Singularity Problem for Interacting Massive Vectors

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Interacting massive spin-1 fields have been widely used in cosmology and particle physics. We obtain a new condition on the validity of the classical limit of these theories related to the nontrivial constraints that exist for vector field components. A violation of this consistency condition causes a singularity in the time derivative of the auxiliary component and could impact, for example, the field's cosmic history and superradiance around black holes. We show that gauge-invariant interactions are generally safe from this problem, even though the mass term explicitly breaks the gauge symmetry. Such restrictions for interactions are expected to exist generically in many other nontrivially constrained systems.

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Introduction.—The lack of direct evidence for weakly interacting massive particles has driven people to explore different dark matter candidates, among which light massive spin-1 (Proca) fields, the so-called "dark photons" or vector dark matter, have been drawing more attention in recent years [1–4]. In general Proca fields could have a variety of nongravitational interactions and thus very rich dynamics. For example, the coupling to an axion field may allow for a significant energy transfer from axions to dark photons, and make the latter the dominant component of dark matter in the present-day Universe [5,6]. If the Proca field possesses a nonlinear self-interacting potential or a nonminimal coupling to gravity, it may drive the cosmic inflation in the early Universe [7,8], and support coherently oscillating localized solitonic field configurations called vector oscillons [9]. The existence of strong self-interactions would also weaken the superradiance bounds on ultralight vectors [10–14]. Moreover, the theory of vector Galileons, whose effective action contains self-interactions with higher-order derivatives, has been constructed by requiring that the equation of motion has second-order time derivatives and yields three healthy propagating degrees of freedom [15,16]. As an IR modification of gravity, it has been shown that these self-interacting Proca fields can lead to a viable cosmic expansion history and even alleviate the Hubble tension without sabotaging the success of general relativity on scales of the Solar System [17–19].

Given these manifold applications, it is necessary to examine the consistency of the interacting Proca theory carefully. One guiding principle that often comes into play is the validity of an effective description of the interaction, which may arise from a low-energy approximation of coupling to other fields or nonminimal coupling to gravity. Another standard lore is that theories with ghosts or energies unbounded from below are usually unstable and problematic [20–22] and the initial conditions must be restricted in "islands of stability" if possible [23,24], although there may be some exceptions [25]. Regarding massive vectors, it is pointed out that if they are nonminimally coupled to gravity, the longitudinal mode may exhibit ghost instabilities and one cannot discuss the vector field dynamics in a healthy way [26,27]. In practice, one performs as many sanity checks as possible to determine the scope of application for the theory in hand.

In this Letter, we will discuss another type of bound that can arise in the classical limit of the theory by demanding the absence of a singularity problem for $\dot{A}_0$, due to the fact that the interacting Proca theory is a nontrivially "constrained" system, where the auxiliary component A_0 cannot always be uniquely solved in terms of the canonical fields. A similar type of constraint is also expected for interacting massive spin-2 fields.

In what follows, we will first clarify three consistency conditions and introduce the singularity problem by taking real-valued self-interacting vectors as an example; then, a specific model is carried out in detail both analytically and numerically. Finally, we discuss the implication of our results and illustrate that the singularity problem can also exist for complex fields and general types of interactions. We adopt the mostly plus convention for the metric.

The singularity problem.—For definiteness, let us consider a real-valued massive vector field $A_\mu = (A_0, \boldsymbol{A})$ with the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A_\mu A^\mu), \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and there is no gauge invariance thanks to the potential V, which includes a mass term along with self-interactions. A concrete example is the Abelian-Higgs model, where a quartic self-interaction is induced by Higgs exchange in the low-energy limit. By varying the action $S = \int d^4x\mathcal{L}$ with respect to the field A_ν, we find the Euler-Lagrange equation $\partial_\mu F^{\mu\nu} - 2V'(A_\mu A^\mu)A^\nu = 0$. In vector notation, it becomes

$$\nabla\cdot\mathbf{\Pi} + 2V'A_0 = 0, \tag{2}$$

$$\dot{\mathbf{\Pi}} + \nabla\times\nabla\times\boldsymbol{A} + 2V'\boldsymbol{A} = 0, \tag{3}$$

