Test of Genuine Multipartite Nonlocality

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While Bell nonlocality of a bipartite system is counterintuitive, multipartite nonlocality in our many-body world turns out to be even more so. Recent theoretical study reveals in a theory-agnostic manner that genuine multipartite nonlocal correlations cannot be explained by any causal theory involving fewer-partite nonclassical resources and global shared randomness. Here, we provide a Bell-type inequality as a test for genuine multipartite nonlocality in network by exploiting a matrix representation of the causal structure of a multipartite system. We further present experimental demonstrations that both four-photon GHZ state and generalized four-photon GHZ state significantly violate the inequality, i.e., the observed four-partite correlations resist explanations involving three-way nonlocal resources subject to local operations and common shared randomness, hence confirming that nature is boundless multipartite nonlocal.

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Introduction.—Nature allows nonlocal correlations between spacelike separated parties that cannot be explained by classical causal models. Nonlocality has been firmly established via the violation of the celebrated Bell inequality [1–3] in a number of experiments with bipartite quantum systems [4–10] and has led to critical applications in quantum information science such as quantum teleportation [11], quantum key distribution [12–14], and quantum randomness [15–17]. Going beyond, understanding nonlocality of a system with three or more parties is an intriguing question, which may potentially impact a broad range of applications such as multipartite cryptography [18], quantum computation [19–21], correlating particles that never interacted [22,23], many-body physics [24–27], and quantum networks that have advanced significantly in the past few years [28–46], besides deepening our understanding of nonlocality.

Multipartite systems have much richer correlation structures compared to bipartite systems. According to Svetlichny's proposal of genuine multipartite nonlocality [47] restricted by no-signaling conditions [48,49], it is possible to construct genuine multipartite correlations with bipartite resources [50]. Actually, Svetlichny's original proposal provided a device-independent witness of genuine multipartite entanglement [51,52], in which he adopted the framework of local operation and classical communications. However, for spacelike separated parties in multipartite Bell scenarios, classical communications are not at their disposal, which enforces no-signaling condition. Furthermore, it is a realistic scenario that all parties may have global access to common shared randomness. Hence, it is of particular interest to ask whether there are multipartite nonlocal correlations that cannot be explained with bipartite and any other fewer-partite nonlocal resources that are subject to local operations (without classical communications) and shared randomness (LOSR) [53–57]. This question led to the latest theoretical advances of genuine LOSR network multipartite entanglement [58] and genuine LOSR network multipartite nonlocality [51,52,59,60].

In [51,52], Coiteux-Roy, Wolfe, and Renou defined genuine LOSR multipartite nonlocality, referred to as genuine multipartite nonlocality in network here, to be those correlations that cannot be simulated with local composition of any k-partite (with $k \in \{1, 2, \ldots, N-1\}$) resources with access to common shared randomness for a network with N parties. From the theory-agnostic perspective, they considered any plausible causal theory, including classical theory, quantum theory, and hypothetical generalized probabilistic theory, that is compatible with device replication. Exploiting the inflation techniques [61–63], they designed a device-independent Bell-type inequality for genuine LOSR multipartite nonlocality [51,52] and proved that N-partite GHZ state can violate their inequality and thus is genuine LOSR multipartite nonlocal for any finite N. This line of research promotes our understanding of nonlocality by revealing that nature is boundlessly nonlocal and in the meantime showcases the usefulness of inflation technique.

In this Letter, we first propose an algebraic approach to inflation technique via matrix representation of the causal structure, i.e., party-resource relations of a general network with N parties. This enables the construction of a new test

