Experimental Refutation of Real-Valued Quantum Mechanics under Strict Locality Conditions

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Quantum mechanics is commonly formulated in a complex, rather than real, Hilbert space. However, whether quantum theory really needs the participation of complex numbers has been debated ever since its birth. Recently, a Bell-like test in an entanglement-swapping scenario has been proposed to distinguish standard quantum mechanics from its real-valued analog. Previous experiments have conceptually demonstrated, yet not satisfied, the central requirement of independent state preparation and measurements and leave several loopholes. Here, we implement such a Bell-like test with two separated independent sources delivering entangled photons to three separated parties under strict locality conditions that are enforced by spacelike separation of the relevant events, rapid random setting generation, and fast measurement. With the fair-sampling assumption and closed loopholes of independent source, locality, and measurement independence simultaneously, we violate the constraints of real-valued quantum mechanics by 5.30 standard deviations. Our results disprove the real-valued quantum theory to describe nature and ensure the indispensable role of complex numbers in quantum mechanics.

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Standard quantum mechanics, in its usual formulation, is highly abstract. Its basis elements include state vectors and operators in the complex Hilbert space [1,2], and the Hilbert space of composite quantum systems is in a tensor-product structure [3]. Complex numbers seem to play a fundamental role in quantum mechanics, which is peculiar as they are simply a mathematical trick in classical mechanics. However, many works have shown the possibility to use only real numbers in an enlarged Hilbert space to simulate quantum systems, indicating that complex numbers are not necessary [4–11]. For example, by embedding a complex density matrix ρ and Hermitian observable H into a double-size real Hilbert space using

$$\tilde{\rho} = \frac{1}{2}\begin{pmatrix} \mathrm{Re}(\rho) & -\mathrm{Im}(\rho) \\ \mathrm{Im}(\rho) & \mathrm{Re}(\rho) \end{pmatrix}$$

and

$$\tilde{H} = \frac{1}{2}\begin{pmatrix} \mathrm{Re}(H) & -\mathrm{Im}(H) \\ \mathrm{Im}(H) & \mathrm{Re}(H) \end{pmatrix},$$

a quantum system in a complex-valued Hilbert space can be simulated by its real-valued analog through $\mathrm{tr}(\rho H) = \mathrm{tr}(\tilde{\rho}\tilde{H})$. The debate on the essential role of complex numbers in quantum mechanics has continued. Importantly, no experimental argument has been proposed so far to distinguish standard quantum mechanics from its real-number explanation.

In 1964, John Bell opened a fundamental paradigm to test local-realist theory using an inequality of two-party quantum correlations [12]. Recently, Renou *et al.* successfully extended this paradigm, identifying a scenario in which the experimental prediction of standard quantum mechanics is inconsistent with its real-number analogy [13]. For simplicity, we use complex-valued quantum mechanics (CQM) for standard quantum mechanics that is formulated in complex Hilbert spaces and uses tensor products to model system composition and real-valued quantum mechanics (RQM) for the corresponding real analogy. With causally independent state preparation and measurement in an entanglement-swapping scenario, Renou *et al.* found that the resulting quantum correlations cannot be reproduced by RQM, even if the real Hilbert space can have infinite dimensions [13]. Also, the tensor-product structure further poses a requirement of independent state preparation in the experimental test [13].

