of vortices per site ρ_v is recorded as a function of Monte Carlo "time" t, which is defined as one averaged updating step per spin. The procedure is repeated for tens of thousands of times with different random number seeds. According to the Ginzburg-Landau theory, the decaying dynamics of the vortices is $\rho_v \sim \ln t/t$, with a logarithmic correction $\ln t$ to the algebraic behavior $1/t$. The data of $\ln(t)/\rho_v$ versus t are plotted in the inset of Fig. 3, and the decay rate λ is extracted. The T dependence of the vortex lifetime $\tau = 1/\lambda$ is shown in Fig. 3. Consistent with the experimental results in the BEC regime [Fig. 2(a)], τ drops as T increases, and, further, the decreasing factor is about 4 from $T = 0.01$ to 0.4.

As an attempt to reveal the underlying mechanism in the intriguing vortex decay dynamics near unitarity and on the BCS side, we further follow the coarse-grained principle and consider a diffusion-annihilation model [33]

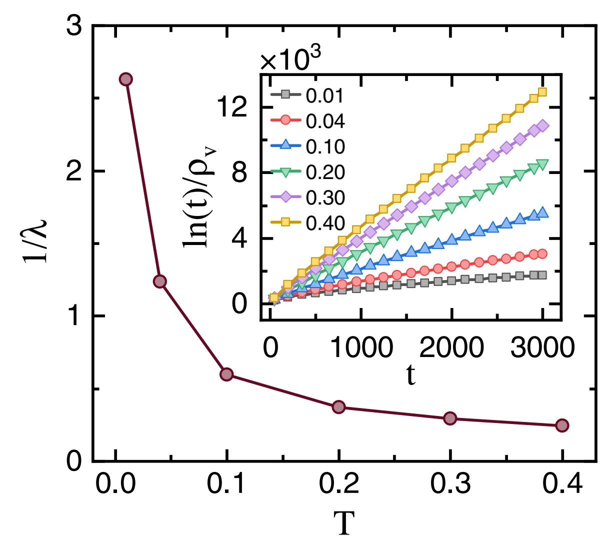

FIG. 3. Temperature dependence of the vortex lifetime $\tau = 1/\lambda$ in the Glauber dynamics of the square-lattice XY model. The inset displays the modified inverse vortex density $\ln(t)/\rho_v$ versus Monte Carlo time t for various temperature T.

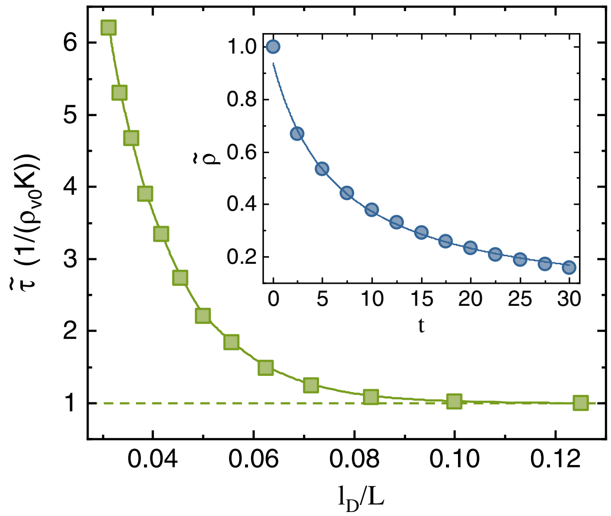

FIG. 4. Normalized decay time $\tilde{\tau}$ [in units of $1/(\rho_{v0}K)$] in the diffusion-annihilation model with $L = 4a_v$. The solid line is a B-spline guide line, and the dashed line is an asymptote at the large diffusion limit. The inset shows an example of time evolution of the dimensionless total vortex density $\tilde{\rho}$.

$$\partial_t n_\pm - D\vec{\nabla}^2 n_\pm = -K n_+ n_-, \tag{1}$$

which is analogous to the equation of the diffusive Coulomb gas dynamics composed of electrons and positrons [34]. The vortex configurations are described by vortex densities $n_+(\vec{x}, t)$ and $n_-(\vec{x}, t)$ with plus and minus representing opposite circulations, respectively. The parameter D characterizes the vortex diffusion due to the collisions of vortices with noncondensed fermions or other thermally excited quasiparticles. The parameter K quantifies the annihilation rate when two vortices with opposite circulations collide. Note that we focus on a balanced case, i.e., $\int d^2\vec{x}[n_+(\vec{x}, t) - n_-(\vec{x}, t)] = 0$, since no net angular momentum is injected during the thermal quenching. Furthermore, the long-range interaction between vortices, given the logarithmic correction that can hardly be observed in our experiments, is neglected.