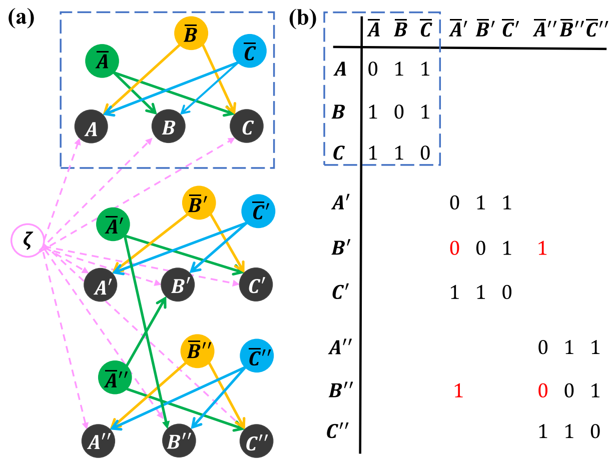

FIG. 1. A triangular network with parties $\mathcal{V} = \{A, B, C\}$ and resources $\mathcal{R} = \{\bar{A}, \bar{B}, \bar{C}\}$ (enclosed by dashed line) and its non-fan-out inflation of order 3 (entire) in (a) with their respective matrix representations Γ (enclosed by dashed line) and Γ' in (b), where blank entries represent zero. All parties have access to common shared randomness (ζ).

of genuine LOSR multipartite nonlocality in terms of Bell-type inequality, in which each party performs two alternative dichotomic measurements. This test outperforms the one proposed in [51] with an improved noise threshold that attains the optimal value found via linear programming in [52] for tripartite GHZ state. Furthermore, we experimentally demonstrate that four-photon and three-photon GHZ states and the respective generalized GHZ states violate the inequalities. Finally, we conclude with a discussion that a large family of quantum pure states can violate the Bell-type inequality besides quantum GHZ state and W state [51,52].

Test in triangular network.—In the framework of LOSR, we consider first a network of three parties, labeled with $\mathcal{V} = \{A, B, C\}$, with global access to common shared randomness ζ as shown in Fig. 1(a). Each pair of parties share a two-way resource, namely, $\omega_{AB} \equiv \bar{C}$, $\omega_{BC} \equiv \bar{A}$, and $\omega_{CA} \equiv \bar{B}$, which can be very general, such as classical, quantum, or no-signaling. As shown in Fig. 1(b), the network can be faithfully represented by a 3×3 incidence matrix Γ with elements $\Gamma_{ij} = 1 - \delta_{ij}$ for $i, j = 0, 1, 2$. A matrix element $\Gamma_{ij} = 1(0)$ indicates that the ith party in the ordered set $\{A, B, C\}$ is (not) sharing the jth resource in the ordered set $\mathcal{R} = \{\bar{A}, \bar{B}, \bar{C}\}$. Following Ref. [51,52], based on the assumption of device replication and causality, a non-fan-out inflation of the triangular network of order 3 is a network of nine parties $\{\mathcal{V}, \mathcal{V}', \mathcal{V}''\}$ connected by resources $\{\mathcal{R}, \mathcal{R}', \mathcal{R}''\}$. Its faithful incidence matrix representation Γ' is presented in Fig. 1(b), in which each row and column have exactly $N-1$ nonzero entries, one for each type of resources and parties, respectively.

In the triangular network, each party performs two alternative dichotomic measurements $\mathcal{V}_{\boldsymbol{x}} = \{A_x, B_y, C_z\}$ (respectively with random and private inputs x, y, $z = 0, 1$) with outcomes $\boldsymbol{a} = \{a, b, c\} \in \{-1, 1\}$, giving rise to a set of correlations $P(\boldsymbol{a}|\mathcal{V}_{\boldsymbol{x}})$. In the inflated network, the measurements for the same type of parties are identical, e.g., measurements A'_x and A''_x performed by parties A', A'' are the same as A_x, so do other parties, respectively. As a result, we obtain an inflated correlation $\mathcal{Q}_3(\boldsymbol{a}\boldsymbol{a}'\boldsymbol{a}''|\mathcal{V}_{\boldsymbol{x}}\mathcal{V}'_{\boldsymbol{x}'}\mathcal{V}''_{\boldsymbol{x}''})$ satisfying a set of compatibility rules. First, it is no-signaling for all parties. Second, as a consequence of no-signaling and causality, as long as two subnetworks are isomorphic the correlations among the corresponding parties are identical. In an inflated network, two subnetworks are isomorphic if they are isomorphic under the dropping of the primes of the parties and resources [52]. For instance we have $\langle A'B'\rangle_{\mathcal{Q}_3} = \langle AB\rangle_{\mathcal{Q}_3}$, $\langle A'C'\rangle_{\mathcal{Q}_3} = \langle AC\rangle_{\mathcal{Q}_3}$, and $\langle ABC\rangle_{\mathcal{Q}_3} = \langle ABC\rangle_P$. And lastly it is nonnegative, e.g.,

