

## Gravitational Bremsstrahlung with Tidal Effects in the Post-Minkowskian Expansion

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We compute the mass and current quadrupole tidal corrections to the four-momentum and energy flux radiated during the scattering of two spinless bodies, at leading order in  $G$  and at all orders in the velocities, using the effective field theory worldline approach. In particular, we derive the conserved stress-energy tensor linearly coupled to gravity generated by the two bodies, including tidal fields, and the waveform in direct space. The integral is solved using scattering amplitude techniques. We show that our expressions are consistent with existing results up to the next-to-next-to-leading order in the post-Newtonian expansion.

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*Introduction.*—The direct detection of gravitational waves from binary black holes [1] and neutron stars [2] has opened an new way to test gravity in the strong-field regime [3] and explore fundamental physics [4]. An important target of current and future observations is the measurement of tidal deformations during the coalescence of compact objects [5–15], which may shed light on the internal structure of neutron stars [16], the nature of black holes [17], or the existence of more exotic astrophysical objects [18–20].

Tidal deformations affect the conservative two-body dynamics as well as the emitted energy in gravitational waves. They have been studied using different analytical techniques, most notably the post-Newtonian (PN) expansion [21–26], the effective one-body approach [27–29], nonrelativistic general relativity [30–35], and the self-force formalism [36–39] (see Ref. [40] for a review).

Another technique that has been employed to study the gravitational two-body problem is the post-Minkowskian (PM) method [41–49], consisting of expanding the gravitational dynamics for small interactions, while keeping the velocities fully relativistic. It has been recently subject of great interest and activity, in particular in association with the effective one-body approach [48,50–55], scattering amplitude techniques [56–73], and worldline approaches [74–88]. Tidal effects have been studied with the PM expansion in [89–100]. These developments concern the scattering of two bodies moving on unbounded orbits but computed observables can be extended to the case of bound orbits by applying the so-called “boundary-to-bound”

(B2B) dictionary, consisting of an analytic continuation between hyperbolic and elliptic motion [101–104].

A long-standing and, until recently, unsolved problem was the calculation of the four-momentum radiated in gravitational waves—the so-called gravitational Bremsstrahlung—during the scattering of two spinless bodies, at leading PM order, i.e., at  $\mathcal{O}(G^3)$ . This was finally obtained very recently in [105,106] via the amplitude-based method of [60], in [70] using the eikonal approach, and in [107] by a classical effective field theory worldline approach. (See also Refs. [97,108–117] for previous work on radiation effects. Earlier pioneering studies include [47,118–123]. Moreover, see Refs. [124,125] for conservative and radiative effects in QED.) Crucially, these calculations strongly benefited from several computational tools developed in the high-energy community [126], such as reduction to master integrals by integration-by-parts identities [127–129] and differential equations [130–133] to solve the latter using the near-static regime as initial conditions.

In particular, in [107] two of us showed that it is possible to use these tools to directly compute radiated observables in the PM expansion without going through the classical limit of scattering amplitudes. Indeed, the emitted four-momentum was obtained by phase-space integration of the graviton momentum weighted by the modulo squared of the classical radiation amplitude [116,117], the latter being derived by matching to the conserved stress-energy tensor linearly coupled to gravity, generated by localized sources. The phase-space integral was then recast as a two-loop integral that we solved with the aforementioned techniques.

In this Letter, we use the same approach but we go beyond the minimally coupled case, and we compute for the first time the effect of tidal deformations on the four-momentum radiated into gravitational waves during the scattering of the two bodies. From this, extending the technique recently developed in [104], we also compute the tidal corrections to the emitted energy flux, which

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is valid for both open and closed orbits. We focus on the leading tidal contributions to the orbital dynamics, i.e., to quadrupolar deformations, but the extension to higher multipoles can be straightforwardly obtained using the same approach.

This Letter is organized as follows. We first define the Feynman rules in the case of tidal couplings, which will allow us to derive the stress-energy tensor linearly coupled to gravity, and the waveform in direct space, at leading PM order. From the stress-energy tensor, we compute, using reverse unitarity, the total four-momentum radiated into gravitational waves, and from this the emitted flux. We then use the B2B dictionary [101–104] to check our results with PN derivations [26].

