Line Operators in Chern-Simons–Matter Theories and Bosonization in Three Dimensions

Barak Gabai,[1] Amit Sever,[2] and De-liang Zhong (钟德亮)[2,*]
[1]*Jefferson Physical Laboratory, Harvard University, Cambridge, Massachusetts 02138 USA*
[2]*School of Physics and Astronomy, Tel Aviv University, Ramat Aviv 69978, Israel*

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We study Chern-Simons theories at large N with either bosonic or fermionic matter in the fundamental representation. The most fundamental operators in these theories are mesonic line operators, the simplest example being Wilson lines ending on fundamentals. We classify the conformal line operators along an arbitrary smooth path as well as the spectrum of conformal dimensions and transverse spins of their boundary operators at finite 't Hooft coupling. These line operators are shown to satisfy first-order chiral evolution equations, in which a smooth variation of the path is given by a factorized product of two line operators. We argue that this equation together with the spectrum of boundary operators are sufficient to uniquely determine the expectation values of these operators. We demonstrate this by bootstrapping the two-point function of the displacement operator on a straight line. We show that the line operators in the theory of bosons and the theory of fermions satisfy the same evolution equation and have the same spectrum of boundary operators.

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Introduction.—Three-dimensional Chern-Simons (CS) theory enjoys level-rank duality, which is well established when the level k and the rank N of the gauge group are finite [1–4]. This duality is believed to extend to a nonperturbative duality between conformal theories that are obtained by coupling CS theory to scalars or fermions in the fundamental representation.

There is extensive and robust evidence for the duality, especially in the large N 't Hooft limit with $\lambda \equiv N/k$ fixed. It includes matching correlation functions of local operators [5–10], the spectrum of monopole and baryon operators [11,12], thermal free energies [13–17], S matrices [18–20], and relating the nonsupersymmetric dualities to well-established supersymmetric ones [21,22] via RG flow [16,23]. Since these tests are performed in the strict planar limit, they do not distinguish between different versions of the duality that differ by half-integer shifts of the Chern-Simons level k and by the gauge group being $SU(N)$ or $U(N)$ [24].

In this Letter, we summarize the results of our study of line operators in the large N limit. They can be either closed, such as Wilson loops, or open, such as Wilson lines stretching between a fundamental and an antifundamental field. We denote the latter as *mesonic line operators*. Detailed derivations of the results presented here will appear in separate publications [30,31].

At leading order in the large N limit, the matter does not contribute to the expectation values of closed Wilson loops. On the other hand, it does contribute to the expectation values of mesonic line operators. Correspondingly, their dependence on the path is not topological and, as will become clear, is directly related to the $1/N$ correction to the closed Wilson loop expectation value. Mesonic line operators overlap with all local operators in the theory, including the single-trace and the multitrace ones.

A generic mesonic operator would experience a RG flow on the line. Here, we focus on the fixed points of that flow, which are the conformal line operators, and study them along arbitrary smooth paths. We classify them, as well as their relevant deformations (when such exist), and speculate about the flows between them.

Since the Wilson line in CS theory is oriented, we have two families of boundary operators, right (fundamental) and left (antifundamental). Both sets are uniquely classified by their conformal dimension and transverse spin. They can be further divided into those that become $SL(2,\mathbb{R})$ primaries and descendants when the line is straight. For any of the conformal line operators and on either the left or the right boundary, we find that there are two towers of primary operators. Operators in the same tower have the same twist. They all have nonzero anomalous dimension and anomalous transverse spin, equal to $\pm\lambda/2$. Correspondingly, if one starts with a boundary operator of integer (half-integer) spin at $\lambda = 0$, one ends up with a boundary operator of half-integer (integer) spin at $|\lambda| = 1$.

To determine the correlation functions and expectation values of the line operators, we need to understand their dependence on the shape of the path. This dependence is

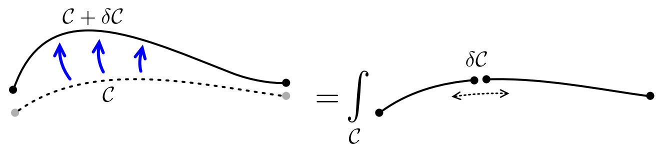

FIG. 1. The evolution equation (21) relates a small smooth deformation of a conformal mesonic line operator to an integrated product of two mesonic line operators.

subject to the *evolution equation*. It is a loop equation of the type that was first introduced for closed Wilson loops in four-dimensional Yang-Mills theory as an alternative nonperturbative approach to solving QCD, (see the review [32] and references therein). Here, in CS-matter theory, the equation is first order. It relates any smooth variation of a (conformal) mesonic line operator to a factorized product of two mesonic line operators, see Fig. 1 [33].

