Doppler Gyroscopes: Frequency vs Phase Estimation

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We consider the fundamental roles of frequency versus phase in parameter estimation, specifically in the Sagnac effect. We describe a novel, ultrasensitive gyroscope based on the extremely steep frequency-dependent gain of a liquid crystal light valve. We provide compelling experimental evidence that the Doppler shift is fundamental in the Sagnac effect giving clarity to a long-debated question. We experimentally show orders of magnitude improvement in sensitivity relative to the standard quantum limit of a gyroscope based on phase estimation.

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The burgeoning field of quantum metrology seeks to find "quantum advantages" over existing classical measurement schemes [1–12]. Owing to its importance in gyroscopes [2] and gravitational wave detection [1,13] as well as its fundamental nature in all branches of interferometry, phase estimation beyond the standard quantum limit has been the prototypical example [1–3,5,7,12]. Pragmatically, due to loss, quantum phase estimation techniques have, so far, only offered a few percent improvement over the standard quantum limit in the few-photon regime [10] or a few dB improvement in the high power regime [9,11]. However, what if phase estimation for a class of experiments is suboptimal? Depending on the measurement apparatus, phase estimation may have different fundamental limits than frequency estimation [14]. Here, we demonstrate that by using an ultra-steep, frequency-dependent gain measurement rather than performing phase estimation in a passive gyroscope, we can achieve orders of magnitude improvement below the phase-estimation standard quantum limit of a single-loop Sagnac interferometer of the same size. Further, we provide important insights into a long-debated question about the role of Doppler shifts in the Sagnac effect.

Gyroscopes are powerful tools in tests of fundamental physics, guidance systems, inertial navigation, accelerometry, geodesy, seismology, and geophysics to name a few. Significant advances in micromechanical [15], atomic [16–18], chip-based systems [19], and ring laser gyros [20–25] have been achieved. It may come as a surprise, then, that there are still open questions about the fundamental underpinnings of gyroscopes. For the sake of clarity, we consider an optical gyroscope to be a system in which a light source and a detector, in the same reference frame, rotating at a constant angular speed, can measure the rotation rate of the system. However, instead of measuring differential phase shifts, as is typically done, we measure differential frequency shifts.

The estimation of the rotation speed is closely related to a long-debated question of the role of Doppler shifts in the Sagnac effect [26]. In a standard Sagnac interferometer, the light is split using a beam splitter into two counterpropagating beams that return to the same beam splitter. Special relativity predicts an in-plane, closed-path relative phase acquired by the two beams in an optical Sagnac, given by

$$\Delta\phi = \frac{8\pi\Omega A}{\lambda c}, \qquad (1)$$

where Ω is the angular frequency, A is the gyroscope area, λ is the wavelength of light, and c is the speed of light. Hence, once the phase is known, the angular rotation frequency can be determined. The shot noise limited rotation sensitivity can then be found by substituting the standard quantum limit for phase, namely

$$\Delta\phi = \frac{1}{2\sqrt{N}}, \qquad (2)$$

where N is the number of photons.

An alternative theory is that the Doppler effect is the fundamental mechanism of the Sagnac effect (see Ref. [26]). However, Malykin elucidates two reasons against the use of the Doppler effect [26]. First, in a closed-loop system, the beam splitter plays both the role of the emitter and the detector. He states, "...the radiation source and detector must be in motion relative to each other if the Doppler effect is to be manifest." He implies that the net Doppler shift *outside* the closed interferometer is zero. However, we feel this is not a good argument against a Doppler shift model, since proponents argue that it is the Doppler shifts between the source and the emitter (leading to a differential phase) that matter and not what happens external to the interferometer. We go beyond this assertion by exploring symmetry-breaking designs where the detector is not the same element as the emitter, allowing us to measure differential frequency shifts even at the detector.

Malykin's second argument is that if there is a material medium, a Doppler-shift theory differs from the standard prediction by a factor of $2n^2$ where n is the index of refraction of the material. Even in vacuum, the result still differs by a factor of 2. The vacuum result is rectified by proponents of the Doppler effect by using the length of the interferometer in the interferometer's frame. However, Malykin points out that it is inconsistent to assume a Doppler shift in the lab frame and the loop length in the rotating frame.