*Leading PM tidal effects.*—We consider the scattering of two gravitationally interacting spinless bodies with mass  $m_1$  and  $m_2$ , approaching each other from infinity. Using the mostly minus metric signature, setting  $\hbar = c = 1$  and defining the Planck mass as  $m_{\text{pl}} \equiv 1/\sqrt{32\pi G}$ , the total action describing the dynamics with tidal effects reads

$$S = -2m_{\text{pl}}^2 \int d^4x \sqrt{-g} R + S_{\text{pp}} + S_{\text{tidal}}. \quad (1)$$

At leading order in their size, the bodies are described by point-particle actions,

$$S_{\text{pp}} = - \sum_{a=1,2} \frac{m_a}{2} \int d\tau_a g_{\mu\nu}(x_a) \mathcal{U}_a^\mu(\tau_a) \mathcal{U}_a^\nu(\tau_a), \quad (2)$$

where  $\tau_a$  and  $\mathcal{U}_a^\mu \equiv dx_a^\mu/d\tau_a$  (with  $\mathcal{U}_a^\alpha \mathcal{U}_a^\alpha = 1$ ) are, respectively, the proper time and the four-velocity of body  $a$ . Note that we have used the Polyakov-like form of the action and fixed the *einbein* to unity, which simplifies the gravitational coupling to the matter sources [77,134,135].

Tidal effects are included by augmenting the point-particle action with nonminimal worldline couplings involving higher-order derivatives of the gravitational field [30]. At leading PM order, only *linear* tidal deformations, i.e., those whose response is linear in the external gravitational field, are relevant. These are described by couplings quadratic in the Weyl tensor  $C_{\mu\alpha\beta}$  evaluated at the particle position. The Weyl tensor can be decomposed in terms of the gravitoelectric and gravitomagnetic fields, defined as

$$E_{\mu\nu} \equiv C_{\mu\alpha\beta} \mathcal{U}^\alpha \mathcal{U}^\beta, \quad B_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\alpha\beta\gamma\mu} C_{\delta\nu}^{\alpha\beta} \mathcal{U}^\gamma \mathcal{U}^\delta, \quad (3)$$

where  $\epsilon_{\alpha\beta\gamma\mu}$  is the Levi-Civita tensor. At lowest order in derivatives, and restricting to parity-even operators for symmetry reasons, the action describing tidal deformations is given by

$$S_{\text{tidal}} = \sum_{a=1,2} \int d\tau_a (c_{E_a^2} E_{\mu\nu}^a E_a^{\mu\nu} + c_{B_a^2} B_{\mu\nu}^a B_a^{\mu\nu}), \quad (4)$$

where  $c_{E_a^2}$  and  $c_{B_a^2}$  are Wilson coefficients related to the relativistic Love numbers  $k_a^{(2)}$  and  $j_a^{(2)}$  [28], respectively, as  $c_{E_a^2} = \frac{1}{6} k_a^{(2)} R_a^5/G$ ,  $c_{B_a^2} = (1/32) j_a^{(2)} R_a^5/G$ , with  $R_a$  the radius of the object  $a$ . Tidal operators can be equally defined by replacing the Weyl tensor in Eq. (3) with the Riemann tensor: the difference can be removed by field redefinitions; see, e.g., Refs. [24,30,136]. Here, we will use the Riemann tensor because it leads to simpler calculations. In full generality, one could also add to Eq. (4) operators that include spatial derivatives, orthogonal to the worldline of the body, of the gravitoelectric or gravitomagnetic field, as well as time derivatives along the worldline [28]. Higher spatial derivatives describe higher-order multipolar deformations of the objects while time derivatives account for the time dependence of the Wilson coefficients; see, e.g., Refs. [7,34].