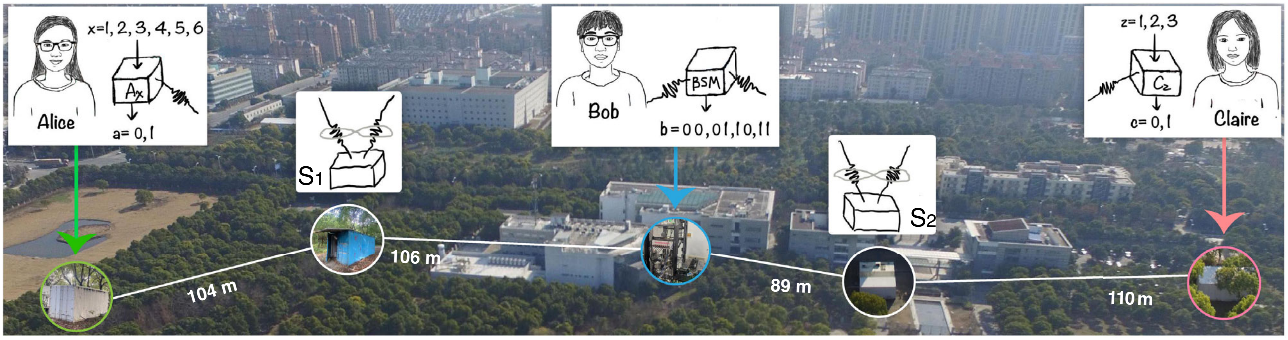

FIG. 1. The scheme to disprove RQM. Two independent sources S_1 and S_2 distribute EPR pairs between Alice and Bob, and Bob and Claire, respectively. Alice and Claire perform measurements (A_x and C_z) on their received photons with random inputs (labeled as x and z) and give measurement outcomes (labeled as a and c). Bob performs a full Bell-state measurement (BSM) on his received two photons, which outputs one of the four measurement results b. The spatial distances between Alice-S_1, S_1-Bob, Bob-S_2, and S_2-Claire are 104, 106, 89, and 110 m, with fiber links of length 112.63, 123.84, 108.75, and 125.48 m, respectively.

Local measurements should also be conducted in a spacelike-separated way to ensure independent measurements.

Experimental violations of Bell-like inequalities are, in general, vulnerable to loopholes. For example, the locality loophole appears if measurement choices and outcomes at one side can influence the outcomes at other sides in time [12], which can be closed by spacelike separating the choice and measurement events [14]. The loophole of measurement independence [15] concerns whether setting choices are independent from any properties of hidden variables, which can be closed only under reasonable assumptions [16]. Moreover, due to unavoidable experimental imperfections, not all the photons can be detected. With the fair-sampling assumption, one can consider that the measured subensemble is representative of the complete ensemble, which opens the detection loophole [17] that can be closed with a sufficiently high detection efficiency [18–20]. Progress has been made to address all possible loopholes in sophisticated experiments, leading to loophole-free Bell tests [21–25].

Recently, the refutation of RQM has been conceptually demonstrated in two different physical platforms, wherein one is a superconducting chip [26] and the other is a photonic system [27]. Note that the causal independence is crucial to disprove RQM [13]; however, no spacelike separation is strictly enforced in these experiments. Also, potential loopholes could be exploited by RQM to replicate the prediction of CQM. Here, we intend to refute RQM in a photonic quantum network where two independent sources deliver photons to three parties under strict locality conditions (in which all parties are spacelike separated and rapid random setting choices are spacelike separated from measurement). With fair-sampling assumptions while closing the loopholes of independent source, locality, and measurement independence simultaneously, we show that RQM is incompatible with our observed data, thus disproving RQM to describe nature.

The Bell-like test proposed by Renou *et al.* is depicted in Fig. 1, involving two independent sources (S_1 and S_2) and three players (Alice, Bob, and Claire). Each source emits an Einstein-Podolsky-Rosen (EPR) entangled state [28] in the form of a Bell state $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Bob performs a full Bell-state measurement (BSM), which randomly outputs four results $b \in \{00, 01, 10, 11\}$ that, respectively, correspond to a projection onto four Bell states $|\Phi^+\rangle$, $|\Psi^+\rangle$, $|\Phi^-\rangle$, and $|\Psi^-\rangle$. Alice and Claire independently perform measurements with random inputs $x \in \{1, 2, 3, 4, 5, 6\}$ and $z \in \{1, 2, 3\}$, respectively. Their measurement outcomes are labeled as $a, c \in \{0, 1\}$. Alice's six measurements are chosen as $A_x \in \{((Z + X)/\sqrt{2}), ((Z - X)/\sqrt{2}), ((Z + Y)/\sqrt{2}), ((Z - Y)/\sqrt{2}), ((X + Y)/\sqrt{2}), ((X - Y)/\sqrt{2})\}$, and Claire's three measurements are chosen as $C_z \in \{Z, X, Y\}$, where Z, X, and Y are Pauli operators. The resulting three-party correlation is defined as the weighted sum of input-output probability distribution $p(abc|xz)$, given by [13]

$$\mathcal{F} = \sum_{abc,xz} \omega_{abc,xz} p(abc|xz), \tag{1}$$

where $w_{abc,xz} = \pm 1$ are the weights, $abc \in \{0, 1\}^{\otimes 4}$ are the bit strings of the measurement results, and $xz \in \{11, 12, 21, 22, 13, 14, 33, 34, 52, 53, 62, 63\}$ are the 12 combinations of measurement settings x and z. $\mathcal{F}$ is upper bounded by 7.66 in RQM, while it can reach a maximum of $6\sqrt{2}(\approx 8.49)$ in CQM, indicating an experimental way to distinguish CQM from RQM.