We show that the evolution equation, combined with the spectrum of boundary operators, uniquely determines the expectation value of any of the mesonic line operators. To demonstrate this, we start with a straight line and deform it smoothly and systematically, order by order in the relative magnitude of the deformation, while imposing the above properties. In particular, we bootstrap the (normalization independent) two-point function of the displacement operator [34]. We find that it is given by

$$\frac{\langle\langle \mathcal{O}_L|\mathbb{D}_i(x_s)\mathbb{D}^i(x_t)|\mathcal{O}_R\rangle\rangle}{\langle\langle \mathcal{O}_L|\mathcal{O}_R\rangle\rangle} = \frac{\Lambda(\Delta)}{x_{st}^4}\left(\frac{x_{10}x_{st}}{x_{1s}x_{t0}}\right)^{2\Delta}, \quad (1)$$

with

$$\Lambda(\Delta) = -\frac{(2\Delta-1)(2\Delta-2)(2\Delta-3)\sin(2\pi\Delta)}{2\pi}. \quad (2)$$

The double brackets in (1) denote expectation values in the presence of the mesonic line operator lying along a straight line stretching between x_0 to x_1. Here, $\mathcal{O}_{L/R}$ are the left and right boundary operators of minimal (and opposite) transverse spin and Δ is their conformal dimension. The λ dependence of Δ, given below, depends on which conformal line operator we consider and whether we use the fermionic or the bosonic descriptions. We have also verified (2) in perturbation theory to all loop orders [30].

We show that the conformal line operators of the bosonic and fermionic theories satisfy the same evolution equation and that their spectra of boundary operators are related to each other through the map $\lambda_f = \lambda_b - \mathrm{sign}(k_b)$ [7]. It follows that their expectation values are related by the same map.

Setup and overview.—The first hint for the existence of a dynamical interplay between fermions and bosons in three dimensions comes from the study of CS theory. This topological gauge theory is governed by the action [35]

$$S_{\mathrm{CS}} = \frac{ik}{4\pi}\int d^3x\epsilon^{\mu\nu\rho}\mathrm{tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right). \quad (3)$$

In this Letter, we work in the Euclidean signature and focus on the 't Hooft limit, in which the rank of the gauge group is large [36]

$$N\to\infty \quad \text{with} \quad \lambda \equiv \frac{N}{k} \in [-1,1] \qquad \text{fixed}. \quad (4)$$

The theory (3) enjoys a level-rank duality under which the parameters in (4) transform as

$$[k,\lambda] \leftrightarrow [-k, \lambda - \mathrm{sign}(k)]. \quad (5)$$

This duality interchanges the weak and strong coupling limits.

In the pure CS theory, the only observables are Wilson loops. When defined with framing regularization, they only depend on the topology of the loops and the self-linking number $\mathfrak{f}$. The latter counts the number of times the framing vector n winds around the loop [37]. For example, the expectation value of an unknotted loop is

$$\langle W^{\mathfrak{f}}_{\mathrm{unknot}}\rangle = e^{i\pi\lambda\mathfrak{f}}\times k\frac{\sin(\pi\lambda)}{\pi}. \quad (6)$$

We can therefore think of the Wilson loop as being a ribbon, parameterized by the framing vector. It is expected that once we attach a Wilson line to an operator in the fundamental representation, this dependence on the framing vector would lead to fractional spin and to statistics ranging between a boson (fermion) at $\lambda = 0$ and a fermion (boson) at $|\lambda| = 1$. Here we will prove this expectation.

Concretely, we study CS theory coupled to fermions or bosons in the fundamental representation. The action in these two cases is given by [38]

$$S_E^{\mathrm{bos}} = S_{\mathrm{CS}} + \int d^3x(D_\mu\phi)^\dagger D^\mu\phi + \frac{\lambda_6}{N^2}(\phi^\dagger\phi)^3, \quad (7)$$

$$S_E^{\mathrm{fer}} = S_{\mathrm{CS}} + \int d^3x\bar{\psi}\cdot\gamma^\mu D_\mu\psi. \quad (8)$$

Both theories are conformal (for tuned λ_6) and have high spin currents that are conserved at leading order in the large N limit (4). The level-rank duality (5) was conjectured to extend to a duality between the theory of fermions (bosons) and the Legendre transform of the theory of bosons (fermions) with respect to the scalar current $J^{(0)}$ [39,41]. The differences between these theories and their Legendre transforms will not be relevant for our primary focus, which is the planar expectation values of mesonic line operators [43].

Without loss of generality, we assume that the level, and correspondingly, the 't Hooft coupling in the bosonic theory is positive [44].

Mesonic line operators in the bosonic theory.—The most familiar line operator along any smooth path $\mathcal{C}$ is a Wilson line. However, on such a line in the bosonic theory, there is a nonzero beta function for the coupling of the adjoint operator $\phi\phi^\dagger$. At the fix points of the corresponding flow, we find operators with the biscalar condensate,

$$\mathcal{W}^\alpha[\mathcal{C}, n] \equiv \left[\mathcal{P}e^{i\int_\mathcal{C}(A\cdot dx + i\alpha\frac{2\pi\lambda}{N}\phi\phi^\dagger|dx|)}\right]_n, \tag{9}$$

where $\alpha = \pm 1$. In what follows, we show that the operator with $\alpha = 1$ is stable. The other operator with $\alpha = -1$ has one relevant deformation that, when turned on, generates a flow that leads to the former [45]. In the next section, we study the $\alpha = 1$ operators, and in the Appendix, we generalize our considerations to the $\alpha = -1$ one. In the Appendix section "Line operator with one degree of freedom," we consider a new conformal line operator with one degree of freedom on the line that is constructed using both of the $\alpha = \pm 1$ operators.