$$\sum_{\substack{\alpha,\beta=\pm 1\\ y=0,1}} \mathcal{Q}_3[(-1)^y\alpha\beta, -\beta, \alpha, \beta|A_1B_yC_1C'_0] \geq 0, \tag{1}$$

from which it follows that

$$2 - \langle A_1(B_0 - B_1)C_1 + (B_0 + B_1)C'_0\rangle_{\mathcal{Q}_3} \geq 0. \tag{2}$$

To proceed we note that

$$\begin{aligned}\langle B_yC'_0\rangle_{\mathcal{Q}_3} &= \langle B'_yC'_0\rangle_{\mathcal{Q}_3} \geq \langle B'_yA'_0\rangle_{\mathcal{Q}_3} + \langle A'_0C'_0\rangle_{\mathcal{Q}_3} - 1\\ &= \langle B_yA_0 + A_0C_0\rangle_P - 1.\end{aligned}$$

Here, the first equality holds because of isomorphism $\{B, C'\}$ and $\{B', C'\}$, both of which do not share a common nonlocal resource and locally they have the same pattern of resources sharing, which is evident in matrix Γ' (see lemma later in the text). The inequality comes from positivity $\sum_{\pm} \mathcal{Q}_3(\mp, \pm, \pm|A'_0B'_yC'_0) \geq 0$. The last equality is due to isomorphisms such as $\{A'B'\}$ with $\{AB\}$ and compatibility. Finally, we obtain the following Bell-type inequality as a test of LOSR genuine three-partite nonlocality in a triangular network:

$$\langle A_0(B_0 + B_1) + A_1(B_0 - B_1)C_1 + 2A_0C_0\rangle_P \leq 4. \tag{3}$$

Some remarks are in order. First, it is straightforward to show that substituting GHZ state $|\mathrm{GHZ}_3\rangle = |000\rangle + |111\rangle/\sqrt{2}$ under local measurements $A_0 = C_0 = Z$, $A_1 = C_1 = X$, and $B_y = (Z + (-1)^yX)/\sqrt{2}$ to the left-hand side of inequality [Eq. (3)] yields $2 + 2\sqrt{2}$, hence violating the inequality. [Note that we use $|0\rangle(|1\rangle)$ to denote photon in horizontal (vertical) polarization state $|H\rangle(|V\rangle)$ and X, Y, Z are Pauli matrices.]

Second, each party performs two alternative dichotomic measurements in our test, in contrast to the original proposals in which one party performs three alternative dichotomic measurements [51,52]. This enables us to find the device-independent maximal violation to the inequality, hence providing a device-independent detection of genuine multipartite nonlocality. It turns out that the maximum is attained at projective measurements performed on a 3-qubit pure state [64]. Thus, for quantum theory (Q), we have (see Supplemental Material [65]) $\langle A_0(B_0+B_1)+A_1(B_0-B_1)C_1+2A_0C_0\rangle_P \overset{Q}{\leq} 2+2\sqrt{2}$. Clearly GHZ state provides the maximal violation. Interestingly the algebraic upper bound of this inequality is 6, which is attained by the extremal Box 8 as documented in Ref. [66].

Third, by symmetry we can obtain other Bell-type inequalities for genuine multipartite nonlocality in network by exchanging some parties, e.g., A and C.