Following [107,117], our first goal is to compute the stress-energy tensor  $T^{\mu\nu}$  defined as the linear term sourcing the gravitational field in the effective action [30,137,138], i.e.,

$$\Gamma[x_a, h_{\mu\nu}] = - \frac{1}{2m_{\text{pl}}} \int d^4x T^{\mu\nu}(x) h_{\mu\nu}(x), \quad (5)$$

with  $h_{\mu\nu} \equiv m_{\text{pl}}(g_{\mu\nu} - \eta_{\mu\nu})$ , which includes contributions from both the bodies and the gravitational self-interactions. To do so, we use a matching procedure consisting in expanding the action [Eq. (1)] for small  $h_{\mu\nu}$  and computing perturbatively all Feynman diagrams with one external graviton. The stress-energy tensor is obtained by matching this result with the one computed using Eq. (5). To proceed, we need to introduce the Feynman rules.

Adding the usual de Donder gauge-fixing term to Eq. (1), from the quadratic part of the gravitational action one can extract the graviton propagator,

$$\text{-----}^{\mu\nu} \text{-----}^{\rho\sigma} = \frac{i}{k^2} \mathbb{P}_{\mu\nu\rho\sigma}, \quad (6)$$

where  $\mathbb{P}_{\mu\nu\rho\sigma} \equiv \eta_{\mu(\rho} \eta_{\sigma)\nu} - (\eta_{\mu\nu} \eta_{\rho\sigma}/2)$ . (The boundary conditions that specify the contour of integration in the complex  $k^0$  plane are discussed in [117].) Furthermore, expanding the Einstein-Hilbert action in Eq. (1) at cubic order we can extract the cubic graviton vertex.

We also need to find the Feynman rules coming from the interaction of gravity with the external sources, i.e., the two bodies. These are of two types: minimal and tidal. For the former, from Eq. (2) one sees that there is only one linear interaction vertex. As discussed in [117] (see also Refs. [77,78]), we isolate the powers of  $G$  by expanding the position and velocity of the bodies around straight trajectories, i.e.,

$$x_a^\mu(\tau_a) = b_a^\mu + u_a^\mu \tau_a + \delta^{(1)} x_a^\mu(\tau_a) + \dots, \quad (7)$$

$$u_a^\mu(\tau_a) = u_a^\mu + \delta^{(1)} u_a^\mu(\tau_a) + \dots, \quad (8)$$

where  $u_a$  is the (constant) asymptotic incoming velocity and  $b_a$  is the body displacement orthogonal to it,  $b_a \cdot u_a = 0$ , while  $\delta^{(1)} x_a^\mu$  and  $\delta^{(1)} u_a^\mu$  are respectively the deviation from the straight trajectory and constant velocity of body  $a$  at order  $G$ , induced by the gravitational interaction. With this expansion we obtain the usual Feynman rules for the leading and next-to-leading PM-order graviton coupling in the point-particle case [117], respectively represented by the diagrams

$$\begin{array}{c} \tau_a \xrightarrow{k} \\ \bullet \text{---} \text{wiggly line} \times \end{array}, \quad \begin{array}{c} \tau_a \\ \bullet \text{---} \text{wiggly line} \text{---} \times \\ \bullet \end{array}, \quad (9)$$

where a filled dot denotes a minimally coupled particle evaluated using the straight worldline and the cross attached to the wiggly line is there to remind us that there is no propagator attached to the straight worldline. Their explicit expressions can be found in [107,117].

Moreover, we need to provide the Feynman rules from tidal contributions. In this case, from Eq. (4) there is no tidal coupling linear in the graviton. Tidal couplings of two gravitons to the body can be directly computed from the action using

$$\begin{aligned} \mathcal{M}_{\mu\nu\alpha\beta}^{E_a}(\ell) &\equiv \frac{2\delta E_{\mu\nu}^a}{\delta h^{\alpha\beta}(\ell)} = \eta_{\mu\sigma}\eta_{\nu\rho} u_a^\sigma u_a^\rho \ell_\alpha \ell_\beta \\ &\quad + (\ell \cdot u_a)^2 \eta_{\alpha(\mu}\eta_{\nu)\beta} - 2(\ell \cdot u_a) u_a^\rho \eta_{\rho(\mu}\eta_{\nu)(\alpha} \ell_\beta), \\ \mathcal{M}_{\mu\nu\alpha\beta}^{B_a}(\ell) &\equiv \frac{2\delta B_{\mu\nu}^a}{\delta h^{\alpha\beta}(\ell)} = \frac{1}{2} l^\rho u_a^\sigma \epsilon_{\rho\sigma\alpha\mu} [\eta_{\nu)\beta} (\ell \cdot u_a) - \eta_{\nu)\rho} u_a^\rho \ell_\beta] \\ &\quad + (\alpha \leftrightarrow \beta), \end{aligned} \quad (10)$$