Our experimental setup is illustrated in Fig. 2. A pulse pattern generator (PPG) in S_2 is used to trigger the PPG in S_1 and provides 250 MHz signals to synchronize all devices (see Refs. [29,30] for more details). Driven by the 250 MHz signal, two distributed feedback (DFB) lasers each emit a 2 ns laser pulse which is further shortened to 80 ps by an intensity modulator (IM). Note that each DFB laser switches on its electric current from much below the threshold to well above the threshold at a rate of 250 MHz, such that the phase of each generated laser pulse is randomized in each source [30]. The laser pulse is

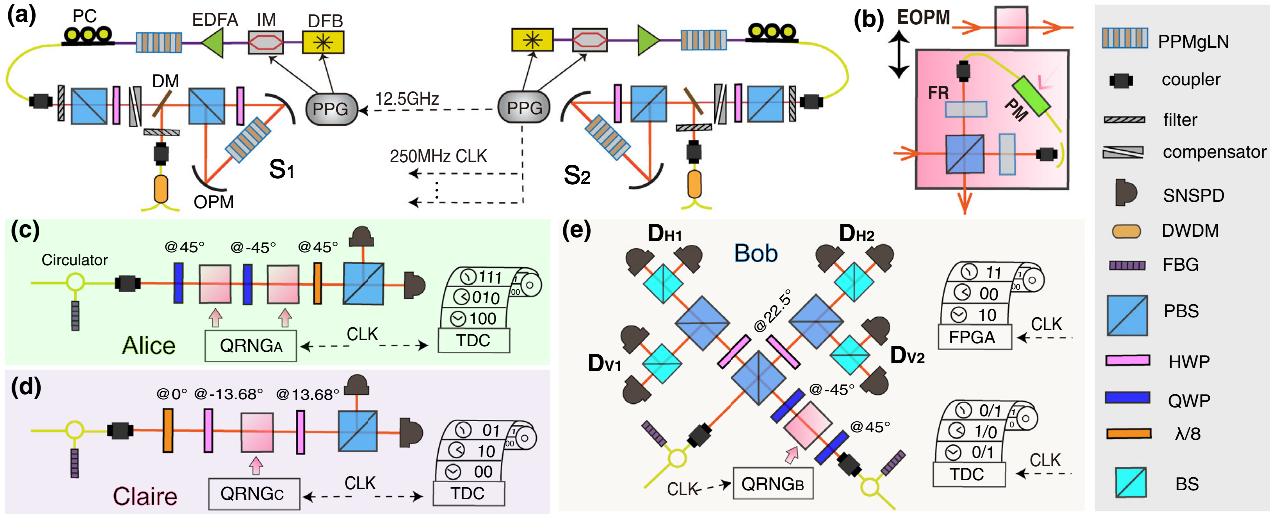

FIG. 2. Experimental setup. (a) In each source, a 1558 nm distributed feedback (DFB) laser is frequency doubled in a periodically poled MgO-doped lithium niobate (PPMgLN) crystal after passing through an erbium-doped fiber amplifier (EDFA), for producing 779 nm pump pulses. The PPMgLN crystal in a polarization-based Sagnac loop is then pumped to create polarization-entangled photon pairs. The pulse pattern generator (PPG) in S_2 offers 250 MHz synchronization signals (CLK, black dashed lines) to all parties and 12.5 GHz signals to trigger the PPG at S_1. (b) An electro-optic polarization modulation (EOPM) consisting of a polarization beam splitter (PBS), two Faraday rotators (FR), and a fiber-coupled electro-optic phase modulator (PM) is implemented for modulating the photon's polarization [29]. (c) Alice performs measurements A_x, which are determined by her quantum random number generator (QRNG$_A$) and a fast polarization modulator (constructed by two EOPMs and fixed wave plates), and records measurement outcomes with a time-to-digital converter (TDC). (d) Similar to (c), Claire performs measurements C_z and records the outcomes with a TDC. (e) A partial BSM analyzer is constructed by three PBSs, two HWPs, four 50:50 beam splitters (BSs), and eight superconducting nanowire single photon detectors (SNSPDs). In front of the first PBS, a polarization modulator is situated there. Bob performs the full BSM of four possible outcomes which are randomly decided by outcomes of QRNG$_B$ and random photon clicks at SNSPDs. The choice of QRNG$_B$ is recorded by a TDC, and the photon detection results are analyzed in real time and recorded by a field-programmable gate array (FPGA). PC, polarization controller; DWDM, dense wavelength-division multiplexer; DM, dichroic mirrors; OPM, off-axis parabolic mirrors; FBG, fiber Bragg grating; QWP, quarter-wave plate; $\lambda/8$, eighth wave plate.