General network.—As isomorphic subnetworks give rise to identical correlations, it is critical to identify isomorphic subnetworks in designing tests for genuine LOSR multipartite nonlocality in network. The following lemma provides a criterion for isomorphism among two-party subnetworks exploiting the matrix representation for inflated network.

Lemma. Consider an inflated network of order k with kN parties $\{N_\mu\}_{\mu=0}^{kN-1}$ specified by $kN\times kN$ incidence matrix Γ. The same type of parties and resources are labeled with indices having the same remainder modular N. Two subnetworks $\{N_\mu, N_\nu\}$ and $\{N_{\mu'}, N_{\nu'}\}$ are isomorphic iff $\vec{\gamma}_{\mu\nu}=\vec{\gamma}_{\mu'\nu'}$, where the N-dimensional vector $\vec{\gamma}_{\mu\nu}$ is defined by components

$$[\vec{\gamma}_{\mu\nu}]_s := \sum_{t\equiv s \ (\mathrm{mod}\ N)} \Gamma_{\mu t}\oplus_2 \Gamma_{\nu t} \tag{4}$$

for $0\leq s\leq N-1$ where $\oplus_2$ denotes addition modular 2.

The proof is straightforward by noting that each component of vector $\vec{\gamma}_{\mu\nu}$ can assume only three possible values: $\{0,1,2\}$. It is zero if and only if they share the corresponding resource and 1 if and only if the corresponding resource is $\bar{N}_\mu$ and $\bar{N}_\nu$ and 2 if and only if the corresponding resources are not shared. For examples, in the inflated network in Fig. 1, we have isomorphic subnetworks $\{B,C'\}\sim\{B',C'\}$ and $\{A,B\}\sim\{A',B'\}$, which is evident given $\vec{\gamma}_{BC'}=\vec{\gamma}_{B'C'}=(2,1,1)$ and $\vec{\gamma}_{AB}=\vec{\gamma}_{A'B'}=(1,1,0)$. Here, we use $\sim$ to denote that the two subnetworks have the same causal structures. Similarly we have $\{A,C\}\sim\{A',C'\}$, while subnetworks $\{B,C\}$ and $\{B',C'\}$ are not isomorphic.

Equipped with this lemma, we can extend the Bell-type inequality for triangular network to a general network with N parties (see Supplemental Material [65]).

Theorem. For a general network with N parties, labeled with $\{A,B,C,D,\ldots,W\}$, with each group of $N-1$ parties sharing a nonlocal resource in addition to global randomness, the correlation produced by two local dichotomic measurements on each party satisfies the following Bell-type inequality:

$$S_N := \langle A_0(B_0+B_1)+2(A_0C_0+C_0D_0+\cdots+V_0W_0) + A_1(B_0-B_1)C_1D_1\ldots W_1\rangle_P \overset{\mathrm{LOSR}}{\leq} 2(N-1), \tag{5}$$

$$\overset{Q}{\leq} 2\sqrt{2}+2(N-2). \tag{6}$$

The LOSR bound is maximally violated by N-qubit GHZ state with a white-noise threshold $\eta_N=[(N-1)/(N-2+\sqrt{2})]$, which improves over previous results [51] (see Supplemental Material [65]). For example, we obtain $\eta_3=0.83$, which is smaller than 0.93 in [51] and coincides with the optimal threshold found via linear programming in [52], hence confirming that our analytical results are optimal. We shall present below experimental verification of our results in the cases of $N=3$, 4.