where we use the flat metric  $\eta_{\mu\nu}$  to raise and lower indices. At leading PM order one obtains

$$\begin{array}{c} \ell_2 \nearrow \text{wiggly line} \kappa\lambda \\ \tau_a \blacksquare \\ \ell_1 \searrow \text{wiggly line} \mu\nu \end{array} = -\frac{i\delta^2 S_{\text{tidal}}}{\delta h^{\mu\nu}(\ell_1)\delta h^{\kappa\lambda}(\ell_2)} \equiv V_{\mu\nu,\kappa\lambda}(\ell_1, \ell_2), \quad (11)$$

where

$$V_{\mu\nu,\kappa\lambda} = i \sum_{X=E,B} \sum_{a=1,2} \frac{c_X^2}{4m_{\text{Pl}}^2} \int d\tau_a e^{i(\ell_1+\ell_2)\cdot(b_a+u_a\tau_a)} \Pi_{\mu\nu,\kappa\lambda}^{X_a}, \quad (12)$$

with

$$\Pi_{\mu\nu,\kappa\lambda}^{X_a}(\ell_1, \ell_2) \equiv \mathcal{M}_{\mu\nu\alpha\beta}^{X_a}(\ell_1) \mathcal{M}_{\kappa\lambda}^{X_a\alpha\beta}(\ell_2). \quad (13)$$

On the left-hand side of Eq. (11), the square denotes a tidally coupled particle evaluated using the straight worldline. We have verified that our expression agrees with that which can be read off from the four-point amplitude at leading PM order obtained in Ref. [89].

*Stress-energy tensor with tidal effects.*—The stress-energy tensor needed to compute the emitted four-momentum is given by the sum of the point-particle and tidal contributions, i.e.,

$$\tilde{T}^{\mu\nu} = \tilde{T}_{\text{pp}}^{\mu\nu} + \tilde{T}_{\text{tid}}^{\mu\nu}, \quad (14)$$

where the tilde denotes the Fourier transform,  $\tilde{T}^{\mu\nu}(k) = \int d^4x T^{\mu\nu}(x) e^{ik\cdot x}$ . The stress-energy tensor in the point-particle case was computed in [107,117] (see also Ref. [75]). Discarding the static contribution, which does not enter the calculation, the leading-order diagrams are represented in Fig. 1. Using the notation  $\int_q \equiv \int [d^4q/(2\pi)^4]$  and  $\delta^{(n)}(x) \equiv (2\pi)^n \delta^{(n)}(x)$ , it can be written as

$$\begin{aligned} \tilde{T}_{\text{pp}}^{\mu\nu}(k) &= \frac{m_1 m_2}{4m_{\text{Pl}}^2} \int_{q_1, q_2} \delta(q_1 \cdot u_1) \delta(q_2 \cdot u_2) \delta^{(4)}(k - q_1 - q_2) \\ &\quad \times \frac{e^{iq_1 \cdot b_1 + iq_2 \cdot b_2}}{q_1^2 q_2^2} [t_{\text{I}}^{\mu\nu}(q_1, q_2) + t_{\text{V}}^{\mu\nu}(q_1, q_2) + t_{\text{II}}^{\mu\nu}(q_1, q_2)], \end{aligned} \quad (15)$$

where  $t_{\text{I}}^{\mu\nu}$  is the contribution from diagram (a),  $t_{\text{V}}^{\mu\nu}$  from the same diagram but with the two particles exchanged and  $t_{\text{II}}^{\mu\nu}$  from (b). We refer the reader to Ref. [107,117] for their explicit expressions.

