Passive Quantum Phase Gate for Photons Based on Three Level Emitters

Björn Schrinski, Miren Lamaison⬤, and Anders S. Sørensen⬤
Center for Hybrid Quantum Networks (Hy-Q), The Niels Bohr Institute, University of Copenhagen,
Blegdamsvej 17, 2100 Copenhagen, Denmark

We present a fully passive method for implementing a quantum phase gate between two photons traveling in a one-dimensional waveguide. The gate is based on chirally coupled emitters in a three level V configuration, which only interact through the photon field without any external control fields. We describe the (non)linear scattering of the emerging polariton states and show that for near resonant photons the scattering dynamics directly implements a perfect control phase gate between the incoming photons in the limit of many emitters. For a finite number of emitters we show that the dominant error mechanism can be suppressed by a simple frequency filter at the cost of a minor reduction in the success probability. We verify the results via comparison with exact scattering matrix theory and show that the fidelity can reach values $\mathcal{F} \sim 99\%$ with a gate success probability of $> 99\%$ for as few as eight emitters.

Introduction.—Light has great advantages as a carrier of quantum information since it travels very fast. Furthermore, light is largely invulnerable to decoherence even at room temperature since photons are absorbed rather than decohered. On the other hand, photons practically do not interact with each other [1], which makes it highly challenging to implement quantum gates, the key building blocks for quantum information processing. Alternative strategies have been introduced by combining sources of single or entangled photons with measurement and feedback [2–5]. These techniques are, however, inherently probabilistic and come with a large overhead in resources [6–8]. One way to avoid these complications is to achieve an indirect interaction via coupling to nonlinear matter. There have been several ideas for how to achieve gate operations in such systems: strong coupling of photons to individual systems such as atoms in optical cavities [9] or optomechanical resonators [10] can induce phase gates with relatively high fidelity for small phase shifts but not for large shifts due to pulse distortions. These distortions can be avoided by mapping the photons into excitations of optical cavities [11] or individual atoms, either coupled to wave guides [12] or optical cavities [13,14], and subsequently scattering a second photon of the system. Similarly, photons can be stored in ensembles of atoms either in free space [15–21] or optical cavities [22]. Gates can then be implemented by relying on direct interaction of the stored excitations in optical lattices [15,18] or through scattering of a second photon pulse and exploiting Rydberg interactions [16,19,22] or stationary light effects [17,21]. Common to these proposals, however, is that they rely on precisely timed laser pulses for photon storage, which complicates their implementation.

Photon gates not depending on precise timing of pulses have been developed using counterpropagating pulses in Rydberg electromagnetically induced transparency systems [23] or through static cross-Kerr interactions between emitters coupled to different one-dimensional waveguides [24,25]. Inspired by the latter, we propose an experimentally viable setup for implementing quantum gates based on counterpropagating photons in waveguides chirally coupled to quantum emitters, such that the emitters decay by only emitting light in one direction [26,27]. Such couplings have previously been demonstrated [28–32] and also used for (non)linear operations [33,34]. Here, we exploit the chiral coupling to achieve a simple implementation of a cascaded quantum system [35–37] without the need for multiple Kerr circulators or multiple arrays of chirally coupled emitters as in Ref. [25]. We furthermore avoid any complication of engineering interactions between emitters by basing our setup on three level dipole emitters in the V configuration coupled to a one-dimensional waveguide; see Fig. 1. This configuration contains an inherent Kerr nonlinearity since the emitter cannot be excited twice, and the only required coupling is between the emitters and the photons. In contrast to previous proposals, the assumed setup thus enables the
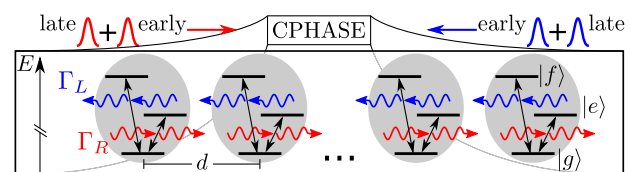

FIG. 1. Photonic phase gate for counterpropagating wave packets in a one dimensional waveguide, coupled to three level dipole emitters with distance d to the next neighbors. The transition $|g\rangle \rightarrow |e\rangle$ $(|g\rangle \rightarrow |f\rangle)$ in the three level systems couples chirally to the right (left) traveling waveguide modes with coupling rate Γ_R (Γ_L).

implementation of a gate completely passively without the need for any control fields, Rydberg electromagnetically induced transparency, or other interactions while still promising excellent fidelity.