The stable mesonic line operator: The mesonic line operator with $\alpha = 1$ is defined by stretching $\mathcal{W} \equiv \mathcal{W}^{\alpha=1}$ between a right (fundamental) boundary operator and a left (antifundamental) boundary operator,

$$M = \mathcal{O}_L \mathcal{W} \mathcal{O}_R. \tag{10}$$

It depends on the shape of the path, the framing vector, and the two boundary operators. In the planar limit, all operators on the line factorize into a product of two boundary operators $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$.

To classify the boundary operators, it is sufficient to consider the case of a straight line along the x^3 direction. An infinite straight line preserves an $SL(2,\mathbb{R}) \times U(1)$ subgroup of the three-dimensional conformal symmetry. The boundary operators are uniquely characterized by two numbers, their $SL(2,\mathbb{R})$ conformal dimension Δ and their $U(1)$ spin in the transverse plane to the line. For example, at tree level, for the right operators,

$$\mathcal{O}^{(n,s)}_{R,\text{tree}} = \frac{1}{\sqrt{N}} \times \begin{cases} \partial^n_{x^3_R}\partial^s_{x^+_R}\phi & s \geq 1 \\ \partial^n_{x^3_R}\partial^{-s}_{x^-_R}\phi & s \leq 0 \end{cases}, \tag{11}$$

and similarly for $\mathcal{O}^{(n,s)}_{L,\text{tree}}$. Here, $x^\pm = (x^1 \pm ix^2)/\sqrt{2}$ parametrize the transverse plane. The operators of minimal twist, $\mathcal{O}^{(0,s)}$, are all $SL(2,\mathbb{R})$ primaries.

At tree level these boundary operators have transverse spin s and dimension $\Delta^{(n,s)}_0 = 1/2 + n + |s|$. However, at loop level, their dimensions and spins receive quantum corrections. We computed their conformal dimensions and anomalous spin explicitly [30]. Working in light cone gauge, we studied the expectation values of all the mesonic line operators (10) along a straight line. It was shown that terms in their perturbative expansions satisfy a recursion relation, which can be solved to give simple expressions. Resummation of these expressions yields an anomalous dimension and anomalous spin equal to $\pm\lambda/2$, as well as the two point function of the displacement operator in (2).

The set of operators with $s \geq 0$ ($s < 0$) all have the same anomalous dimension. They are related to each other by the covariant path derivatives, denoted $\delta_{x^\mu_{R/L}}$, as follows [48]:

$$\begin{aligned} \mathcal{O}_L\mathcal{W}\mathcal{O}^{(0,s+1)}_R &= \delta_{x^+_R}\mathcal{O}_L\mathcal{W}\mathcal{O}^{(0,s)}_R, \qquad s \geq 1, \\ \mathcal{O}_L\mathcal{W}\mathcal{O}^{(0,-s-1)}_R &= \delta_{x^-_R}\mathcal{O}_L\mathcal{W}\mathcal{O}^{(0,-s)}_R, \qquad s \geq 0, \end{aligned} \tag{12}$$

and

$$\begin{aligned} \mathcal{O}^{(0,s+1)}_L\mathcal{W}\mathcal{O}_R &= \delta_{x^+_L}\mathcal{O}^{(0,s)}_L\mathcal{W}\mathcal{O}_R, \qquad s \geq 0, \\ \mathcal{O}^{(0,-s-1)}_L\mathcal{W}\mathcal{O}_R &= \delta_{x^-_L}\mathcal{O}^{(0,-s)}\mathcal{W}\mathcal{O}_R, \qquad s \geq 1. \end{aligned} \tag{13}$$

Here, the use of equal signs instead of a proportionality relation is a relative choice of normalization. At the bottom of these four towers of primaries we have the boundary operators

$$\left\{\mathcal{O}^{(0,0)}_L, \mathcal{O}^{(0,-1)}_L\right\} \quad \text{and} \quad \left\{\mathcal{O}^{(0,0)}_R, \mathcal{O}^{(0,1)}_R\right\}. \tag{14}$$

Similarly, the descendants are obtained from the primaries by acting with the $SL(2,\mathbb{R})$ raising generator. Their form depends on the conformal frame. Since we let the endpoints vary and do not keep the line straight, we use a simpler classification of the descendants by the number of longitudinal path derivatives,

$$\mathcal{O}^{(n+1,s)}_L\mathcal{W}\mathcal{O}_R \equiv \delta_{x^3_L}\mathcal{O}^{(n,s)}_L\mathcal{W}\mathcal{O}_R, \tag{15}$$

and similarly for the right operator.