Experiments.—A schematic of implementing a quantum network distributing four-photon GHZ state to four parties —Alice (A), Bob (B), Charlie (C), and David (D)—is depicted in Fig. 2. We first prepare two EPR sources. We use a pulse pattern generator (PPG) to send out trigger pulses at a rate of 250 MHz. In each source, the trigger pulse signals a distributed feedback (DFB) laser to emit a laser pulse at $\lambda=1558$ nm. We shorten the pulse width from 2 ns to 90 ps with an intensity modulator (IM). After passing through an erbium-doped fiber amplifier (EDFA), a periodically poled MgO-doped lithium niobate (PPLN) waveguide to double the frequency, and a dense wavelength division multiplex (DWDM) filter to remove the residual pump light, we use the produced pulse at $\lambda_p=779$ nm to drive a Type-0 spontaneous parametric downconversion (SPDC) process in a piece of PPLN crystal in a Sagnac interferometer, which emits probabilistically a pair of photons in EPR state $|\Phi^+\rangle=(|HH\rangle+|VV\rangle)/\sqrt{2}$ [67] at the phase-matched wavelength 1556 nm (signal) and 1560 nm (idler). Interfering signal photons from the two EPR sources on a polarizing beam splitter (PBS), we create four-photon GHZ state, $|\mathrm{GHZ}_4\rangle=(|HHHH\rangle+|VVVV\rangle)/\sqrt{2}$, after postselection [68]. We pass photons through fiber Bragg gratings (FBGs) with bandwidths of 3.3 GHz before entering single-photon detectors to suppress the spectral distinguishability between photons from different EPR sources and keep the photon-pair production rate of each EPR source at 0.0025 per trigger to strongly mitigate the multiphoton effect. Quantum tomography measurements indicate that the state fidelity is greater than 0.99 for the two-photon states produced at EPR sources with respect to the ideal Bell state $|\Phi^+\rangle$ and is 0.9740 ± 0.0043 for the produced four-photon state with respect to the ideal state $|\mathrm{GHZ}_4\rangle$, respectively.

We install a quantum random number generator at each party [67,69–75], which privately and randomly feeds a 2-bit random number ($x_\alpha\in\{0,1\}$) to the party to switch