We will show that this setup implements any desired two-photon phase gate controlled by the energy of the photons. If the photons are on resonance, the underlying physics can be sketched quite easily: photons on resonance receive a phase shift of π every time they scatter off an emitter [38]. Because of the chiral coupling, the photons cannot be reflected but rather travel from emitter to emitter. This leads to an overall phase of $(-1)^N$ accumulated by photons leaving on the opposite side of the array with N the number of emitters. If two counterpropagating photons are inside the emitter chain at the same time, they have to pass one another at some point but cannot simultaneously excite the very emitter at which they cross, leading to an additional phase factor of (-1). In a time-binned superposition where the late state of the right propagating photon arrives simultaneously with the early state of the left propagating photon (see Fig. 1), the emitter array therefore implements the transformation

$$
\begin{aligned}
|\text{early, early}\rangle &\rightarrow |\text{early, early}\rangle \\
|\text{late, late}\rangle &\rightarrow |\text{late, late}\rangle \\
|\text{early, late}\rangle &\rightarrow |\text{early, late}\rangle \\
|\text{late, early}\rangle &\rightarrow -|\text{late, early}\rangle,
\end{aligned} \tag{1}
$$

where the first (last) entry in the state refer to the right (left) propagating photon. This creates a controlled Z-phase gate flipping the sign in case of two photons being simultaneously in the system.

The description above relies on photons simultaneously being resonant and inside the emitter chain. Fundamentally this cannot be achieved with a single emitter. The finite size of the photon wave packets thus leads to degradation of the gates due to dispersion, a smeared out phase, and inelastic scattering [9,10,39–46], which is the reason why similar proposals only achieve high fidelities for small phases [9,10]. Multiple emitters are thus required to ensure that the photons meet each other inside the chain despite the large time uncertainty from being narrow in frequency. In the following, we give a concise analytical description of the process, which enables us to address this and other imperfections that potentially compromise the fidelity of the gate.

System.—The system in question consists of a one-dimensional waveguide coupled to many three level emitters with next neighbor distance d; see Fig. 1. We consider two counterpropagating photonic modes with field operators $\mathcal{E}_R(z)$ and $\mathcal{E}_L(z)$, respectively, with commutation relation $[\mathcal{E}_{R/L}(z_1), \mathcal{E}_{R/L}^\dagger(z_2)] = c\delta(z_1 - z_2)$, where c is the group velocity of light in the waveguide (assumed identical in both directions). Transitions between different levels of

an emitter are described by operators $\sigma_{ab}^{(\mu)} = |a\rangle_\mu \langle b|_\mu$. Applying the rotating wave approximation, and rescaling the photon energy by the transition energies so that all energies are relative to resonance, we have the Hamiltonian $H_{\text{tot}} = H_p + H_{\text{int}}$ with

$$
H_p = -i\hbar \int dz [\mathcal{E}_R^\dagger(z)\partial_z \mathcal{E}_R(z) - \mathcal{E}_L^\dagger(z)\partial_z \mathcal{E}_L(z)] \tag{2}
$$

and

$$
H_{\text{int}} = \sum_\mu [\mathcal{E}_R(z_\mu)\sigma_{eg}^{(\mu)} g_R + \mathcal{E}_L(z_\mu)\sigma_{fg}^{(\mu)} g_L] + \text{H.c.} \tag{3}
$$

For our purposes we assume an identical coupling rate to the left and right traveling modes $g_R = g_L = g_0$. Further, we assume the photons to travel quasi-instantaneously from emitter to emitter relative to the lifetime of excitations. This allows us to eliminate the photonic degrees of freedom [44] and to describe the dynamics within the emitter array on the level of polaritons, i.e., coupled light-matter states. The effective Hamiltonian acting on the polaritons then reads

$$
\begin{aligned}
H = &-i\frac{\Gamma_0}{2}\sum_\mu [\sigma_{eg}^{(\mu)}\sigma_{ge}^{(\mu)} + \sigma_{fg}^{(\mu)}\sigma_{gf}^{(\mu)}] \\
&- i\Gamma_0 \sum_{\mu<\nu} e^{ik_0|z_\mu - z_\nu|} [\sigma_{eg}^{(\nu)}\sigma_{ge}^{(\mu)} + \sigma_{fg}^{(\mu)}\sigma_{gf}^{(\nu)}],
\end{aligned} \tag{4}
$$

with k_0 the wave number at the resonance frequency and $\Gamma_0 = 2\pi g_0^2$ the decay rate of the emitter into the waveguide. This effective Hamiltonian is non-Hermitian, reflecting that excitations can leave the system at the edges.