then fed into a PPMgLN crystal for second-harmonic generation to create a 779 nm pump laser. By pumping a PPMgLN crystal in a polarization-based Sagnac loop, EPR state $|\Phi^+\rangle$ is created via the type-0 spontaneous parametric down-conversion (SPDC) process, as shown in Fig. 2(a). We use horizontal and vertical polarization to encode the qubit $|0\rangle$ and $|1\rangle$, respectively. The lack of coherence between the pump pulses and disconnecting the two SPDC processes on each experimental trial ensure the independent state preparations [30].

To vary the direction of local polarization analysis for Alice and Claire, high-speed polarization analyzers are implemented with fixed wave plates, EOPMs [Fig. 2(b)], a PBS, and two SNSPDs, as shown in Figs. 2(c) and 2(d) (for detailed configurations, see Ref. [29]). The setting choices for A_x and C_z are determined by their fast quantum random number generators (QRNGs) [31,39,40] (for details, see Ref. [29]). Note that Alice's and Claire's QRNGs output eight and four random bits, respectively; however, we can discard unnecessary ones to have only six and three choices under fair-sampling assumptions. For Bob, with a standard linear optical Bell-state analyzer, only two of four Bell states can be distinguished [41,42], e.g., a partial BSM for $|\Phi^\pm\rangle$ with a successful rate of 50%. One could also have a partial BSM for $|\Psi^\pm\rangle$. We can, therefore, combine two partial BSMs as a full BSM with an effective efficiency of 50%. The full BSM is implemented in two steps: (i) *premeasure*—i.e., the entangling measurement projects the states to a two-dimensional subspace determined by a fast QRNG; (ii) *measure*—i.e., the subspace is further projected to one unique Bell state in the subspace registered by random photon clicks. With the arrangement in Fig. 2(e), we describe how the two steps work. In premeasure, Bob's QRNG outputs a random bit to trigger the EOPM sitting between two quarter-wave plates (QWPs). This will project the incoming state onto a two-dimensional subspace in the partial BSM either for $|\Phi^\pm\rangle$ or for $|\Psi^\pm\rangle$, depending on Bob's QRNG. In measure, four pseudo-photon-number-resolving detectors (PRNRDs, denoted as D_{H1}, D_{V1}, D_{H2}, and D_{V2}) constructed by a 50:50 beam splitter and two SNSPDs are used to register random photon clicks and additionally remove erroneous coincidences caused by the multiphoton emission from SPDC. Coincidence detection between either D_{H1} and D_{H2} or D_{V1} and D_{V2} indicates that