Instead of resolving the theoretical inconsistencies, we endeavor to show that Doppler shifts do exist within the system. It is our opinion that measuring the phase alone leads to ambiguous interpretations. To parse out the role of the Doppler effect, we need a system that measures *only* differential frequency shifts, but *cannot* measure differential phase.

For our spectral estimation technique, we incorporate an ultrasensitive, wave-mixing spectrometer based on an extremely steep frequency-dependent gain to measure frequency offsets. We use a liquid crystal light valve (LCLV) to measure the relative Doppler shift of two counterpropagating paths, but not the relative phase. Two-beam interference in the crystal creates a self-induced index grating from which the beams scatter. The various scattering orders from the two-beam interference inside an LCLV can lead to phase-insensitive steep spectral gain dependence [14] or extremely slow or fast group velocities [27]. It was shown [14] that the shot noise limited spectral sensitivity is given by

$$\Delta f = \frac{1}{|\chi|\sqrt{N}}, \tag{3}$$

where $|\chi|$ is the slope of the spectral gain and N is the number of measured photons. Because the slope of the gain curve can be large, we can achieve very high spectral resolution. This is the type of "slow light" advantage that was sought for almost two decades [28–31].

Now, we consider the Doppler shift from each reflecting surface of a rotating interferometer, like the one shown in Fig. 1. We recently showed [32] that the Ashworth-Davies [33] nonrelativistic Doppler shift Δf_m of a laser of wavelength λ from a mirror moving with angular velocity Ω and radius R is given by

$$\Delta f_m = \frac{2\Omega R}{\lambda}\cos(\phi)\cos(\alpha), \tag{4}$$

where, with respect to the mirror's surface normal, α is the direction of propagation of the mirror and ϕ is the angle of incidence.

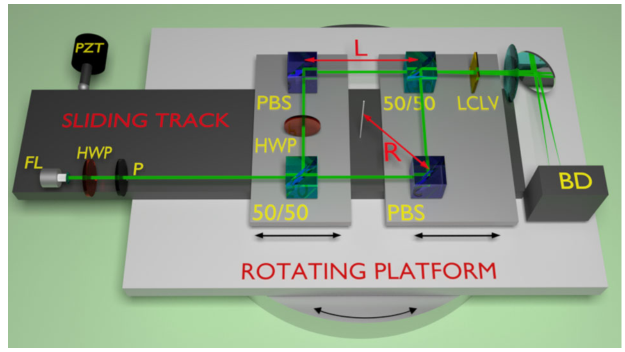

FIG. 1. Experimental setup. A laser beam is coupled into a square Mach-Zehnder interferometer on top of a rotating platform. The size and distance of the interferometer relative to the axis of rotation can be controlled by moving small tables with the beam splitters. A liquid crystal light valve acts as the relativistic "detector" by providing spectrally dependent gain, which is then measured by balanced detectors. All elements are on the rotating platform.

Consider a simple Doppler model in which the axis of rotation is at the center of a square Mach-Zehnder interferometer like the one shown in Fig. 1. The distance from the axis of rotation to each reflective element is then $R = L/\sqrt{2}$, where L is the length of a side of the square. In this scenario $\alpha = 0$ for the first 50/50 beam splitter and $\alpha = 90°$ for the two corner mirrors (we use polarizing beam splitters (PBS) effectively as mirrors). The final beam splitter has an angle $\alpha = \epsilon/2$, where $\epsilon \ll \phi$. The incidence angle is the same for all surfaces, namely $\phi = 45°$ for all surfaces, except the last beam splitter, which is $\phi = 45° + \epsilon/2$. In this scenario $\alpha = 0°$ for the first beam splitter $\alpha = \epsilon/2$ for the final beam splitter. Keeping only terms first order in ϵ, the relative Doppler shifts between the two paths are given by