We are interested in a setup including many emitters and thus solve Eq. (4) in the limit of infinitely many interaction sites, where we can neglect losses at the edges. For a single excitation propagating to the right, this leads to a dispersion relation,

$$
\omega_1 = \omega(k_1) = -\frac{\Gamma_0}{2}\frac{\cos[(k_1 - k_0)d/2]}{\sin[(k_1 - k_0)d/2]}, \tag{5}
$$

for the momentum state excited by $\sigma_{eg}^{k_1} = \sqrt{d}\sum_\mu e^{ik_1 z_\mu}\sigma_{eg}^\mu$ [and analogously $\omega_2 = \omega(-k_2)$ for the left traveling one]. This results in an individual phase $\varphi_\ell = kNd$ accumulated while passing the whole array of emitters, with

$$
e^{i\varphi_\ell} = \left(\frac{i\omega_\ell + \Gamma_0/2}{i\omega_\ell - \Gamma_0/2}\right)^N, \tag{6}
$$

where $\ell \in 1, 2$. This coincides with the exact result [41] for finite chains of N emitters and gives a phase shift of $(-1)^N$ when on resonance ($\omega_\ell = 0$).

In the case of two polaritons, we can solve the scattering problem by switching to center-of-mass momentum
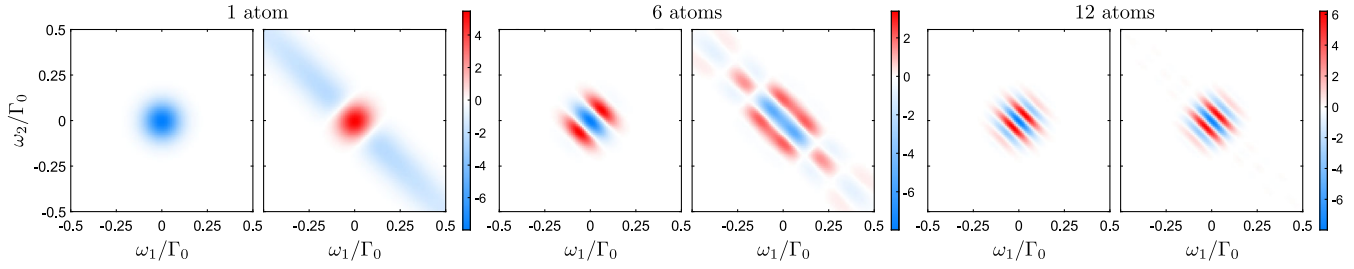
FIG. 2.   Real part of the two-dimensional output state of an initial two-dimensional Gaussian state with width $\sigma = 0.05\Gamma_0$ for both ω_1 and ω_2. For different emitter numbers (1,6,12) we compare our analytical approximation (left) with exact scattering matrix results (right).

$K = (k_1 + k_2)/2$ and relative momentum $q = (k_1 - k_2)/2$, as well as the center of mass $z = (z_1 + z_2)/2$ and relative position $\Delta = z_2 - z_1$ of the two excitations [46–48]. The eigenstates of the Hamiltonian [Eq. (4)] with eigenenergies $\omega_{q,K} = \omega_1 + \omega_2$ then read

$$|\psi_{q,K}\rangle = \sum_{z,\Delta} f(q, \Delta) e^{2iKz} \sigma_{eg}^{[(z-\Delta/2)/d]} \sigma_{fg}^{[(z+\Delta/2)/d]} |0\rangle, \qquad (7)$$

with

$$f(q, \Delta) = \begin{cases} e^{iq\Delta} & \text{for } \Delta > 0 \\ t_{\text{el}} e^{iq\Delta} + t_{\text{in}} e^{iq'\Delta} & \text{for } \Delta < 0. \end{cases} \qquad (8)$$

See Ref. [48] for details of this and the calculations below.

This expression describes elastic scattering with amplitude

$$t_{\text{el}} = |t_{\text{el}}| e^{i\varphi_{1,2}} = 1 - \frac{2\Gamma_0(\Gamma_0 - i\omega_1 - i\omega_2)}{\Gamma_0^2 + 2\omega_1^2 + 2\omega_2^2}, \qquad (9)$$

preserving the relative momentum q due to the chirality of the process. In addition, we have an amplitude

$$t_{\text{in}} = \frac{2i\Gamma_0(\omega_1 - \omega_2)^2}{(\omega_1 + \omega_2 - \Gamma_0)(\Gamma_0^2 + 2\omega_1^2 + 2\omega_2^2)} \qquad (10)$$

for scattering into a degenerate q' with $\omega_{q,K} = \omega_{q',K}$. This redistributes the energies of the outgoing photons

$$\omega_1' = \frac{\Gamma_0^2 + 2(\omega_1 + \omega_2)\omega_2}{2(\omega_2 - \omega_1)}$$

$$\omega_2' = -\frac{\Gamma_0^2 + 2(\omega_1 + \omega_2)\omega_1}{2(\omega_2 - \omega_1)}, \qquad (11)$$

while preserving the total energy $\omega_1 + \omega_2 = \omega_1' + \omega_2'$. We hence refer to it as inelastic scattering. Importantly, these fulfill the continuity equation $|t_{\text{el}}|^2 + |t_{\text{in}}|^2 v_{q',K}/v_{q,K} = 1$ where $v_{q,K} = \partial_q \omega_{q,K}$.