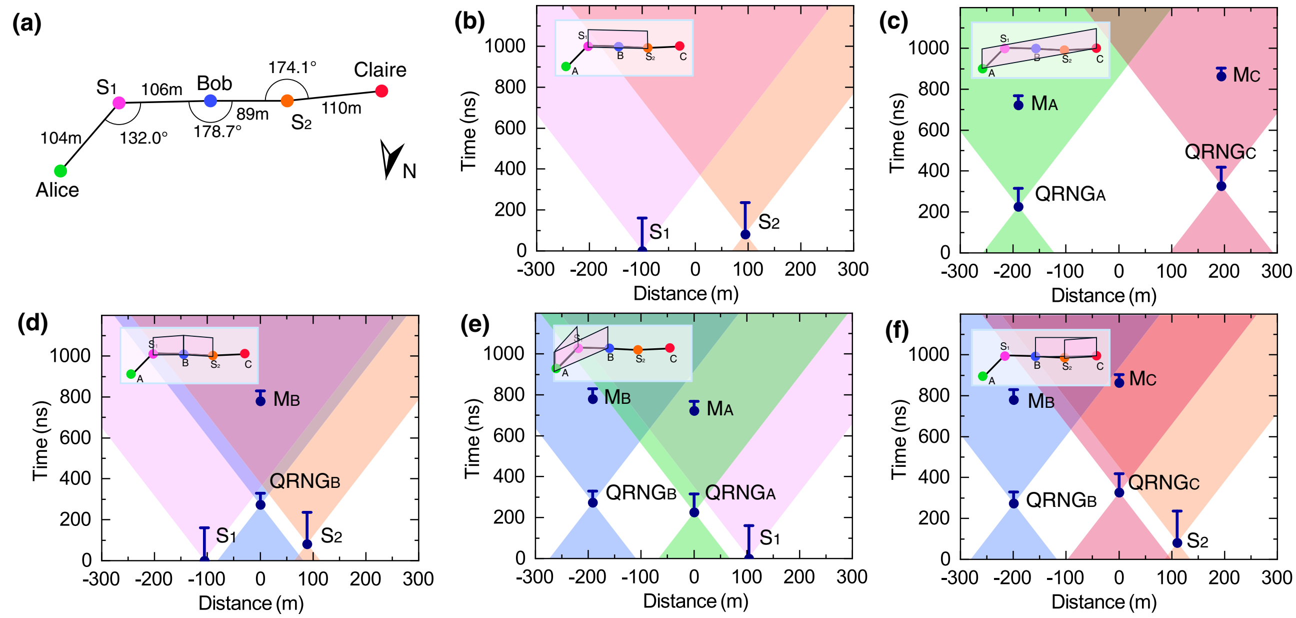

FIG. 3. Space-time configuration of relevant events in our experiment. Origins of the axes are displaced to reflect the relative space and time difference between them. (a) Relative spatial configuration of the two independent sources (S_1 and S_2) and three players (Alice, Bob, and Claire). (b) Spacelike separation between state emission events from sources S_1 and S_2. (c) Spacelike separation between setting choice events QRNG$_A$ and QRNG$_C$ and between setting choice event QRNG$_A$ (QRNG$_C$) and measurement event M_C (M_A). (d) Spacelike separation between setting choice event QRNG$_B$ and state emission events from sources S_1 and S_2. (e) Spacelike separation between setting choice events QRNG$_A$ and QRNG$_B$ and between setting choice event QRNG$_A$ (QRNG$_B$) and measurement event M_B (M_A) shown on the left side of the vertical axis, with the state emission event S_1 on the right side. (f) Similar to (e), QRNG$_C$ is the setting choice event of Claire and the state emission event is S_2. Blue vertical bars indicate the time elapsing for events, with the start and end marked by circles and a horizontal line, respectively. All time-space relations are drawn to scale. Hence, further relations can be inferred, e.g., the spacelike separation between QRNG$_A$-S_2 and QRNG$_C$-S_1 can be implied by (e) and (f) (for details, see Ref. [29]).

the premeasure subspace collapses to a unique Bell state in either $|\Phi^+\rangle$ or $|\Psi^+\rangle$, while coincidence detection between either D_{H1} and D_{V2} or D_{V1} and D_{H2} shows that the subspace collapses to a Bell state in either $|\Phi^-\rangle$ or $|\Psi^-\rangle$. Hence, we have a full BSM that gives four measurement outcomes. In the experiment, we employ a locally predefined coincidence window of 4 ns that is synchronized by the 250 MHz system clock and, thus, not vulnerable to the coincidence-time loophole [43]. All the photon and setting data stored locally in the measurement stations were collected by a separate computer that evaluates the three-party correlation $\mathcal{F}$.