$$\Delta f \approx \frac{\sqrt{2}L[\cos(\phi) - \cos(\phi + \epsilon/2)]}{\lambda}, \tag{5}$$

yielding the differential frequency approximation

$$\Delta f \approx \frac{L\Omega\epsilon}{2\lambda}. \tag{6}$$

The final beam splitter, with its small relative angle, breaks the symmetry in the system. It should also be noted that the frequency differential is zero when the angle between the output beams $\epsilon = 0$ as observed for closed systems. This model predicts, using parameters from our system (see Sec. IV in the Supplemental Material [34] for further information), an estimated shot noise limited rotation sensitivity down to $\Omega \approx 100$ pRad/s/Hz$^{-1/2}$ (approximately 5 orders of magnitude below the standard quantum limit for phase estimation of an interferometer of the same size and same number of measured photons).

For our experiment, light from a 532 nm fiber-coupled laser was launched on a rotation mount. A half-wave plate (HWP) and polarizer (P) were used to adjust the beam intensity and make the light vertically polarized (necessary for the LCLV). The 50/50 beam splitter and a polarizing beam splitter were placed on a small movable platform on top of a track. The other 50/50 beam splitter and the other polarizing beam splitter were placed on another movable platform on the same track. The two independently movable platforms allowed us to measure the sensitivity of the system versus the position of the axis of rotation and the interferometer's size. The two polarizing beam splitters were used in place of mirrors, allowing for amplitude control of the two paths. The beams were then directed from the polarizing beam splitters and the final 50/50 beam splitter such that they overlapped on the LCLV, but had a relative angle of approximately 10 mRad. The beam waists at the crystal were approximately 2.5 mm with combined intensity of between 1.5 and 2.5 mW impinging on the crystal. The beams then pass through a lens and are sent to a balanced detector (BD) in the focal plane of the lens where the beams have separated. The half-wave plate inside the Mach-Zehnder was adjusted until the beams were intensity-balanced in the detector. The differential balanced detector signal was passed through a low-noise preamp. From the fiber launch to the photodetectors, all equipment is on a single rotating platform atop a turntable driven by a piezoactuator. We note that we observe the effect with the laser also in the rotating platform frame.

To test the properties of this system, we used a linear piezoelectric actuator to exert a transverse force on the sliding track that also acts as a lever arm. The horizontal distance from the piezoactuator to the axis of rotation was 28 cm. The actuator has a linear response of approximately 60 nm/V. We used a range of amplitudes and frequencies for driving sinusoidal, triangular, and square wave oscillations. To demonstrate that we are measuring a frequency offset and not a phase offset, we show that constant phase offsets (*not* phase gradients) do not contribute to the LCLV response. It is important to note that for demonstrating that the system is not sensitive to a constant phase offset we are *not* using a setup in Fig. 1. Rather, the experimental setup for this experiment is based on [14] in which a mirror is linearly translated with respect to the lab frame in which the detector and laser are stationary. Figure 2(a) shows the response of the LCLV to sudden changes in phase (square wave driven piezo). The square wave peak-to-peak oscillation corresponds approximately to 17° of phase shift. It can be seen that the after an initial sudden change in phase (Doppler shift) the system relaxes once again to the equilibrium position (zero) even though the phase offset remains constant until the next fluctuation. This shows that the LCLV does not respond to constant phase offsets as would be the case in standard Sagnac interferometers undergoing uniform rotation.

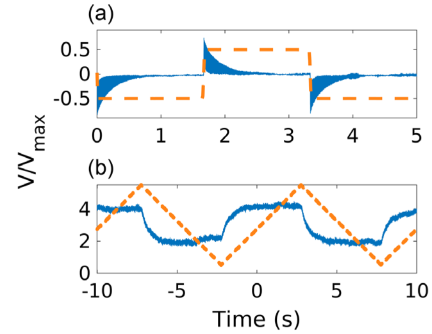

FIG. 2. LCLV response vs input signals. In (a), an experiment was performed similar to the one in [14] in which the laser and detector are in the laboratory frame and a mirror undergoes linear translations according to a piezoelectric input signal. However, the LCLV response always reequilibrates to zero, meaning the LCLV, after relaxing, has no response to constant phase. The experimental setup (b) uses the turntable shown in Fig. 1 unlike (a). A triangle wave is applied to the piezoactuator that rotates the turntable, yielding the expected square wave signal.