From the scattering amplitudes [Eqs. (9) and (10)] we observe that with both incoming photons on resonance ($\omega_1 = \omega_2 = 0$) we get $t_{\text{el}} = -1$, $t_{\text{in}} = 0$ corresponding to a

perfect Z-phase gate as described in the introduction. Further, by going off resonance while keeping $\omega_1 = \omega_2 = \omega$, we can ensure elastic scattering while achieving any desired phase shift

$$t_{\text{el}} = e^{i\varphi_{1,2}} = \frac{i\omega - \Gamma_0/2}{i\omega + \Gamma_0/2}, \qquad (12)$$

controlled by the energy of the photons in close analogy to the single polariton phase shift [Eq. (6)].

In addition to the scattering phase, photons traveling to the right (left) will also acquire phases φ_1 (φ_2) due to the combination of propagation [Eq. (6)] and entering and leaving the medium for which the mapping between photons and polaritons is given in Refs. [46,48]. These phases are the same for all components of the state and thus only lead to an overall phase that can be ignored. The inelastic component acquires a more complex phase, but this will not influence the results below.

For the implementation of a gate, we wish to suppress the inelastic scattering. For this reason we consider wave packets centered around the same detuning ω, depending on the desired phase shift. This (i) suppresses the amplitude of inelastic scattering [Eq. (10)] and (ii) ensures that inelastically scattered photons are far away in energy; cf. Eq. (11). The latter makes it easy to filter out these inelastic photons, allowing us to absorb the inelastic scattering into a slightly decreased overall success rate of the gate. We note that alternatively one could try to maximize the inelastic scattering component by choosing $\omega_1 = -\omega_2 = \Gamma_0/2$, which produces $t_{\text{el}} = 0$; the result is an energy swap between the two chiral modes, $\omega_1' = \omega_2$ and $\omega_2' = \omega_1$. Potentially this dynamics could also be used to implement quantum protocols.

Numerical verification.—The analytical results presented above describe the dynamics of an infinite chain. To investigate the performance for a finite chain, we compare our results to exact scattering matrices derived from input-output theory [44]. In Fig. 2, we show the result of this for two Gaussian input states with widths $\sigma = 0.05\Gamma_0$. Naturally, our approximation is worst for setups with very few emitters. Especially the high probability for inelastic scattering, as prominently indicated

by the "jets" emerging on the sides of the initial state in Fig. 2, limits the performance of the gate with few emitters. These jets are completely gone for 12 emitters and furthermore the output states coincide quantitatively with the analytical results. In the latter case, the energy regions (sparsely) populated by inelastic scattering lie outside of the plotted area and are thus easily filtered out by frequency filters. The oscillations arise from the phase [Eq. (6)] accumulated by each photon.

Fidelity.—To characterize the performance of the gate operation we exploit the Choi-Jamiolkowski isomorphism [49,50] and consider the input states to be maximally entangled states:

$$|\Phi\rangle \sim (|00\rangle_{12} + |11\rangle_{12}) \otimes (|00\rangle_{34} + |11\rangle_{34}). \quad (13)$$

Here, the gate only affects the qubits 1 and 3 while 2 and 4 denote auxiliary systems. The Choi-Jamiolkowski fidelity is then the overlap between the actual output state and the ideal gate operation, $\mathcal{F} = |\langle\Phi_{\text{ideal}}|\Phi_{\text{out}}\rangle|^2$. The fidelity will depend on which wave function we choose for the ideal state. Here, we consider the mode functions for the computational basis states $|0\rangle$ and $|1\rangle$ to be the dispersed states of the photons traversing the array individually, i.e., multiplying the input 2D wave function with $e^{iN(k(\omega_1)+k(\omega_2)-2k_0)d}$. In principle higher fidelity could be obtained by carefully optimizing this reference state.

With this choice the fidelity for perfect chirality is given by

$$\mathcal{F} = \frac{1}{4(3 + t_{\text{norm}})}|3 + e^{-i\alpha}t_{\text{av}}|^2 \quad (14)$$

for a desired phase α, obtained by adjusting $\varphi_{1,2}$ in Eq. (12), and

$$t_{\text{av}} = \int d\omega_1 d\omega_2 |\psi_1(\omega_1)|^2 |\psi_2(\omega_2)|^2 t_{\text{el}}(\omega_1, \omega_2)$$

$$t_{\text{norm}} = \int d\omega_1 d\omega_2 |\psi_1(\omega_1)|^2 |\psi_2(\omega_2)|^2 |t_{\text{el}}(\omega_1, \omega_2)|^2. \quad (15)$$

Here, the normalization in the denominator corrects for the success probability of the gate, assuming large detuning of the inelastic scattering so that it can be filtered out and herald any failing of the gate.