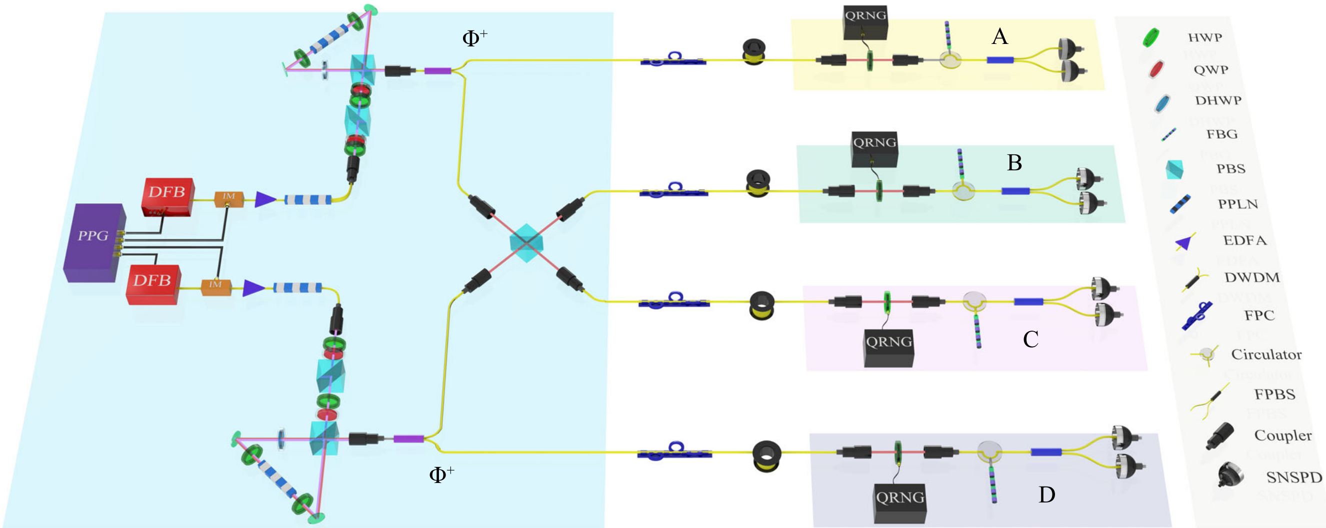

FIG. 2. Schematic of the experiment. We prepare two EPR sources by driving two SPDC processes in parallel (see text for details). In each SPDC process, we inject a laser pulse at 779 nm into a PPLN crystal in a Sagnac interferometer [67], which probabilistically emits a pair of polarization-entangled photons in EPR state $|\Phi^+\rangle$ at paired wavelengths 1556 nm (signal) and 1560 nm (idler). We interfere signal photons from the two sources on a PBS and obtain four-photon GHZ state sharing between parties A, B, C, and D after postselection. A quantum random number generator (QRNG) is used to instruct each party to perform two alternative dichotomic measurements to the photon at his disposal. DHWP, dual-wavelength HWP for 1560 nm and 779 nm; DWDM, dense wavelength division multiplexing; FPBS, fiber PBS; FPC, fiber polarization controller; SNSPD, superconducting nanowire single-photon detector.

between measurement bases Z ($x_\alpha = 0$) and X ($x_\alpha = 1$) for Alice, Charlie, and David, and between $(Z + X)$ ($x_\alpha = 0$) and $(Z - X)$ ($x_\alpha = 1$) for Bob to perform measurement to her or his share of photon, where the Pauli matrices X, Z, and $Z \pm X$ are implemented by a half-wave plate (HWP) at

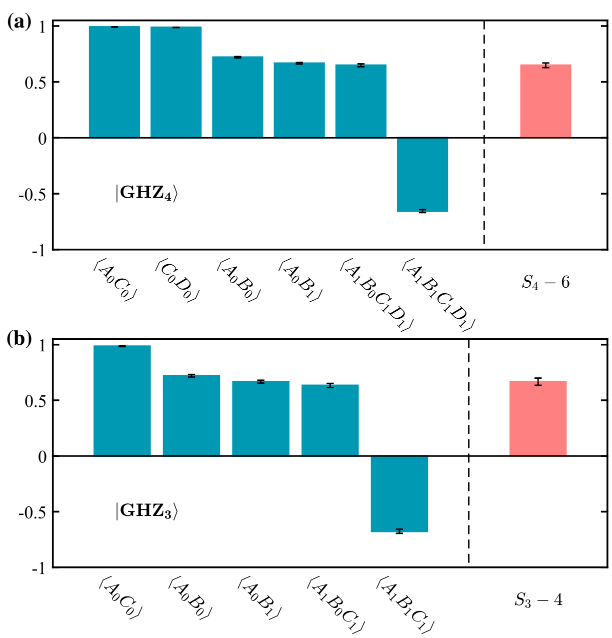

FIG. 3. Experimental measurements of correlation function S_N (left panel) and violations of inequality [Eq. (5)], $S_N - 2(N-1)$, (right panel) with GHZ states $N = 4$ in (a) and $N = 3$ in (b). Error bars represent 1 standard deviation in experiments.

each party, respectively. The generation of random numbers is synchronized to the PPG. We switch the measurement settings every 30 s, reserving the first 10 s to reset the measurement settings, including quantum random number generation and wave plate rotation, and the remaining 20 s for data collection. We collect 33252 four-photon coincidence events over 16 measurement setting combinations in 14741 switching cycles. We compute the correlation function $S_4 = 6.6484 \pm 0.0209$, which surpasses the LOSR bound 6 by more than 30 standard deviations as shown in Fig. 3(a), i.e., the observed correlation cannot be reproduced by involving any three-way nonlocal resources with local operations and unlimited shared randomness. Hence, the observed correlation is genuinely LOSR four-partite nonlocal.

Each of David's successful detections of a photon probabilistically heralds the presence of a three-photon GHZ state shared between Alice, Charlie, and Bob. We show in Fig. 3(b) the correlation function $S_3 = 4.6674 \pm 0.0323$ surpasses the LOSR bound 4 by more than 20 standard deviations, i.e., the observed correlation cannot be reproduced by involving any two-way nonlocal resources with local operations and unlimited shared randomness.