To satisfy the locality and measurement independence constraints, it is essential to spacelike separate each setting choice (labeled as QRNG$_A$, QRNG$_B$, and QRNG$_C$) from the measurement of other stations (denoted as M_A, M_B, and M_C), as well as from the photon emission (labeled as S_1 and S_2). By employing QRNG and spacelike separating setting choice and measurement on one side from the measurement on other sides, we close the locality loophole. Note that we consider a reasonable assumption that hidden variables are created together with the state creations [21–25], as it is impossible to rule out all loopholes [44] due to the fact that they can be correlated at the birth of the Universe. By spacelike separating two creation events and spacelike separating setting choice events from the entanglement creation event, we close loopholes of independent source and measurement independence. We characterize all relevant events and describe in Fig. 3 (for details, see Ref. [29]).

We then perform tomographic measurements for the prepared two independent EPR states and obtain fidelity of 0.9852(6) and 0.9892(9), respectively, at the average photon-pair number per pulse ∼0.01. The Hong-Ou-Mandel measurement with photons from the two independent sources [45] gives a visibility of 0.943(20). To compute the statistical significance of our measured violation, we have collected 77 326 four-photon coincidence detection events in 460 810 s. We then estimated the three-party correlation for each of Bob's outcomes with its related 12 setting combinations and obtain a value of 7.80(6), 7.89(6), 7.78(6), and 7.84(6) for the corresponding states $|\Phi^+\rangle$, $|\Psi^+\rangle$, $|\Phi^-\rangle$, and $|\Psi^-\rangle$, respectively. Summing over all the obtained correlations, we finally get an average result of 7.83(3), which exceeds the real bound of 7.66 over 5.30 standard deviations, as described in Fig. 4. To rigorously rule out the possibility of RQM without assuming any *a priori* statistics behavior, we further perform a hypothesis testing, where we take the null hypothesis that

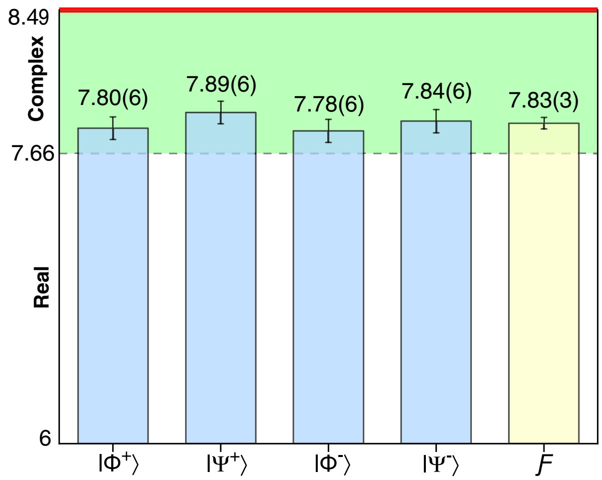

FIG. 4. Experimental results. The upper bounds of $\mathcal{F}$ for classical mechanics, RQM, and CQM are 6, 7.66, and 8.49, respectively. The results for Bob's four measurement outcomes are 7.80(6), 7.89(6), 7.78(6), and 7.84(6). The average result is 7.83(3), which violates the RQM bound by 5.30 standard deviations. Error bars indicate one standard deviation based on Poisson statistics.

RQM governs our experiment. Following a recently developed martingale binomial approach [32–34], we calculate that the P value, or the maximum probability to witness a Bell value at least as large as observed in our experiment assuming the correctness of the null hypothesis, is as small as 8.22×10^{-53} [29]. Our results provide strong evidence to reject RQM.

We have for the first time experimentally refuted RQM under strict locality conditions. By erasing any quantum coherence between the sources and spacelike separating state emission events, we close the independent source loophole. Employing QRNGs, we spacelike separate the setting choices, measurements, and emission events to close the locality and measurement independence loopholes simultaneously. In addition, we close the coincidence-time loophole by using locally defined time slots. Our demonstration requires fair-sampling assumptions and is subject to detection loopholes, which could be closed using high-efficiency photon sources and detectors [18–20]. Our current implementation exploited two partial BSMs determined by a fast QRNG and random photon clicks to certify the presence of four outcomes. It has been suggested that BSM's four outcomes can be reinterpreted as a QRNG's four random setting choices, such that one could use a partial BSM with QRNGs to refute RQM [27], which will be interesting to explore in the future.

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