Z-phase gate.—As discussed above, the scattering phase approaches minus unity if both photons are on resonance leading to perfect gate operation. This is illustrated in Figs. 3(a) and 3(b), where we show fidelity and success probability for an initial Gaussian distribution on resonance and varying width. As shown with lines in the figure, the analytical results for infinite chains rapidly approach the ideal limit as the incident states become narrow in frequency. For finite chains, we have excellent agreement as long as the pulses are sufficiently short in time (wide in
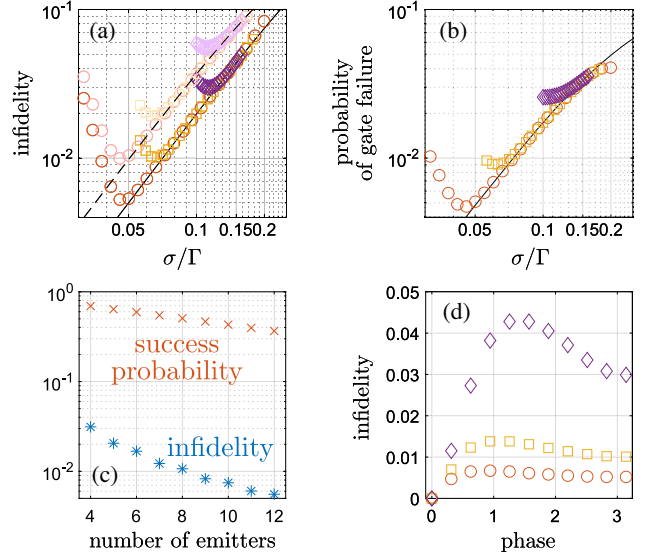

FIG. 3. (a) Infidelity $1 - \mathcal{F}$ and (b) probability of (heralded) gate failure as a function of the width σ of an incident Gaussian pulse for a perfectly chiral setup. The solid line is for in infinite chain while the dark symbols represent numerical results for 4 (diamonds), 8 (squares), and 12 (circles) emitters. The dashed line and lighter symbols in (a) result from not filtering out the inelastic scattering. (c) Imperfect coupling: results assuming that photons are emitted backward or to the side at identical rates rate $\Gamma_B = \Gamma_S = 0.01\Gamma_0$. The success probability decays exponentially with the number of emitters while the fidelity still increases. We chose $k_0 d = \pi$ to avoid Bragg scattering (see main text). (d) Gate operation for arbitrary phase φ by going off resonance [Eq. (12)]. The plot shows the lowest achievable infidelity for different numbers of 4, 8, and 12 emitters using the same symbols as in (a).

frequency). As the pulses become wider in time, however, they can no longer fit inside the medium for a finite number of emitters. This leads to an optimal width of the incident pulses and a higher fidelity for a larger number of emitters. As a specific example, for 12 emitters and an initial width of $\sigma = 0.05\Gamma_0$, a pulse width comparable to one recently used for a related quantum dot experiment with a single emitter [51], we have a near perfect fidelity $\mathcal{F} = 99.48\%$ with only a minor reduction in success probability of the whole gate $\mathcal{R} = (3 + t_{\text{norm}})/4 = 99.51\%$.

If inelastic scattering events are not filtered out, we obtain a success probability of unity but the infidelity is higher. As shown by the dashed line in Fig. 3(a), $1 - \mathcal{F}$ is doubled in the limit of many emitters. The breakdown for finite emitters occurs at the same pulse width. Overall this thus leads to a factor of 2 increase in infidelity.

The performance of the gate relies on the chirality of the setup. To investigate this we assume that photons are emitted to the side or opposite to the intended direction in the waveguide at rates Γ_S and Γ_B, respectively. The forward coupling is then characterized by a coupling efficiency $\beta = (\Gamma_0 + \Gamma_B)/\Gamma_{\text{tot}}$ and directionality $D = (\Gamma_0 - \Gamma_B)/(\Gamma_0 + \Gamma_B)$, where $\Gamma_{\text{tot}} = \Gamma_0 + \Gamma_S + \Gamma_B$. For the perfectly chiral case

$\beta = 1$, $D = 1$ the positions of the emitters are completely irrelevant, but when $\Gamma_B \neq 0$ interference between scattering events introduces a dependence on the exact placement. To minimize the effect of backscattering we assume $k_0 d = n\pi$ with $n \in \mathbb{Z}$. (Note that although this is equal to the usual condition for Bragg scattering, the strong chiral coupling modifies the Bragg condition to occur at $k_0 = \pi/2 + n\pi$ for our setup on resonance.) In Fig. 3(c) we present numerically optimized results for a varying number of emitters and $\Gamma_S = \Gamma_B = 0.01\Gamma_{\text{tot}}$ corresponding to $\beta = 0.99$ and $D = 0.98$. As seen in the figure, nonperfect chirality reduces the success probability but the fidelity is almost unaffected. In practice this means that one will have to use a limited number of emitters for imperfect coupling, but it does not prevent the application of the gate.