The spin $\mathfrak{s}$ of the boundary operators can be computed explicitly [30] or alternatively, be determined from the anomalous dimension as we now explain. First, we notice that operators that are related to each other by a path derivative, (12), (13), and (15), must have the same anomalous spin. Second, we notice that the conformal line operator $\mathcal{W}$ can be lifted into locally supersymmetric line operators in the $\mathcal{N} = 2$ supersymmetric Chern-Simons theory. Under this lift, the four operators (14) are mapped into boundary operators in Bogomol'nyi–Prasad–Sommerfield (BPS) supermultiplets [49]. It turns out that at leading order in the large N limit, the expectation values of the locally supersymmetric mesonic line operators in the $\mathcal{N} = 2$ theory are the same as they are in the nonsupersymmetric theory, (7) or (8). The BPS condition for either the left or the right boundary operators leads to the relation

$$\Delta^{(n,s)} = \frac{1}{2} + n + |\mathfrak{s}|. \tag{16}$$

This implies that the anomalous part of the transverse spin equals the anomalous dimension that we have derived before and is given by

$$\mathfrak{s}_L = s_L + \lambda/2, \qquad \mathfrak{s}_R = s_R - \lambda/2, \tag{17}$$

where $s_{L/R}$ are the tree level spins.

This result confirms the expectation in which the framing dependence of a closed Wilson loop in CS theory turns bosons into fermions and vice versa.

The operator on the line with the minimal dimension is $\mathcal{O}_L^{(0,0)} \times \mathcal{O}_R^{(0,0)}$. It has conformal dimension $\Delta_{\rm inner}^{\rm min} = 1 + \lambda$. Hence, the line operator (9) with $\alpha = 1$ does not have a relevant deformation. As in (17), all results we find for the left operators are related to those for the right ones by parity, which flips the sign of the transverse spin.

The evolution equation: Next, we would like to understand the dependence of the mesonic line operator on the path. Under a small smooth deformation of the path $x(\cdot) \mapsto x(\cdot) + v(\cdot)$, the change in the line operator can be expressed in terms of the displacement operator $\mathbb{D}_\mu(s)$ as

$$\delta\mathcal{W} = \int ds|\dot{x}(s)| v^\mu(s)\mathcal{P}[\mathbb{D}_\mu(s)\mathcal{W}], \tag{18}$$

where the deformation is parametrized such that $v(s)$ is a normal vector. We find that the displacement operator is chiral, with its two components given by [57]

$$\begin{aligned} \mathbb{D}_+ &= -4\pi\lambda\mathcal{O}_R^{(0,1)}\mathcal{O}_L^{(0,0)} \\ \mathbb{D}_- &= -4\pi\lambda\mathcal{O}_R^{(0,0)}\mathcal{O}_L^{(0,-1)} \end{aligned} \quad \text{for } \alpha = 1, \tag{19}$$

with the understanding that the framing vector, being continuous, is the same on the right and left. Note that while the left and right boundary operators have nonzero anomalous dimensions and anomalous spins, these exactly cancel out in the combinations in (19).

This form of the displacement operator is derived by computing the Schwinger-Dyson equation for the line operator defined in (9), with no self-intersections. Alternatively, we notice that $\mathbb{D}$ in (19) is the unique operator on the line with exact dimension $\Delta(\mathbb{D}_\pm) = 2$ and transverse spin equal to one. We can therefore reverse the logic and use (19) as the definition of the deformed operator.

The factorized form of the displacement operator leads to a closed equation for the mesonic line operators. We label them using the shorthand notation

$$M_{st}^{(s_L,s_R)}[x(\cdot)] \equiv \mathcal{O}_L^{(0,s_L)}\mathcal{W}_{st}[x(\cdot)]\mathcal{O}_R^{(0,s_R)}, \tag{20}$$

where $x(\cdot)$ is a smooth path between $x_L = x(s)$ and $x_R = x(t)$. In this notation, the evolution equation takes the form

$$\begin{aligned} \delta M_{10}^{(s_L,s_R)}[x(\cdot)] &= [\text{boundary terms}] \\ &- 4\pi\lambda \int_0^1 ds|\dot{x}_s|\left[v_s^+ M_{1s}^{(s_L,1)} M_{t0}^{(0,s_R)} + v_s^- M_{1s}^{(s_L,0)} M_{t0}^{(-1,s_R)}\right], \end{aligned} \tag{21}$$

where $u_s \equiv u(s)$. The boundary terms can be determined by consistency of the equation, see the section "*The line bootstrap*" and [31].

The boundary equation: Similar to the line evolution equation, the boundary operators also satisfy a Schwinger-Dyson type equation. It relates $SL(2,\mathbb{R})$ primaries from the same tower as

$$\begin{aligned} \delta_{x_R^+}\mathcal{O}_L\mathcal{W}\mathcal{O}_R^{(0,-s-1)} &= \beta\mathcal{O}_L\mathcal{W}\mathcal{O}_R^{(2,-s)}, \qquad s \geq 0, \\ \delta_{x_R^-}\mathcal{O}_L\mathcal{W}\mathcal{O}_R^{(0,s+1)} &= \bar{\beta}\mathcal{O}_L\mathcal{W}\mathcal{O}_R^{(2,s)}, \qquad s \geq 1, \end{aligned} \tag{22}$$

and similarly for the left operators. On the right-hand side we have the unique boundary operator with the correct dimension and transverse spin. In the section "*The line bootstrap*" we bootstrap the proportionality coefficients to be given by [58]

$$\beta = \bar{\beta} = -\frac{1}{2}. \tag{23}$$

Mesonic line operators in the fermionic theory.—In the fermionic theory, the Wilson line operator that only couples to the gauge field

$$W[x(\cdot)] = \mathcal{P}e^{i\int A_\mu \dot{x}^\mu ds} \tag{24}$$

is a conformal line operator. The corresponding mesonic line operators take the form (10) with

$$\mathcal{O}_R^{(n,s)} = \frac{1}{\sqrt{N}} \times \begin{cases} D_3^n D_+^{|s|-\frac{1}{2}}\psi_+(x_R) & s \geq +\frac{1}{2} \\ D_3^n D_-^{|s|-\frac{1}{2}}\psi_-(x_R) & s \leq -\frac{1}{2} \end{cases}, \tag{25}$$

and similarly for the left boundary. Here, the tree level spin s takes half integer values and D_μ is the covariant derivative.