where the prime denotes the derivative of the potential in terms of $A_\mu A^\mu$ and we have defined the conjugate field $\Pi_\mu \equiv \partial\mathcal{L}/\partial\dot{A}^\mu = F_{0\mu}$, so

$$\dot{\boldsymbol{A}} = \mathbf{\Pi} + \nabla A_0. \tag{4}$$

One more useful equation can be obtained by noting that $F^{\mu\nu}$ is antisymmetric, so that $\partial_\mu(V'A^\mu) = 0$, that is,

$$-(V' - 2V''A_0^2)\dot{A}_0 - 2A_0V''(\boldsymbol{A}\cdot\dot{\boldsymbol{A}}) + \nabla\cdot(V'\boldsymbol{A}) = 0. \tag{5}$$

In the language of Hamiltonian mechanics, $\Pi_0 = 0$ is a primary constraint, Eq. (2) is a secondary constraint obtained by requiring $\dot{\Pi}_0 = \delta H/\delta A^0 = 0$, and Eq. (5) is a tertiary constraint obtained by requiring the secondary constraint to be preserved in time [28]. The foundation for these derivations is the stationary action principle, in which we have implicitly assumed that the field A_μ is continuous; otherwise, the infinitesimal variation δA_μ is ill-defined. By applying the above formalism, therefore, we require a "consistent" classical system to satisfy at least three conditions everywhere: (i) the field $A_\mu(t,\boldsymbol{x})$ is real-valued; (ii) the field $A_\mu(t,\boldsymbol{x})$ is continuous; and (iii) the second-class constraints, e.g., Eqs. (2) and (5), are respected.

These conditions are not trivial, and we may gain some insights about them by using Eqs. (3)–(5) and numerically evolving $\mathbf{\Pi}, \boldsymbol{A}$, and A_0. Given appropriate initial conditions, suppose that $V' - 2V''A_0^2$ never becomes 0, then the infinitesimal variations $\delta\mathbf{\Pi}$, $\delta\boldsymbol{A}$, and δA_0 are always well-defined in a infinitesimal time interval δt, and the field A_μ will remain smooth and unique all the time. This is indeed the case for free massive fields, where $V(A_\mu A^\mu) = m^2A_\mu A^\mu/2$. For theories with self-interactions, however, a singularity is encountered if $V' - 2V''A_0^2$ becomes 0 at some spacetime point unless $-2A_0V''(\boldsymbol{A}\cdot\dot{\boldsymbol{A}}) + \nabla\cdot(V'\boldsymbol{A})$ also vanishes in an appropriate way to ensure a finite $\dot{A}_0$, which otherwise causes a discontinuity in A_0 and violates at least one of the consistency conditions. Thus, maintaining the continuity of A_0 at this point needs an overconstraint and requires fine-tuning of initial conditions. It is seen that any plausible interacting Proca theories should ensure that such a problem is avoided in its validity, to wit, the field value should never cross the boundary in field space $\{|A_0|, |\boldsymbol{A}|\}$ specified by

$$V' - 2V''A_0^2 = 0. \tag{6}$$

One may think that the problem identified here can be easily avoided if we use Eq. (2) instead of Eq. (5) to obtain A_0. As will be shown shortly, this difficulty is actually independent of whether and how we evolve the system numerically.

A concrete model.—In order to understand the significance of this singularity bound, it is illuminating to consider the simplest possibility of a self-interaction:

$$V(A_\mu A^\mu) = \frac{m^2}{2}A_\mu A^\mu + \frac{\lambda}{4}(A_\mu A^\mu)^2, \tag{7}$$

where A_0 can be solved in closed form. We are going to show that if we stick with conditions (i) and (iii), then condition (ii) will necessarily be violated if the field system hits the boundary [Eq. (6)] during its evolution.