The contribution of the tidal operators to the stress-energy tensor has no static piece. The leading PM term can be obtained from the diagram (c) in Fig. 1 and it is symmetric under exchange of the two particles. We obtain

$$\begin{aligned} \tilde{T}_{\text{tid}}^{\mu\nu} &= \frac{m_1 m_2}{4m_{\text{Pl}}^2} \int_{q_1, q_2} \delta(q_1 \cdot u_1) \delta(q_2 \cdot u_2) \delta^{(4)}(k - q_1 - q_2) \\ &\quad \times \frac{e^{iq_1 \cdot b_1 + iq_2 \cdot b_2}}{q_1^2 q_2^2} \sum_{a=1,2} \sum_{X=E,B} t_{X_a}^{\mu\nu}(q_1, q_2), \end{aligned} \quad (16)$$

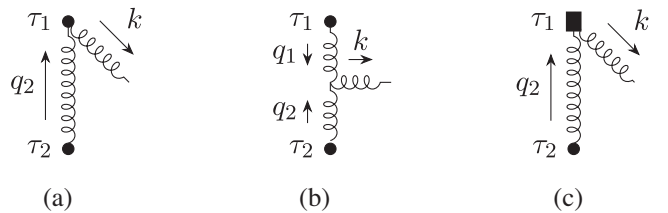


FIG. 1. The Feynman diagrams needed for the computation of the stress-energy tensor: (a) and (b) are the point-particle contributions, and (c) is the tidal one. The symmetric terms are obtained by exchange of  $1 \leftrightarrow 2$ .





TABLE I. Functions specifying the radiated four-momentum in Eq. (19).

$$\begin{aligned}
 f_1^E &= \{1/[2(\gamma+1)^3\sqrt{\gamma^2-1}]\}[937\gamma^9 + 1551\gamma^8 - 2463\gamma^7 - 5645\gamma^6 + 20415\gamma^5 + 65965\gamma^4 - 349541\gamma^3 + 535057\gamma^2 - 360356\gamma + 92160] \\
 f_1^B &= \{(\gamma-1)/[4(\gamma+1)^3\sqrt{\gamma^2-1}]\}[1559\gamma^8 + 3716\gamma^7 - 1630\gamma^6 - 11660\gamma^5 - 28288\gamma^4 + 155292\gamma^3 - 543442\gamma^2 + 535212\gamma - 180775] \\
 f_2^E &= 30\sqrt{\gamma^2-1}(21\gamma^4 - 14\gamma^2 + 9) \\
 f_2^B &= 210(\gamma^2-1)^{3/2}(1+3\gamma^2), \quad f_3^X = -f_2^X[\gamma(2\gamma^2-3)/4(\gamma^2-1)] \\
 \mathcal{F}^E &= \{3(\gamma-1)^2/[(\gamma+1)^3\sqrt{\gamma^2-1}]\}[42\gamma^8 + 210\gamma^7 + 315\gamma^6 - 105\gamma^5 - 944\gamma^4 - 1528\gamma^3 + 22011\gamma^2 - 33201\gamma + 16272] \\
 \mathcal{F}^B &= -\{3(\gamma-1)^3(105\gamma^5 + 630\gamma^4 + 1840\gamma^3 + 3690\gamma^2 - 17769\gamma + 15984)/[(\gamma+1)^3\sqrt{\gamma^2-1}]\}
 \end{aligned}$$

where

$$\mathcal{G}(\mathcal{E}^X, \mathcal{F}^X) \equiv \sum_X [\kappa_{X^2} \mathcal{E}^X + \lambda_{X^2} (\mathcal{F}^X - \mathcal{E}^X)], \quad (23)$$

and we have introduced the dimensionless parameters [95]

$$\lambda_{X^2} \equiv \frac{1}{G^4 m^5} \left( \frac{c_{X_1^2} m_2}{m_1} + \frac{c_{X_2^2} m_1}{m_2} \right), \quad (24)$$

$$\kappa_{X^2} \equiv \frac{1}{G^4 m^4} \left( \frac{c_{X_1^2}}{m_1} + \frac{c_{X_2^2}}{m_2} \right). \quad (25)$$