Hence, the signal must be from frequency shifts and not from phase.

We now turn to the results for the rotating platform shown in Fig. 1 undergoing small oscillatory motions. As can be seen in Fig. 2(b), a 100 mHz triangle wave with peak-to-peak angular displacement of approximately 1 μRad is applied to the piezoactuator. At the turning points, there is a sudden acceleration at which there is a rapid change in response. After a finite crystal relaxation time, the system settles to the new equilibrium. For a system of constant rotational speed, the system approaches a constant voltage offset proportional to the speed as expected.

Figure 3 shows the driving signal and the system response of a 0.003 m^2 interferometer, from a 200 mHz sine wave (a 20 second interval is shown in the inset). The driving signal resulted in a 1.3 μRad/s amplitude for the angular velocity. The length of the total measurement was

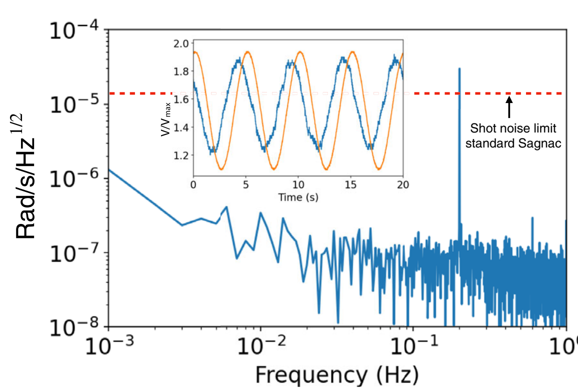

FIG. 3. Amplitude spectral density of the signal response (shown in blue in the inset) of a 0.003 m^2 interferometer undergoing 200 mHz sinusoidal oscillations from the driven piezoelectric.

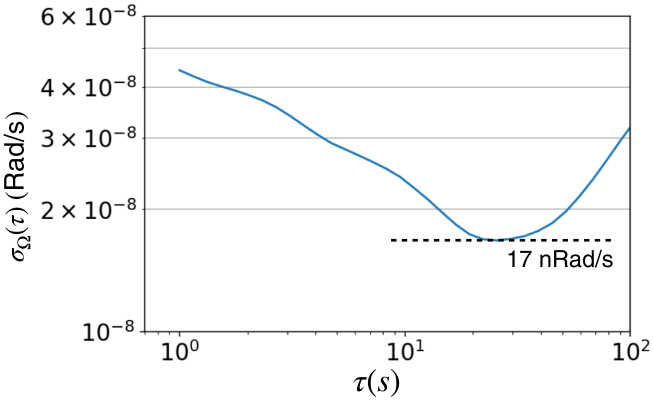

FIG. 4. Allan deviation for 0.003 m^2 interferometer. The bias drift is observed to be approximately 17 nRad/s.

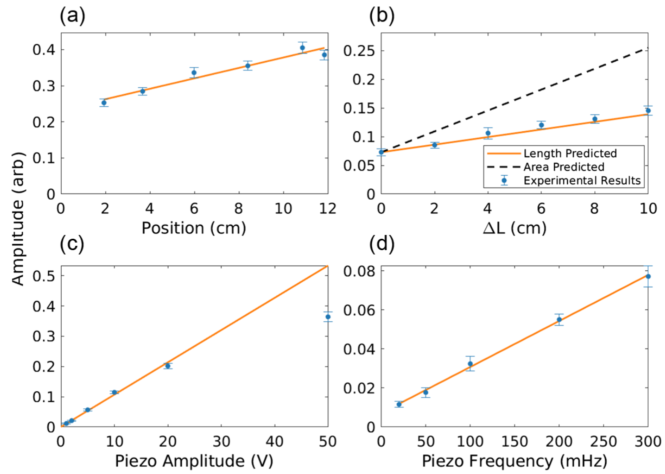

FIG. 5. (a) System response as a function of a 0.003 m^2 interferometer being moved relative to the axis of rotation. (b) Dependence of the interferometer sensitivity vs a linear change in length. It can be seen that the behavior shows that the system sensitivity is a function of the length and not the area of the interferometer. (c),(d) show that the system is linear in the piezo driving amplitude and frequency, respectively.