The secondary constraint [Eq. (2)] in this case becomes

$$A_0^3 + c_1A_0 + c_2 = 0, \tag{8}$$

where $c_1 = -m^2/\lambda - \boldsymbol{A}^2$ and $c_2 = -\nabla\cdot\mathbf{\Pi}/\lambda$. The general solution of this cubic equation can be given by the Cardano's formula,

$$A_0^{(1)} = u + v, \tag{9}$$

$$A_0^{(2)} = \frac{-1 + i\sqrt{3}}{2}u + \frac{-1 - i\sqrt{3}}{2}v, \tag{10}$$

$$A_0^{(3)} = \frac{-1 - i\sqrt{3}}{2}u + \frac{-1 + i\sqrt{3}}{2}v, \tag{11}$$

where $u = \sqrt[3]{-c_2/2 + \sqrt{\Delta}}$ and $v = \sqrt[3]{-c_2/2 - \sqrt{\Delta}}$. The $A_0^{(1)}$ is a real root and the other two are complex conjugate if $\Delta > 0$. All three are real roots with $A_0^{(2)}$ and $A_0^{(3)}$ being the same if $\Delta = 0$. And all three are different real roots if $\Delta < 0$ [29]. Here, the discriminant is defined as

$$\Delta \equiv \left(\frac{c_2}{2}\right)^2 + \left(\frac{c_1}{3}\right)^3 = \left(\frac{\nabla\cdot\mathbf{\Pi}}{2\lambda}\right)^2 - \left(\frac{m^2}{3\lambda} + \frac{\boldsymbol{A}^2}{3}\right)^3. \tag{12}$$

The value of Δ in terms of $\nabla\cdot\mathbf{\Pi}$ and $\boldsymbol{A}$ is shown in Fig. 1.

A few subtleties need to be clarified when we apply the Cardano's formula [Eqs. (9)–(11)]. First, we have defined the square root of any number by its principal value. Second, we have defined the cube root of any number by its principal value when $c_2 \leq 0$ and its antiprincipal value

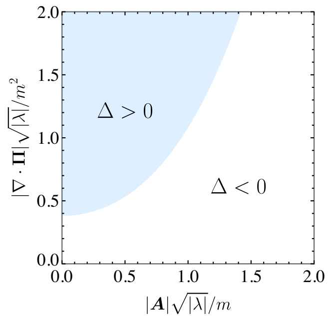

FIG. 1. The value of the discriminant Δ in terms of $\nabla \cdot \mathbf{\Pi}$ and $\boldsymbol{A}$, defined by Eq. (12), for repulsive ($\lambda > 0$, left) and attractive ($\lambda < 0$, right) self-interactions. There are one, two, or three different real roots of A_0 in Eq. (8) when Δ is $>$, $=$ or < 0.

when $c_2 > 0$. The (anti)principal cube root returns the real cube root for a real number, and the root with the (smallest) greatest real part for a complex number. These conventions are adopted such that $A_0^{(1)}$ is always real and all three roots are continuous everywhere except at $\nabla \cdot \mathbf{\Pi} = 0$, where A_0 can actually remain continuous by switching roots.

If a field system crosses the boundary $\Delta = 0$ (and $\nabla \cdot \mathbf{\Pi} \neq 0$) during its evolution, then the real roots $A_0^{(2,3)}$ will be annihilated or created depending on which region in Fig. 1 the system is in before the crossing. It is easy to see that the discontinuity of A_0 is an inevitable consequence if A_0 follows either $A_0^{(2)}$ or $A_0^{(3)}$, and if the system hits $\Delta = 0$ from the white region where $\Delta < 0$.

Now we will show that A_0 cannot remain continuous if the system hits the boundary specified by Eq. (6). To do this, we can judiciously rewrite the discriminant Δ in terms of $|A_0|$ and $|\boldsymbol{A}|$ by using the secondary constraint [Eq. (2)]. The value of Δ in terms of $|A_0|$ and $|\boldsymbol{A}|$ is shown in Fig. 2. At $\Delta = 0$, the three roots [Eqs. (9)–(11)] become $|A_0^{(1)}| = 2A_{0,\mathrm{crit}}$, $|A_0^{(2,3)}| = A_{0,\mathrm{crit}}$, where [30]

$$A_{0,\mathrm{crit}}(\boldsymbol{A}) = \sqrt{\frac{m^2 + \lambda \boldsymbol{A}^2}{3\lambda}}, \tag{13}$$

and $A_0 = 2A_{0,\mathrm{crit}}$ and $A_0 = A_{0,\mathrm{crit}}$ are visualized as the gray dashed and solid black curves respectively in Fig. 2. Note that only $A_0 = A_{0,\mathrm{crit}}$ (solid black line) corresponds to the boundary $V' - 2V'' A_0^2 = 0$, and the adjacent regions separated by this line both have $\Delta < 0$, which justifies the foregoing claim.