Expanding for small velocities  $v \equiv \sqrt{\gamma^2-1}/\gamma$ , we find

$$\begin{aligned}
 \mathcal{E}^E &= 288v^3 + \frac{2143}{7}v^5 + \frac{14542}{21}v^7 + \mathcal{O}(v^9), \\
 \mathcal{E}^B &= -98v^5 + \frac{585}{4}v^7 + \mathcal{O}(v^9), \\
 \mathcal{F}^E &= 288v^3 + 336v^5 + \frac{3027}{4}v^7 + \mathcal{O}(v^9), \\
 \mathcal{F}^B &= -210v^5 - \frac{669}{4}v^7 + \mathcal{O}(v^9), \quad (26)
 \end{aligned}$$

which shows that the current (magnetic) quadrupole is 1PN order higher than the mass (electric) one, as expected.

Finally, the emitted energy from a two-body encounter can be used to derive the energy loss for closed orbits by the use of the B2B relation [101–104],  $\Delta E^{(\text{closed})}(\gamma, J) = \Delta E^{(\text{open})}(\gamma, J) - \Delta E^{(\text{open})}(\gamma, -J)$ , where the emitted energy on the right-hand side must be expressed in terms of the angular momentum  $J = |\mathbf{b}|m\nu\sqrt{\gamma^2-1}/h(\nu, \gamma)$  (with  $h = E/m$ ) and analytically continued to bound orbits with  $h < 1$ , corresponding to  $\gamma = 1 + [(h^2 - 1)/2\nu] < 1$ . This yields

$$\Delta E_{\text{tid}}^{(\text{closed})} = \frac{15\pi G^7 m^{15} \nu^9 (1-\gamma^2)^{7/2}}{64J^7 h^8} \mathcal{G}(\tilde{\mathcal{E}}^X, \tilde{\mathcal{F}}^X), \quad (27)$$

where  $\tilde{\mathcal{E}}^X = -2\mathcal{E}^X$  and  $\tilde{\mathcal{F}}^X = -2\mathcal{F}^X$ , with  $\mathcal{E}^X$  and  $\mathcal{F}^X$  subject to the replacement  $(\gamma^2-1)^{n/2} \rightarrow (1-\gamma^2)^{n/2}$ . In the following we show that this expression is consistent with known results in the PN approximation.

*Radiated flux.*—The instantaneous flux is defined as  $F \equiv dE/dt$ . Focussing on the tidal correction,  $F_{\text{tid}}$ , and integrating this relation for half of the scattering trajectory, we obtain

$$\Delta E_{\text{tid}}(\gamma) = \int_{|b|}^{\infty} \frac{dr}{\dot{r}} F_{\text{tid}}(r, \gamma). \quad (28)$$

We have assumed that the expression of the flux is in isotropic gauge; thus, we have dropped the dependence on  $J$  in  $F_{\text{tid}}$ . From Eq. (22), the leading-order tidal contribution to the flux scales as  $G^7$  so that its dependence on  $r$  is fully determined:  $F_{\text{tid}}(r, \gamma) \propto r^{-8}$ . By integrating the right-hand side of Eq. (28) with this ansatz, and using  $\dot{r}$  for straight orbits at this PM order, we find

$$F_{\text{tid}}(r, \gamma) = \frac{G^7 m^8 3\nu^3 \sqrt{\gamma^2-1}}{r^8 4h^3 \xi} \mathcal{G}(\mathcal{E}^X, \mathcal{F}^X), \quad (29)$$

where  $\xi \equiv E_1 E_2 / E$ , and  $E_a$  is the initial asymptotic energies of body  $a = 1, 2$ . This result extends the one for point-particles computed in [104]. As discussed there, due to the absence of a term higher in  $G$ , the leading PM computation is insufficient to reconstruct the leading PN flux but it provides the full velocity—or reduced-energy—series to order  $G^3$ .

*Consistency check.*—We can compare our result for small velocities to the emitted flux and energy in one period derived in the PN expansion in the large eccentricity limit, i.e., to leading order in large  $J$ .