With the total vortex density $\rho_v = n_+ + n_-$ and the vortex polarization $m = n_+ - n_-$, we have

$$\begin{aligned} &\partial_t \rho_v - D\vec{\nabla}^2 \rho_v = -K(\rho_v^2 - m^2), \\ &\partial_t m - D\vec{\nabla}^2 m = 0. \end{aligned} \tag{2}$$

We further introduce four dimensionless variables, $\rho_v = \rho_{v0}\tilde{\rho}$, $m = \rho_{v0}\tilde{m}$, $\partial_t = (\rho_{v0}K)\partial_{\tilde{t}}$, and $\vec{\nabla}^2 = (D^{-1}\rho_{v0}K)\partial^2_{\tilde{\mathbf{x}}}$, where $\rho_{v0} = (1/L^2)\int d^2x\rho(\vec{x}, t = 0)$ is the initial total vortex density and L is the linear system size. Note that D, K, ρ_{v0}, and L are independent parameters in our simulation. Then, Eq. (2) can be reformulated as

$$\begin{aligned} &\partial_{\tilde{t}}\tilde{\rho} - \partial^2_{\tilde{\mathbf{x}}}\tilde{\rho} = -(\tilde{\rho}^2 - \tilde{m}^2), \\ &\partial_{\tilde{t}}\tilde{m} - \partial^2_{\tilde{\mathbf{x}}}\tilde{m} = 0. \end{aligned} \tag{3}$$

Given the generic dimensionless differential equation (3), the normalized vortex lifetime $\tilde{\tau}$ obeys a scaling form

$$\tilde{\tau} = f\left(\frac{\xi}{l_D}, \frac{a_v}{l_D}, \frac{L}{l_D}\right)[\rho_{v0}K]^{-1}, \tag{4}$$

where ξ is the vortex correlation length, $l_D = \sqrt{(D/\rho_{v0}K)}$ is the diffusion length, a characteristic length scale representing the relative strength of diffusion, and $a_v = L/\sqrt{N}$ is the average initial intervortex distance.

We numerically solve Eq. (3) and average over different initial vortex configurations. As in the inset of Fig. 4, the $1/t$ decay behavior of $\tilde{\rho}$ is well captured by our model. From the fit, we obtain the normalized decay time $\tilde{\tau}$ [in units of $1/(\rho_{v0}K)$] as a function of l_D (Fig. 4). It can be seen that $\tilde{\tau}$ has a sharp dependence on l_D when the diffusion is slow—$l_D/L \ll 1$. In contrast, when the diffusion is fast ($l_D/L \to 1$), $\tilde{\tau}$ saturates to a constant, yielding an inverse relation between τ and K.

The results in Fig. 4 can be understood by an intuitive physical picture. Given a pair of randomly placed vortex and antivortex, they will first undergo a random diffusion process due to thermal fluctuations, and, when being sufficiently close to each other, they form a quasibound state, survive for a while, and then disappear via a two-body collision. The decay time is effectively the sum of the times taken by both processes. Thus, if the diffusion is fast, the vortex lifetime is mostly determined by the stability of the quasibound state and is mainly from the annihilation process. In contrast, if the random motion of vortices is slow, a significantly long time might be taken before vortices are brought close to each other.

From these observations, we argue that the different temperature dependences of the vortex lifetime τ in Fig. 2 can be explained as follows. In the BEC regime, almost all the fermions are paired into bosonic molecules, while at unitarity or in the BCS regime, there is a substantial fraction of unpaired fermions. In a very recent study, as the system is tuned from BEC to the BCS regime, the sharp increase of vortex diffusion has been observed, due to the rapid increase of mutual collisions between vortices and unpaired fermions [13]. This suggests that, in the BCS regime, the diffusion process is relatively fast, and thus, the relatively slow annihilation process contributes mostly to the vortex lifetime. As T is lowered, the annihilation rate K is enhanced since the superfluid fraction increases [35], leading to a shorter lifetime. In the BEC regime, the vortex lifetime is from the relatively slow diffusion process and thus becomes longer as T decreases.

In conclusion, we carry out a systematical experimental study on the temperature-dependent decay dynamics of quasi-2D spontaneous vortices in the BCS-BEC crossover. In the BEC regime, the vortex lifetime decreases with the increase of system temperature. In contrast, at unitarity and in the BCS regime, the vortex lifetime increases with the increase of system temperature. Based on a phenomenological diffusion-annihilation model and the existing vortex diffusion experiment [13], we propose a qualitative explanation for the counterintuitive experimental observations. Our experiment is an important step in the study of intriguing vortex dynamics in the BCS-BEC crossover. Further theoretical investigation, e.g., with the density functional theory [36], and experimental studies, e.g., in a 2D uniform trap [37–39], may bring us a quantitative understanding of the quantum vortex turbulence [1,2] in the atomic Fermi gas.

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[*]These authors contributed equally to this work.

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