We have repeated the explicit computation of the anomalous dimensions, the anomalous spins, the lift to a locally supersymmetric line operator in the $\mathcal{N} = 2$ theory, as well as the analysis of the BPS boundary operators [30]. We find that under the duality map between the 't Hooft couplings in the fermionic and bosonic theories, $\lambda_f = \lambda_b - 1$, the spectrum of boundary operators as well as their transverse spins exactly match the ones we have obtained for the $\alpha = 1$ operator in (16) and (17).

The form of the evolution equation in the two theories also matches. The only difference is in the coefficient, being $4\pi\lambda_b$ in (21) and $4\pi\lambda_f$ in the fermionic theory. This factor, however, depends on the normalization of the mesonic line operators.

The bosonic operator with $\alpha = -1$ also has a dual description in the fermionic theory. It is constructed in the Appendix section "The condensed fermionic line operator."

The line bootstrap.—In this section, we explain how the evolution equation and the spectrum of boundary operators can be used to evaluate the expectation value of mesonic line operators. Here, we summarize our results and explain the main ideas that lead to them. For concreteness, we use the labeling of the operators in the bosonic theory with $\alpha = 1$. The construction, however, does not depend on this.

The expectation value of the mesonic line operators along a straight line, $\mathcal{M}^{(s,s')} \equiv \langle M^{(s,s')} \rangle$, is fixed by conformal symmetry to take the form [59]

$$\mathcal{M}^{(s,s')} = \frac{c_s(\lambda)\delta_{s,-s'}}{4\pi|x_L - x_R|^{1+|2s+\lambda|}}. \tag{26}$$

In our normalization of the boundary operators (12) and (13), the normalization constants c_s are not independent [60]. Using the chiral form of the evolution equation (21) and demanding that the expectation values are invariant under constant translation in the transverse plane, we find that

$$\begin{aligned} c_{s+1} &= -\beta(s+1+\lambda)(s+2+\lambda)c_s, \qquad s \geq 0, \\ c_{-s-1} &= -\bar{\beta}(s+1-\lambda)(s+2-\lambda)c_{-s}, \qquad s \geq 1, \end{aligned} \tag{27}$$

where β and $\bar{\beta}$ are defined in (22). Demanding that the expectation values are also invariant under rigid rotations fixes them to be given by (23).

So far, the results (27) and (23) were obtained using only deformations for which the contribution of the displacement operator (19) drops out. To proceed, we must include deformations to which they do contribute. As we next describe, this is done using a form of conformal perturbation theory on the straight line.

We deform away from the straight line as $x(\cdot) \mapsto x(\cdot) + v(\cdot)$. At any order in v, we add all operators of the corresponding dimension and spin to the local action of the straight line and its boundaries. This includes scheme dependent counterterms that cancel power divergences arising from the integration of the displacement operator. We then fix their coefficients systematically, imposing the correct spectrum of boundary operators, the conformal symmetry of the line, and the evolution equation.

Demanding that the straight line transforms covariantly under conformal transformations is sufficient to fix all the coefficients at order v, but not at order v^2. We then demand in addition that if we first preform an arbitrary smooth deformation, and, on top of that, we apply a conformal transformation then the deformed line transforms covariantly. These conditions turn out to fix all the second-order coefficients. We then evaluate the two-point function of the displacement operator and find that it is given by (2) with $\Delta = (1 + \lambda_b)/2$. The analysis in the fermionic theory is manifestly identical, with the only difference being in the normalization convention.

Going to higher orders in the deformation is tedious but systematic. It can be used to unambiguously evaluate the expectation value order by order in the deformation from the straight line. It follows that the expectation values of the line operators in the bosonic and fermionic theories are related to each other by the duality map $\lambda_f = \lambda_b - \text{sign}(k_b)$. This is because their spectrum of boundary operators are related to each other by this map, and the forms of their evolution equations are the same.

Another conclusion from the derivation above is a match of the $1/N$ corrections to the expectation values of closed line operators between the bosonic and fermionic theories. That is a direct outcome of the fact that their deformations are governed by the same local displacement operators. In other words, smooth deformations of a closed loop (and circular in particular) are equal to $1/N$ times factorized expectation values of mesonic line operators.

Discussion.—In this Letter, we have classified the conformal line operators of large N Chern-Simons theory coupled to fermions or bosons in the fundamental representation. We have computed the spectrum and transverse spins of their boundary operators at finite 't Hooft coupling. In particular, their displacement operators factorize into a product of fundamental and antifundamental boundary operators. Together, the spectrum and the form of the displacement operator were shown to fix the expectation value of the mesonic line operators uniquely. We have found that the line operators of the theory coupled to bosons and the ones of the theory coupled to fermions are related to each other through the strong-weak duality map $\lambda_f = \lambda_b - \text{sign}(k_b)$.