The tidal effects on the gravitational wave energy flux for spinless bodies has been computed up to the next-to-next-to-leading PN order in [26] (see Refs. [24,25] for a derivation of the equations of motion and Hamiltonian in this case, respectively; see also Ref. [97] for a calculation of the PM Hamiltonian and the emitted energy for quasicircular orbits at leading PN order, with interactions cubic in the curvature and tidal effects). Although in that reference the results were given only for quasicircular orbits, their authors have kindly provided us with an expression of the flux  $F_{\text{tid}}^{(\text{PN})}$  and the conserved energy  $E$  and angular momentum  $J$  for generic orbits, written in terms of  $r$ ,  $\dot{r}$ , and  $\dot{\phi}$ , respectively the two-body distance, the radial velocity, and the angular velocity in the

center-of-mass frame. We have used the expressions for  $E$  and  $J$  to replace  $\dot{r} = \dot{r}(r, E, J)$  and  $\dot{\phi} = \dot{\phi}(r, E, J)$  in the flux and we have computed the emitted energy for *generic closed orbits* by integrating it in the variable  $r$  over one period.

The resulting energy reduces to that given in [26] for circular orbits. Moreover, it is consistent with the expansion Eq. (26) [taking into account the factor of  $-2$  according to Eq. (27)]. Since all the powers of  $\gamma$  in Table I intervene in this expansion, this is a rather nontrivial check of our calculation. Moreover, the PN flux  $F_{\text{tid}}^{(\text{PN})}$  coincides with the low-velocity expansion of Eq. (29), up to total derivatives in the balance equations—the so-called Schott terms. Although the two fluxes are written in different gauges [in harmonic and isotropic gauge, respectively, in Ref. [26] and in Eq. (29)] the gauge difference is 2PM orders higher and can be neglected. For the reader's convenience, we report the explicit expression of the PM flux in the ancillary file submitted with the arXiv version of this article.

*High-energy limit.*—Going back to the energy loss for hyperboliclike orbits, Eq. (22), for large  $\gamma$  we find  $\mathcal{E}_{\text{HE}}^X = (a_X + b_X \log \gamma) \gamma^5 + \mathcal{O}(\gamma^3)$  and  $\mathcal{F}_{\text{HE}}^X = c_X \gamma^6 + d_X \gamma^4 + \mathcal{O}(\gamma^2)$ , with  $a_E = 937/2 - 945 \log 2$ ,  $a_B = 1559/4 - 945 \log 2$ ,  $b_E = b_B = 315$ ,  $c_E = 126$ ,  $c_B = 0$ ,  $d_E = -504$ , and  $d_B = -315$ . While  $\mathcal{E}_{\text{HE}}^E$  and  $\mathcal{E}_{\text{HE}}^B$  scale in the same way with  $\gamma$ ,  $\mathcal{F}_{\text{HE}}^E$  and  $\mathcal{F}_{\text{HE}}^B$  behave differently. Our perturbative expansion is valid for  $\gamma(Gm/|\mathbf{b}|) \ll 1$  [54,149,150] (see also Ref. [122]). In this regime  $\Delta E_{\text{tid}} \ll \Delta E \sim (Gm/|\mathbf{b}|)^3 (m/h) \gamma^3 \ll E$ .

*Conclusion.*—We have computed the four-momentum and the flux emitted in gravitational waves by the scattering of tidally interacting bodies at leading order in the post-Minkowskian approximation. Our computation uses the worldline effective field theory approach and the results obtained are, to our knowledge, new. We focused on electric and magnetic-type quadrupolar effects but our computations can be straightforwardly extended to higher multipoles or to higher orders in the curvature fields.

We have derived the emitted energy for bound orbits using the B2B dictionary and verified that it is consistent with PN results for eccentric orbits. Considering the ultrarelativistic limit of the energy loss, we observe that the contributions of the electric and magnetic component scale differently unlike the case of the conservative scattering angle. It would be interesting to use the derived PM flux to study the corresponding modifications of the waveform.

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