On the other hand, it is always safe to cross the gray dashed line, since in this case A_0 follows the root $A_0^{(1)}$ and $A_0^{(1)}$ is real and continuous. But there is no guarantee that the evolution will be healthy if the root $A_0^{(1)}$ is chosen for A_0 initially, because A_0 switches roots at $\nabla \cdot \mathbf{\Pi} = 0$. In order to avoid the singularity problem and also allowing field values to be small, we conclude that the field evolution should be restricted in the nonmeshed region in Fig. 2.

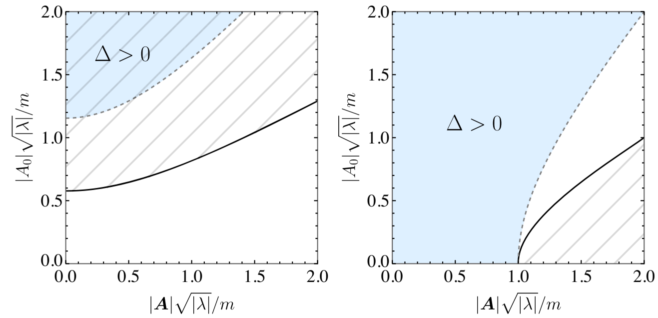

FIG. 2. The value of the discriminant Δ in terms of A_0 and $\boldsymbol{A}$ for repulsive ($\lambda > 0$, left) and attractive ($\lambda < 0$, right) self-interactions. The colored and white regions correspond to $\Delta > 0$ and $\Delta < 0$ as in Fig. 1, and the gray dashed and black solid curves represent $A_0 = 2A_{0,\mathrm{crit}}$ and $A_0 = A_{0,\mathrm{crit}}$ at which $\Delta = 0$. A consistent classical system should never cross the black solid curve, which is exactly the one specified by Eq. (6) (see the texts for proof). Allowing field values to be small, the system during the evolution should never enter into the meshed region.

A minimal model of Eq. (7) is carried out numerically in $1+1$-dimensional spacetime to support the above analysis. We present field-space trajectories for repulsive self-interactions in Fig. 3. The case of attractive self-interactions is similar, and thus only shown in the Supplemental Material [31], Sec. I, where numerical details are also provided.

Up to this point, it looks like that the "discontinuity problem" is a more appropriate name inasmuch as the temporal component A_0 cannot be continuous when the field system hits the boundary specified by Eq. (6). In fact, the discontinuity is just an artificial phenomenon because in principle we can stick with the conditions (i) and

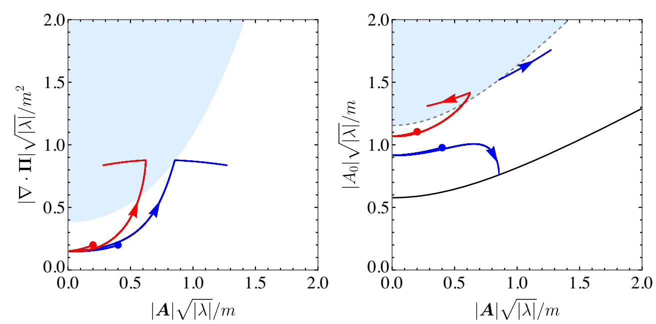

FIG. 3. Field-space trajectories of a numerical example for repulsive self-interactions, where the system crosses the black solid boundary specified by Eq. (6). The colored and white regions, and the gray dashed and black solid curves have the same meaning as in Figs. 1 and 2. The blue and red trajectories represent the time evolution of fields at two adjacent spatial locations starting from the solid point. As shown by the blue trajectory, when the system meets the black solid boundary, the value of A_0 can no longer remain continuous and suddenly jumps to the gray dashed line, which violates the consistency conditions. Numerical details and an example for attractive self-interactions are provided in the Supplemental Material [31], Sec. I.