To complete the derivation of the duality at the planar level, one should also match the connected piece of the correlation functions between mesonic line operators. The path dependence of this piece is controlled by the expectation values studied here. We expect that the additional information about the known spectrum of single trace operators will be sufficient to determine these connected correlators uniquely. There are two future directions that we hope to report on in the near future. First, it would be interesting to find an explicit finite coupling solution for the expectation value of the mesonic line operators. Second, there is much evidence that the bosonic and fermionic vector models are holographically dual to parity breaking versions of Vasiliev's higher-spin theory [61], see Refs. [7,13,42,62–65]. One of our main motivations for this Letter is to better understand and, optimistically, even to derive this duality.

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Appendix: The unstable mesonic line operator.—By repeating the resummation of the perturbation theory for the operator (9) with $\alpha=-1$, we find the same dimensions and spins as for the $\alpha=1$ boundary operators with the only exception been that for $s=0$

$$\tilde{\Delta}^{(n,0)}_{L/R}(\lambda)=\Delta^{(n,0)}_{L/R}(-\lambda), \tag{A1}$$

where the tilde is added to distinguish from the $\alpha=1$ line.

The relation between the anomalous dimension and the anomalous spin is also confirmed by an explicit computation and by lifting the line operator (9) with $\alpha=-1$ to a different supersymmetric line operator in the $\mathcal{N}=2$ theory and repeating the analysis of its boundary operators.

As for the $\alpha=1$ case, here there are also four towers of $SL(2,\mathbb{R})$ primaries that are related by path derivatives. At the bottom we have the operators

$$\left\{\tilde{\mathcal{O}}^{(0,0)}_L,\tilde{\mathcal{O}}^{(0,1)}_L\right\} \quad \text{and} \quad \left\{\tilde{\mathcal{O}}^{(0,0)}_R,\tilde{\mathcal{O}}^{(0,-1)}_R\right\}. \tag{A2}$$

The operator on the line with the minimal dimension, $\tilde{\mathcal{O}}^{(0,0)}_L\times\tilde{\mathcal{O}}^{(0,0)}_R$, now has conformal dimension $\tilde{\Delta}^{\min}_{\text{inner}}=1-\lambda$. It is the unique relevant deformation of the $\alpha=-1$ line operator. In perturbation theory, turning this deformation on is equivalent to changing the coefficient in front of the biscalar condensate in (9). Doing so with a positive coefficient generates a flow between the $\alpha=-1$ and the $\alpha=1$ line operators. Deforming by it with the opposite sign generates a flow to an (almost) trivial line. The dual picture of this flow is discussed in the Appendix section "The condensed fermionic line operator."

Finally, the displacement operator now takes the form

$$\begin{aligned}\tilde{\mathbb{D}}_+&=+4\pi\lambda\tilde{\mathcal{O}}^{(0,0)}_R\tilde{\mathcal{O}}^{(0,1)}_L\\ \tilde{\mathbb{D}}_-&=+4\pi\lambda\tilde{\mathcal{O}}^{(0,-1)}_R\tilde{\mathcal{O}}^{(0,0)}_L\end{aligned}\qquad \text{for } \alpha=-1, \tag{A3}$$

and the evolution equation is modified accordingly.

Line operator with one degree of freedom: In this Appendix, we construct a conformal line operator with a worldline fermion. Equivalently, we add a two-dimensional Hilbert space on the line that carries the transverse spin $1/2$ and is coupled nontrivially to the CS-matter fields.

The line operators with $\alpha=\pm1$ (9) have boundary operators with the same anomalous spins and the same absolute value of anomalous dimensions. As a result, they can be combined to generate a new conformal line operator. It is defined as

$$\mathbb{W}[\mathcal{C},n]\equiv\left[\mathcal{P}e^{i\int_{\mathcal{C}}dx^\mu(A_\mu\mathbb{I}+i\frac{2\pi\lambda}{N}\phi^\dagger\phi\sigma_\mu)}\right]_n, \tag{A4}$$

where we have introduced a two-dimensional spin $1/2$ space on the line. At the upper component (spin $+1/2$) we have the $\alpha=1$ connection and the $\alpha=-1$ at the lower component (spin $-1/2$). For a straight line, this operator simply factorized into the $\alpha=1$ and $\alpha=-1$ line operators. However, as we deform away from the straight line, the coupling $dx\cdot\sigma$ in (A4) couples the two nontrivially. Correspondingly, the displacement operator contains an off-diagonal component

$$\begin{aligned}\mathbb{D}_-&=4\pi\lambda\begin{pmatrix}-\mathcal{O}^{(0,0)}_R\mathcal{O}^{(0,-1)}_L & \frac{1}{\sqrt{2}}\delta_3(\mathcal{O}^{(0,0)}_R\tilde{\mathcal{O}}^{(0,0)}_L)\\ 0 & \tilde{\mathcal{O}}^{(0,-1)}_R\tilde{\mathcal{O}}^{(0,0)}_L\end{pmatrix},\\ \mathbb{D}_+&=4\pi\lambda\begin{pmatrix}-\mathcal{O}^{(0,1)}_R\mathcal{O}^{(0,0)}_L & 0\\ \frac{1}{\sqrt{2}}\delta_3(\tilde{\mathcal{O}}^{(0,0)}_R\mathcal{O}^{(0,0)}_L) & \tilde{\mathcal{O}}^{(0,0)}_R\tilde{\mathcal{O}}^{(0,1)}_L\end{pmatrix}.\end{aligned} \tag{A5}$$