To investigate the effect of other imperfections we consider a proof of concept realization with $N = 4$ emitters and parameters corresponding to state of the art experiments with directionality and couplings of $D = 0.98$ [52] and $\beta = 0.98$ [53], respectively. In theory even higher values are possible [27]. The full details are presented in Ref. [48]. As an example we incorporate an average fluctuation of Γ_{tot} with width $\sigma_{\Gamma_{\text{tot}},\text{dB}} = 1.2$ dB and of the resonance energy at each emitter with width $\sigma_\Delta = 0.2\Gamma_{\text{tot}}$. We also implement random distances d between emitters and show that any mismatch in the pulse timing can be neglected for typical delay lines [48]. We average all fluctuating parameters over 10^3 realizations, resulting in $\mathcal{F} > 0.92$ and $\mathcal{R} > 0.51$. For comparison, in linear optics, controlled phase gates are fundamentally limited by $\mathcal{R} \leq 1/9$ [54].

P-phase gate.—The proposed setup is ideally suited to induce a complete phase change by working on resonance, which suppresses inelastic scattering and minimizes the influence of finite numbers of emitters. According to Eqs. (10) and (12), we can achieve a phase gate with an arbitrary phase α by going off resonance with

$$\omega_1 = \omega_2 = \frac{i}{2} \frac{1 + e^{i\alpha}}{1 - e^{i\alpha}} \qquad (16)$$

while simultaneously suppressing inelastic scattering. According to the analytical approximation, assuming infinitely many emitters, this indeed leads to a perfect gate operation for any values of α.

In reality, moving away from resonance jeopardizes the approximation of having many emitters since off-resonant photons interact less strongly with each emitter. This leads to a higher group velocity, which in turn means that we have to choose pulses more narrow in time to localize the pulses inside the medium. This shifts the optimum pulse length to larger widths σ in frequency for a finite number of emitters. In Fig. 3(d) we show the fidelity for different numbers of emitters and gate phases maximized over the energy width of the input states. As seen in the figure, the

infidelity increases as we go away from $\alpha = \pi$ until it again decreases as we approach $\alpha \to 0$, where the gates do nothing. For $N > 10$ the infidelity is, however, on the permille scale for all values of α.

Conclusion.—We have presented a simple, completely passive approach to implement a gate between two photons. While the gate has ideal performance in the limit of many emitters it shows excellent behavior even for a limited number of emitters. The rapidly evolving field of chiral quantum optics promises proof of principle experiments with only a couple of emitters, which would already suffice to reach fidelities above 90%. The developed photon gate could immediately be applied to facilitate photonic based quantum information processing, e.g., it allows the implementation of efficient Bell-state measurements for photons, thereby enhancing the communication rate of ensemble based quantum repeaters [55].

[1] D. d'Enterria and G. G. da Silveira, Observing Light-by-Light Scattering at the Large Hadron Collider, Phys. Rev. Lett. **111**, 080405 (2013).

[2] E. Knill, R. Laflamme, and G. J. Milburn, A scheme for efficient quantum computation with linear optics, Nature (London) **409**, 46 (2001).

[3] H. J. Briegel, D. E. Browne, W. Dür, R. Raussendorf, and M. Van den Nest, Measurement-based quantum computation, Nat. Phys. **5**, 19 (2009).

[4] M. Gimeno-Segovia, P. Shadbolt, D. E. Browne, and T. Rudolph, From Three-Photon Greenberger-Horne-Zeilinger States to Ballistic Universal Quantum Computation, Phys. Rev. Lett. **115**, 020502 (2015).

[5] K. Azuma, K. Tamaki, and H.-K. Lo, All-photonic quantum repeaters, Nat. Commun. **6**, 1 (2015).

[6] D. E. Browne and T. Rudolph, Resource-Efficient Linear Optical Quantum Computation, Phys. Rev. Lett. **95**, 010501 (2005).

[7] M. Pant, H. Krovi, D. Englund, and S. Guha, Rate-distance tradeoff and resource costs for all-optical quantum repeaters, Phys. Rev. A **95**, 012304 (2017).

[8] M. Pant, D. Towsley, D. Englund, and S. Guha, Percolation thresholds for photonic quantum computing, Nat. Commun. **10**, 1070 (2019).

[9] C. Chudzicki, I. L. Chuang, and J. H. Shapiro, Deterministic and cascadable conditional phase gate for photonic qubits, Phys. Rev. A **87**, 042325 (2013).

[10] J. Gea-Banacloche and N. Német, Conditional phase gate using an optomechanical resonator, Phys. Rev. A **89**, 052327 (2014).

[11] M. Heuck, K. Jacobs, and D. R. Englund, Controlled-Phase Gate Using Dynamically Coupled Cavities and Optical Nonlinearities, Phys. Rev. Lett. **124**, 160501 (2020).