1000 seconds. The associated amplitude spectral density of the signal is shown. It can be seen that over a large range of the spectrum, the noise floor is more than 2 orders of magnitude below the shot noise limited (the standard quantum limit) response of a standard passive gyroscope of the same area and approaches 3 orders of magnitude close to 1 Hz. Figure 4 shows the Allan deviation. The bias drift of the system is calculated to be approximately 17 nRad/s. For emphasis, we note that we are still several orders of magnitude above the shot noise limit of the frequency differential measurement of the LCLV, implying that many orders of magnitude improvement are still possible.

We tested the dependence of the system on several parameters: the position of the axis of rotation, the size of the interferometer, and the amplitude and frequency of the piezoactuated movement. For the experiments shown in Fig. 5, triangle waves of various amplitudes and frequencies were used. In Fig. 5(a), the two movable platforms were translated together, while preserving the interferometer area, by a distance ΔL to find the system sensitivity to position of the interferometer relative to the axis of rotation. It can seen that there is a linear dependence on distance from the axis of rotation, thus differing from other passive optical gyroscopes. Figure. 5(b) shows the dependence of the interferometer's sensitivity on the size of the interferometer. We kept one platform fixed and moved the other platform a linear distance ΔL. We plot two theoretical behavior lines to show the system sensitivity is linear in the length and not the area, which also differs from other passive optical gyroscopes. Figures 5(c) and 5(d) show that the response of the system is linearly dependent in the amplitude and frequency of the driving oscillation, as expected. It can be seen in Fig. 5(c) that if the amplitude is too large, when the crystal gain curve is nonlinear, the linear behavior stops. Nonlinear behavior also occurred for oscillation frequencies above 1 Hz (not shown), which is approximately the bandwidth of the gain curve.

A brief discussion of some practical aspects is in order. First, the LCLV, in its current form, is very sensitive to vibrations. Second, the sensitivity and bias of the LCLV is a strong nonlinear function of the laser power. However, with intensities of even just a few tens of mW/cm^2, the transparent electrodes on the LCLV can overheat, causing a liquid phase transition. This is a technical not a fundamental limitation, which can be improved. Third, air current fluctuations create temporal phase fluctuations, which appear as frequency shifts in the system. Fourth, it is likely that bias drift can be greatly improved. We believe the primary forms of bias drift were from intensity fluctuations and crystal temperature variations, both of which can be actively stabilized. Fifth, we expect the sensitivity of our device can be greatly improved if it is used in a high finesse resonator or loop design. Sixth, there are passive fiber optic gyros with similar sensitivities to ours, but with enclosed areas tens to hundreds of thousands of times larger (see Ref. [35] and references therein).

The last and perhaps most important item is that we have not yet measured any signal related to the Earth's rotation. This should be manifest by a large frequency offset bias (expected to be several hundred thousand Hertz, assuming a linear increase in sensitivity with distance) and a sensitivity to interferometer tilt, neither of which we have observed. We are puzzled by this and are looking for answers to large distance rotations. We note that such a large bias would be so far outside the spectral gain window that the system could not work. This dilemma could point to (1) a flaw in the theory, (2) new physics, or (3) the system behaving like translational rather than rotational motion in the large distance limit. We hope this spurs additional theoretical activity in the community.

We believe we have provided compelling evidence of the possibility for enhancing sensitivity relative to some phase estimation problems by considering frequency estimation instead. We have also provided strong evidence of the role of Doppler shifts as being fundamental within the Sagnac effect. An obvious question is in what other systems such techniques could be applied. For example, it may be possible that ring laser gyros could be improved if the differential spectral estimation of the LCLV can be shown to be better than the typical heterodyne beatnote analysis.

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