On the diagonal we see the displacement operators (19) and (A3). On the off-diagonal we have the longitudinal derivative of the operators $\mathcal{O}^{(0,0)}_R\tilde{\mathcal{O}}^{(0,0)}_L$ and $\tilde{\mathcal{O}}^{(0,0)}_R\mathcal{O}^{(0,0)}_L$, respectively. For these, the cancellation of the anomalous dimensions between the left and right operators comes about due to the flip in the sign of α, $\Delta^{(0,0)}=(1+\lambda)/2$, $\tilde{\Delta}^{(0,0)}=(1-\lambda)/2$. Moreover, their ±1 spin now arises due to the flip in the components of the two-dimensional space on the line. The displacement operator (A5) can also be shown to factorize into left times right (two components) boundary operators and will be considered in more detail in [31].

The condensed fermionic line operator: The bosonic theory also has the conformal line operator with $\alpha=-1$ and we can ask what is its dual fermionic description? A hint comes from looking at the spectrum of boundary operator at $\lambda_b=1$. According to $\alpha=-1$ spectrum and (17), in the free fermionic theory, we expect to have right and left boundary operators of dimension zero and spins $s_L=-s_R=\frac{1}{2}$, respectively. Another clue comes from the form of the displacement operator (A3) that factorizes at tree level to a dimension zero times a dimension two boundary operator.

The corresponding conformal line operator is somewhat unusual, having integrated fundamental fields in the exponent that interpolate between regions of the line with and without a Wilson line, see Fig. 2. To write it in a compact form, we introduce a two-dimensional space on the line [66]. Its upper (lower) component stands for the regions

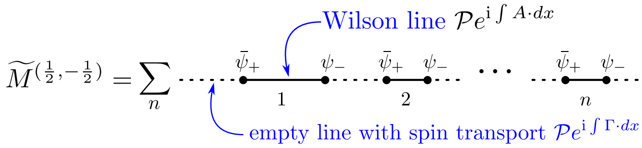

FIG. 2. The condensed fermionic line operator is defined perturbatively by integrating fundamental and antifundamental fermions along the path, (A7). Neighboring ordered pairs of fundamental and an antifundamental are connected by Wilson lines. In between these pairs we have a topological transport of the spin component (A9). We call these segments "*empty*" because they do not have fields inserted in them.

with (without) a Wilson line. Using this convention, the mesonic line operator takes the form

$$\tilde{M}^{(\frac{1}{2},-\frac{1}{2})}[x(\cdot)]=\left[\mathcal{P}e^{i\int\tilde{\mathcal{A}}_\mu\dot{x}^\mu ds}\right]_{22}, \tag{A6}$$

where $\tilde{\mathcal{A}}(s)$ is the 2×2 matrix

$$\tilde{\mathcal{A}}_\mu\equiv\begin{pmatrix} A_\mu & iP^-_\mu\psi \\ -i\frac{4\pi}{k}\bar{\psi}P^-_\mu & \frac{4\pi}{k}\frac{1}{\epsilon}\gamma_\mu+\Gamma_\mu\end{pmatrix}, \tag{A7}$$

and in (A6) we have taken the 2-2 component of the matrix. Here,

$$P^\pm_\mu(s)\equiv\frac{1}{2}(e_\mu(s)\pm\gamma_\mu),\quad \text{with}\quad e=\dot{x}/|\dot{x}|, \tag{A8}$$

is a projector to the spinor $\pm$ ($\mp$) component on the right (left). The term proportional to $1/\epsilon$ is a counterterm for subtracting a power divergence, with ϵ being a point splitting regulator on the line. Finally, $\Gamma_\mu\dot{x}^\mu$ is a spinor connection that is responsible for a topological transporting of the $\pm$ spinorial component along the empty regions. It is given by,

$$\Gamma_\mu\dot{x}^\mu=-\frac{i}{2}\epsilon_{\mu\nu\rho}(e^\mu\dot{e}^\nu\gamma^\rho-\dot{n}^\mu n^\nu e^\rho e\cdot\gamma). \tag{A9}$$

The four towers of boundary operators are obtained by taking path derivatives of the four operators

$$\left\{\tilde{\mathcal{O}}_L^{(0,\frac{1}{2})},\tilde{\mathcal{O}}_L^{(0,\frac{3}{2})}\right\}\quad\text{and}\quad\left\{\tilde{\mathcal{O}}_R^{(0,-\frac{3}{2})},\tilde{\mathcal{O}}_R^{(0,-\frac{1}{2})}\right\}, \tag{A10}$$

with $\tilde{M}^{((3/2),-(3/2))}$ defined as

$$\tilde{M}^{(\frac{3}{2},-\frac{3}{2})}[x(\cdot)]=\tilde{\mathcal{O}}_L^{(0,\frac{3}{2})}\left[\mathcal{P}e^{i\int\tilde{\mathcal{A}}_\mu\dot{x}^\mu ds}\right]_{11}\tilde{\mathcal{O}}_R^{(0,-\frac{3}{2})}, \tag{A11}$$

and

$$\tilde{\mathcal{O}}_L^{(0,\frac{3}{2})}=D_+(\bar{\psi}P^+_\nu e^\nu_L)/\sqrt{N},$$
$$\tilde{\mathcal{O}}_R^{(0,-\frac{3}{2})}=D_-(e^\rho_R P^-_\rho\psi)/\sqrt{N}. \tag{A12}$$