(ii) instead, and then we will reach a conclusion that the second-class constraints cannot be obeyed. The real problem is that at least one of the consistency conditions will be violated if the boundary [Eq. (6)] is hit, which is closely related to a singularity in $\dot{A}_0$.

Conclusions and discussions.—We have demonstrated that there exists a generic constraint on field values for interacting Proca theory. Such a constraint can be obtained by observing that a classical massive spin-1 field $A_\mu(t,\boldsymbol{x})$ should be both real-valued and continuous, and the second-class constraints of the system be preserved everywhere. Ensuring these conditions is crucial to get realistic results in some contexts, where nongravitationally interacting classical fields may play a pivotal role, such as density perturbations in dark photon production [6], vector oscillons [9,32] and black hole superradiance of vectors [10–14]. Taking nonderivative self-interactions as a concrete example, we have shown that the field system during its evolution should never meet the singularity bound specified by Eq. (6), otherwise the effective description becomes inconsistent.

Loosely speaking, trajectories in phase space (if we could ever visualize them for partial differential equations) would intersect at the singularity bound [Eq. (6)], indicating that A_0 can no longer be solved uniquely. This situation is usually avoided in physics equations because of the Picard-Lindelof theorem (also called the existence and uniqueness theorem), which states that the existence and uniqueness of solutions are guaranteed if the derivative of the variable is continuously differentiable. To fully appreciate this point, in Supplemental Material [31], Sec. II, we provide a toy model with second-class constraints where the phase portrait can be shown explicitly. We also note there the existence of a basin of attraction toward the singularity bound. If the same goes for interacting massive vectors, the allowed field space is further restricted.

Our procedure to obtain the singularity bound is systematic, and also works for more general interactions (such as derivative interactions and interactions with external fields)—finding the tertiary constraint in the theory and then picking out the coefficient of the time derivative of the auxiliary component. (This procedure may even work for spin-2 fields, and we leave this investigation for future work.) Following the procedure, we can also find that the singularity bound equally exists for complex fields. To see this, we may separate the real and imaginary part of a complex field $A_\mu = R_\mu + iI_\mu$, then the theory of A_μ becomes a theory of two interacting real fields R_μ and I_μ, which are both constrained by demanding the absence of the singularity for $\dot{R}_0$ and $\dot{I}_0$.

We note that the singularity problem can be avoided by gauge-invariant interactions, e.g., those only involving $F_{\mu\nu}$, although this may not be the only solution. This is because the gauge-invariant part in action must satisfy $\partial_\mu(\delta S_{\mathrm{GI}}/\delta A_\mu) = 0$, while we have $\partial_\mu(\delta S/\delta A_\mu) = 0$ if the equation of motion is satisfied [35]. Thus, the tertiary constraint like Eq. (5) can be obtained solely from gauge-symmetry-breaking terms. This is an example where gauge invariance plays a role even in theories without gauge invariance.

The existence of the singularity problem in a theory indicates that the theory cannot be the complete story. Learning from perturbative unitarity [37–39], the standard solution would be to introduce new particles or to look for a UV completion above the scale where the singularity bound is met. For example, we can introduce a Higgs boson to rescue the quartic theory [Eq. (7)]. We leave such considerations for future work.

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Note added.—In recent Letters [13,40], the authors also consider self-interactions that are described by Eq. (1) and interpret the singularity problem discussed here as a ghost instability or a loss of hyperbolicity by rewriting the field equations for A_μ in the form of wave equations (up to some terms with derivatives) and by identifying an effective metric $\hat{g}_{\mu\nu}$. The condition $\hat{g}_{00} = 0$ turns out to be the same as Eq. (6). We acknowledge that these works are complementary to this Letter and may provide some physical intuition of the singularity problem for self-interacting massive vectors. There are some caveats, though: (i) The vector field would become ghosts by diverging, at which point the entire theory actually breaks down (so the field would not get a chance to acquire kinetic terms with a wrong sign). (ii) The loss of hyperbolicity is often dangerous but not always fatal for physical systems (see Refs. [41–43] for examples).

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[30] We are only interested in the case where $A_{0,\mathrm{crit}}$ is real.
[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.151101 for numerical details and an example for attractive self-interactions.
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