[12] D. E. Chang, A. S. Sørensen, E. A. Demler, and M. D. Lukin, A single-photon transistor using nanoscale surface plasmons, Nat. Phys. **3**, 807 (2007).

[13] L.-M. Duan and H. J. Kimble, Scalable Photonic Quantum Computation through Cavity-Assisted Interactions, Phys. Rev. Lett. **92**, 127902 (2004).

[14] L.-M. Duan, B. Wang, and H. J. Kimble, Robust quantum gates on neutral atoms with cavity-assisted photon scattering, Phys. Rev. A **72**, 032333 (2005).

[15] M. Mašalas and M. Fleischhauer, Scattering of dark-state polaritons in optical lattices and quantum phase gate for photons, Phys. Rev. A **69**, 061801(R) (2004).

[16] I. Friedler, D. Petrosyan, M. Fleischhauer, and G. Kurizki, Long-range interactions and entanglement of slow single-photon pulses, Phys. Rev. A **72**, 043803 (2005).

[17] A. André, M. Bajcsy, A. S. Zibrov, and M. D. Lukin, Nonlinear Optics with Stationary Pulses of Light, Phys. Rev. Lett. **94**, 063902 (2005).

[18] A. V. Gorshkov, J. Otterbach, E. Demler, M. Fleischhauer, and M. D. Lukin, Photonic Phase Gate via an Exchange of Fermionic Spin Waves in a Spin Chain, Phys. Rev. Lett. **105**, 060502 (2010).

[19] A. V. Gorshkov, J. Otterbach, M. Fleischhauer, T. Pohl, and M. D. Lukin, Photon-Photon Interactions via Rydberg Blockade, Phys. Rev. Lett. **107**, 133602 (2011).

[20] J. D. Thompson, T. L. Nicholson, Q.-Y. Liang, S. H. Cantu, A. V. Venkatramani, S. Choi, I. A. Fedorov, D. Viscor, T. Pohl, M. D. Lukin *et al.*, Symmetry-protected collisions between strongly interacting photons, Nature (London) **542**, 206 (2017).

[21] I. Iakoupov, J. Borregaard, and A. S. Sørensen, Controlled-Phase Gate for Photons Based on Stationary Light, Phys. Rev. Lett. **120**, 010502 (2018).

[22] S. Das, A. Grankin, I. Iakoupov, E. Brion, J. Borregaard, R. Boddeda, I. Usmani, A. Ourjoumtsev, P. Grangier, and A. S. Sørensen, Photonic controlled-phase gates through rydberg blockade in optical cavities, Phys. Rev. A **93**, 040303(R) (2016).

[23] B. He, A. V. Sharypov, J. Sheng, C. Simon, and M. Xiao, Two-Photon Dynamics in Coherent Rydberg Atomic Ensemble, Phys. Rev. Lett. **112**, 133606 (2014).

[24] D. J. Brod, J. Combes, and J. Gea-Banacloche, Two photons co-and counterpropagating through N cross-kerr sites, Phys. Rev. A **94**, 023833 (2016).

[25] D. J. Brod and J. Combes, Passive CPHASE Gate via Cross-Kerr Nonlinearities, Phys. Rev. Lett. **117**, 080502 (2016).

[26] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature (London) **541**, 473 (2017).

[27] N. V. Hauff, H. Le Jeannic, P. Lodahl, S. Hughes, and N. Rotenberg, Chiral quantum optics in broken-symmetry and topological photonic crystal waveguides, Phys. Rev. Research **4**, 023082 (2022).

[28] I. J. Luxmoore, N. A. Wasley, A. J. Ramsay, A. C. T. Thijssen, R. Oulton, M. Hugues, S. Kasture, V. G. Achanta, A. M. Fox, and M. S. Skolnick, Interfacing Spins in an InGaAs Quantum Dot to a Semiconductor Waveguide Circuit Using Emitted Photons, Phys. Rev. Lett. **110**, 037402 (2013).

[29] I. Söllner, S. Mahmoodian, S. L. Hansen, L. Midolo, A. Javadi, G. Kiršanskė, T. Pregnolato, H. El-Ella, E. H. Lee, J. D. Song *et al.*, Deterministic photon–emitter coupling in chiral photonic circuits, Nat. Nanotechnol. **10**, 775 (2015).

[30] R. Coles, D. Price, J. Dixon, B. Royall, E. Clarke, P. Kok, M. Skolnick, A. Fox, and M. Makhonin, Chirality of nanophotonic waveguide with embedded quantum emitter for unidirectional spin transfer, Nat. Commun. **7**, 1 (2016).

[31] P.-O. Guimond, H. Pichler, A. Rauschenbeutel, and P. Zoller, Chiral quantum optics with V-level atoms and coherent quantum feedback, Phys. Rev. A **94**, 033829 (2016).