We have repeated the derivation of the boundary dimensions and the evolution equation for this condensed fermion operator. The results match those of the $\alpha=-1$ operator, with the replacement of $\lambda_b\to 1+\lambda_f$ in the spectrum, and $\lambda_b\to\lambda_f$ in the displacement operator, (A3). The operator in (A6) also has a lift into a locally supersymmetric line operator in the $\mathcal{N}=2$ theory. The corresponding circular 1/2 BPS operator was considered in the context of quiver gauge theories, with the fermion in the bifundamental representation [67–69]. The lift of the operator in (A6) is obtained by taking the rank of one of the gauge groups to one. This limit was discussed previously in [70]. The resulting anomalous spin is given by (17), with the tree level spin being shifted by one half with respect to the bosonic theory. The operators (A10) match with the ones in (A2) and the rest are related to these by path derivatives.

Nonunitary conformal line operators: The conformal line operator defined in (A6) and (A7) has the components of the fermion ψ_- and $\bar{\psi}_+$ condensed in the exponent. We can instead use the components ψ_+ and $\bar{\psi}_-$ as

$$\check{\mathcal{A}}_\mu\equiv\begin{pmatrix} A_\mu & iP^+_\mu\psi \\ -i\frac{4\pi}{k}\bar{\psi}P^-_\mu & \frac{4\pi}{k}\frac{1}{\epsilon}\gamma_\mu+\Gamma_\mu\end{pmatrix}, \tag{A13}$$

The resulting line operator is also conformal. The spectrum of boundary operators is related to the ones of the line operator (A7) by flipping the tree level spins as well as the anomalous dimensions

$$\check{\Delta}_R^{(n,s)}=\begin{cases}|\frac{1}{2}-s|+n+\lambda/2 & s\le\frac{1}{2}\\ |\frac{1}{2}+s|+n-\lambda/2 & s\ge\frac{3}{2}\end{cases}, \tag{A14}$$

and

$$\check{\Delta}_L^{(n,s)}=\begin{cases}|\frac{1}{2}+s|+n+\lambda/2 & s\ge-\frac{1}{2}\\ |\frac{1}{2}-s|+n-\lambda/2 & s\le-\frac{3}{2}\end{cases}. \tag{A15}$$

The anomalous spin is unchanged and is given by (17). At the bottom of these four towers we now have the operators

$$\left\{\check{\mathcal{O}}_L^{(0,-\frac{1}{2})},\check{\mathcal{O}}_L^{(0,-\frac{3}{2})}\right\}\quad\text{and}\quad\left\{\check{\mathcal{O}}_R^{(0,\frac{3}{2})},\check{\mathcal{O}}_R^{(0,\frac{1}{2})}\right\}. \tag{A16}$$

The corresponding displacement operator is

$$\check{\mathbb{D}}_+=-4\pi\lambda\check{\mathcal{O}}_R^{(0,\frac{3}{2})}\check{\mathcal{O}}_L^{(0,-\frac{1}{2})},\qquad\check{\mathbb{D}}_-=-4\pi\lambda\check{\mathcal{O}}_R^{(0,\frac{1}{2})}\check{\mathcal{O}}_L^{(0,-\frac{3}{2})}. \tag{A17}$$

If we quantize the theory radially in the presence of a straight conformal line operator, then reflection positivity restricts the dimensions of boundary operators to be positive [71]. The dimensions of the operators $\check{\mathcal{O}}_L^{(0,-(1/2))}$ and $\check{\mathcal{O}}_R^{(0,(1/2))}$ are, however, negative, (given by $\lambda/2$ of the fermionic theory). This is because conjugation in radial quantization relate ψ_- to $\bar{\psi}_+$ and ψ_+ to $-\bar{\psi}_-$. As a result, the line operator with (A7) is unitary while the one with (A13) is not.

*Corresponding author.
zdlzdlzdl@gmail.com

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[34] The displacement operator displaces the contour in an orthogonal direction. It is defined in equation (18).
[35] Here, $A_\mu = A_\mu^I T^I$, with T^I generators of the gauge group in the fundamental representation, with $\mathrm{tr}(T^I T^J) = \frac{1}{2}\delta^{IJ}$, and $\epsilon^{123} = 1$ is the antisymmetric tensor.

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[39] Legendre transform here represents a double trace deformation triggered by $(J^{(0)})^2$, where $J^{(0)}$ is given by $\phi_i^\dagger\phi^i$ in the bosonic case, and by $\bar{\psi}_i\psi^i$ in the fermionic case. Details can be found in e.g., [40].

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