Discussions and conclusions.—Besides GHZ state and W state [51,52], we now show that a large family of pure states can violate inequality [Eq. (5)]. First, it is straightforward to show that the generalized N-qubit GHZ state $|\xi_N\rangle = \sqrt{[(1+\xi)+2]}|0\rangle^{\otimes N} + \sqrt{[(1-\xi)/2]}|1\rangle^{\otimes N}$ violates inequality [Eq. (5)] with the same measurement settings as those for GHZ state, whenever $|\xi| < \xi_c$ with $|\xi| \in [-1, 1]$

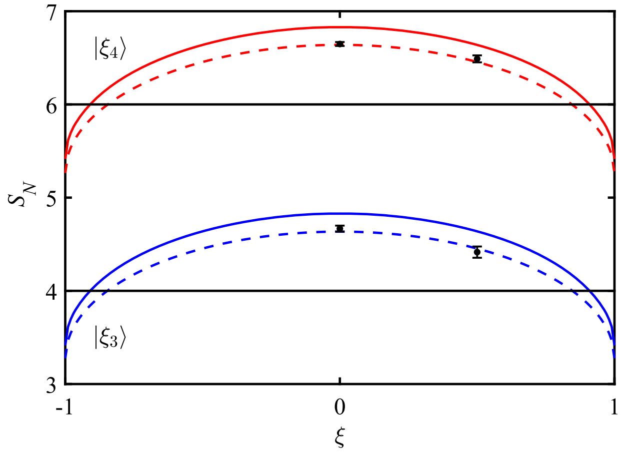

FIG. 4. Correlation function S_N for generalized GHZ states $|\xi_3\rangle$ (blue, bottom) and $|\xi_4\rangle$ (red, upper) with the respective LOSR bounds (horizontal lines). Smooth lines, ideal theory; dashed lines, theory considering white noise (η_N) in the experiment; filled dots, $S_4 = 6.6484 \pm 0.0209$ (6.4890 ± 0.0375) and $S_3 = 4.6674 \pm 0.0323$ (4.4153 ± 0.0603) when $\xi = 0$ (0.5), measured with 1 standard deviation in experiment.

and $\xi_c = 0.91$ independent of N. We present in Fig. 4 experimental demonstrations for $N = 3$ and $N = 4$ with $\xi = 0.5$ along with theoretical predictions. The results uphold a good agreement.

Furthermore, under local unitaries, the most general three-qubit pure state can be cast into the canonical form [76]

$$|\Psi_3\rangle = h_0|000\rangle + h_1 e^{i\phi}|100\rangle + h_2|101\rangle + h_3|110\rangle + h_4|111\rangle$$

with $h_i \geq 0$ and $\sum_i h_i^2 = 1$ and $\phi \in [0, \pi]$. Performing the same measurements to this state as that for GHZ state shall violate inequality [Eq. (3)] if the state satisfies the condition $\sqrt{2}[(h_0+h_4)^2+2h_3^2+h_0^2+h_4^2-1]+4(h_0^2+h_4^2+h_2^2)-2>4$.

It is reasonable for one to anticipate that the study of multipartite nonlocality may be as fruitful as that of Bell nonlocality [3]. Hence, it will be interesting to explore more states and new approaches suitable for the test of genuine LOSR multipartite nonlocality, for example, the Hardy type of nonlocality tests, which have been used to detect genuine multipartite nonlocality in Svetlichny's original definition with no-signaling restrictions [77,78]. The matrix representation of the causal relation of networks introduced in this Letter may provide a convenient tool in these explorations. While our experiment presents a test of the genuine multipartite nonlocality, we note that the measurement events of one party are not spacelike separated from those of other parties and the photon detection efficiencies are low; hence, our experiment has the loopholes of locality and fair sampling assumption [3]. The next task along this line of research may be to conduct an experimental test of genuine multipartite nonlocality without these loopholes, like the loophole-free test of Bell inequality [6–10], and a strategy to do so was just proposed in [79].

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Note added.—While finishing this manuscript, we became aware of a related work by Huan Cao *et al.* [79].

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