[32] S. Barik, A. Karasahin, S. Mittal, E. Waks, and M. Hafezi, Chiral quantum optics using a topological resonator, Phys. Rev. B **101**, 205303 (2020).

[33] J. Volz, M. Scheucher, C. Junge, and A. Rauschenbeutel, Nonlinear π phase shift for single fibre-guided photons interacting with a single resonator-enhanced atom, Nat. Photonics **8**, 965 (2014).

[34] I. Shomroni, S. Rosenblum, Y. Lovsky, O. Bechler, G. Guendelman, and B. Dayan, All-optical routing of single photons by a one-atom switch controlled by a single photon, Science **345**, 903 (2014).

[35] K. Stannigel, P. Rabl, and P. Zoller, Driven-dissipative preparation of entangled states in cascaded quantum-optical networks, New J. Phys. **14**, 063014 (2012).

[36] T. Ramos, H. Pichler, A. J. Daley, and P. Zoller, Quantum Spin Dimers from Chiral Dissipation in Cold-Atom Chains, Phys. Rev. Lett. **113**, 237203 (2014).

[37] H. Pichler, T. Ramos, A. J. Daley, and P. Zoller, Quantum optics of chiral spin networks, Phys. Rev. A **91**, 042116 (2015).

[38] D. E. Chang, L. Jiang, A. Gorshkov, and H. Kimble, Cavity QED with atomic mirrors, New J. Phys. **14**, 063003 (2012).

[39] J. H. Shapiro, Single-photon kerr nonlinearities do not help quantum computation, Phys. Rev. A **73**, 062305 (2006).

[40] J. Gea-Banacloche, Impossibility of large phase shifts via the giant kerr effect with single-photon wave packets, Phys. Rev. A **81**, 043823 (2010).

[41] S. Fan, Ş. E. Kocabaş, and J.-T. Shen, Input-output formalism for few-photon transport in one-dimensional nanophotonic waveguides coupled to a qubit, Phys. Rev. A **82**, 063821 (2010).

[42] S. Xu, E. Rephaeli, and S. Fan, Analytic Properties of Two-Photon Scattering Matrix in Integrated Quantum Systems Determined by the Cluster Decomposition Principle, Phys. Rev. Lett. **111**, 223602 (2013).

[43] S. Xu and S. Fan, Input-output formalism for few-photon transport: A systematic treatment beyond two photons, Phys. Rev. A **91**, 043845 (2015).

[44] T. Caneva, M. T. Manzoni, T. Shi, J. S. Douglas, J. I. Cirac, and D. E. Chang, Quantum dynamics of propagating photons with strong interactions: A generalized input–output formalism, New J. Phys. **17**, 113001 (2015).

[45] Y. Ke, A. V. Poshakinskiy, C. Lee, Y. S. Kivshar, and A. N. Poddubny, Inelastic Scattering of Photon Pairs in Qubit Arrays with Subradiant States, Phys. Rev. Lett. **123**, 253601 (2019).

[46] B. Schrinski and A. S. Sørensen, Polariton interaction in one-dimensional arrays of atoms coupled to waveguides, arXiv:2110.15878.

[47] B. Bakkensen, Y.-X. Zhang, J. Bjerlin, and A. S. Sørensen, Photonic bound states and scattering resonances in waveguide QED, arXiv:2110.06093.

[48] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.130502 for calculational details and additional discussion about experimental limitations.

[49] A. Jamiołkowski, Linear transformations which preserve trace and positive semidefiniteness of operators, Rep. Math. Phys. **3**, 275 (1972).

[50] M.-D. Choi, Completely positive linear maps on complex matrices, Linear Algebra Appl. **10**, 285 (1975).

[51] M. L. Chan, A. Tiranov, M. H. Appel, Y. Wang, L. Midolo, S. Scholz, A. D. Wieck, A. Ludwig, A. S. Sørensen, and P. Lodahl, On-chip spin-photon entanglement based on single-photon scattering, arXiv:2205.12844.

[52] B. Le Feber, N. Rotenberg, and L. Kuipers, Nanophotonic control of circular dipole emission, Nat. Commun. **6**, 6695 (2015).

[53] M. Arcari, I. Söllner, A. Javadi, S. Lindskov Hansen, S. Mahmoodian, J. Liu, H. Thyrrestrup, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, Near-Unity Coupling Efficiency of a Quantum Emitter to a Photonic Crystal Waveguide, Phys. Rev. Lett. **113**, 093603 (2014).

[54] T. C. Ralph, N. K. Langford, T. B. Bell, and A. G. White, Linear optical controlled-not gate in the coincidence basis, Phys. Rev. A **65**, 062324 (2002).

[55] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, Quantum repeaters based on atomic ensembles and linear optics, Rev. Mod. Phys. **83**, 33 (2011).