Scalable Spin Squeezing from Spontaneous Breaking of a Continuous Symmetry

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Spontaneous symmetry breaking is a property of Hamiltonian equilibrium states which, in the thermodynamic limit, retain a finite average value of an order parameter even after a field coupled to it is adiabatically turned off. In the case of quantum spin models with continuous symmetry, we show that this adiabatic process is also accompanied by the suppression of the fluctuations of the symmetry generator —namely, the collective spin component along an axis of symmetry. In systems of $S = 1/2$ spins or qubits, the combination of the suppression of fluctuations along one direction and of the persistence of transverse magnetization leads to spin squeezing—a much sought-after property of quantum states, both for the purpose of entanglement detection as well as for metrological uses. Focusing on the case of XXZ models spontaneously breaking a U(1) [or even SU(2)] symmetry, we show that the adiabatically prepared states have nearly minimal spin uncertainty; that the minimum phase uncertainty that one can achieve with these states scales as $N^{-3/4}$ with the number of spins N; and that this scaling is attained after an adiabatic preparation time scaling linearly with N. Our findings open the door to the adiabatic preparation of strongly spin-squeezed states in a large variety of quantum many-body devices including, e.g., optical-lattice clocks.

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Introduction.—Many-body entanglement [1] is at the heart of the fundamental complexity of quantum states [2,3], and it is the basis mechanism by which a closed quantum many-body system relaxes to a stationary regime after having been driven away from equilibrium [4]. In this respect, the generation of many-body entangled quantum states is one of the main goals of a new generation of quantum devices, going from quantum simulators [5,6] to quantum computers [7], whose common trait is the ability to perform coherent unitary evolutions of quantum many-body systems. Yet, certifying (let alone putting to use) many-body entanglement is a task which is restricted to a small class of quantum many-body states, most prominently those whose entanglement can be detected via the measurement of the lowest moments of the quantum-noise distribution [8]. In the context of ensembles of N qubits ($S = 1/2$ spins), characterized by the collective-spin operator $\boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{S}_i$ (where the $\boldsymbol{S}_i$'s are $S = 1/2$ quantum spin operators), one of the best examples of such states is offered by *spin-squeezed* ones, whose entanglement is detected via the spin-squeezing parameter [9],

$$\xi_R^2 = \frac{N \min_\perp \mathrm{Var}(J^\perp)}{\langle \boldsymbol{J} \rangle^2}, \tag{1}$$

where $\min_\perp$ expresses the minimization of the variance on the collective-spin components perpendicular to the average spin orientation $\langle \boldsymbol{J} \rangle$. A state with $\xi_R^2 < 1/k$ ($k \geq 1$) is entangled, with an entanglement depth (least number of entangled spins) of $k + 1$ [10,11]. The ξ_R^2 parameter is also a fundamental figure of merit of the sensitivity of the state to rotations, expressing the reduction in phase-estimation error for a Ramsey interferometric protocol with respect to a factorized state [9], and it offers the possibility to improve the efficiency of quantum devices such as atomic clocks [12–15] or quantum sensors [16–18] by using entanglement as a resource.

For the reasons listed above, devising many-body mechanisms that lead to the controlled preparation of spin-squeezed states [19] is a very significant endeavor —and particularly so when the squeezing parameter can be parametrically reduced by increasing the number of resources. This situation leads to *scalable squeezing*—generically $\xi_R^2 \sim N^{-\alpha}$ ($\alpha > 0$)—which allows one to surpass the standard quantum limit for the scaling of the phase-estimation error with the number of qubits [11]. The two main mechanisms that have been identified and implemented so far in this direction are the preparation of spin-squeezed states via unitary evolutions with long-range interactions [20–24] and via the nondemolition measurement of a spin component [25,26]. Yet a third way to spin squeezing is offered by *adiabatic preparation*, which relies upon the identification of spin squeezing in the ground state of quantum spin Hamiltonians implemented, e.g., by quantum simulation platforms. A clear example in this direction is offered by Ising quantum critical points [27], which exhibit scalable squeezing at and above the upper critical dimension ($d \geq 3$ for short-range interactions) [28].

In this work we unveil another fundamental link between many-body physics of spin models and the generation of spin squeezing, namely, the appearance of squeezing in the presence of spontaneous breaking of a continuous spin symmetry in the ground state. Without loss of generality, in the following we will be concerned with symmetry under U(1) rotations $U_z(\phi) = e^{-i\phi J^z}$ generated by the collective spin component J^z—this property is also present for SU(2)-symmetric Hamiltonians. On a finite-size system and in the absence of any symmetry-breaking field, the ground state of a U(1)-symmetric Hamiltonian has $\mathrm{Var}(J^z) = 0$, namely, it exhibits so-called Dicke squeezing [11] [Fig. 1(a)], lacking nonetheless a finite net magnetization $\langle \boldsymbol{J} \rangle = 0$. At the same time, the low-lying energy spectrum of such an Hamiltonian exhibits a so-called Anderson tower of states, whose energy collapses as $1/N$ onto that of the ground state [29–32]. Hence, a field $\Omega \sim 1/N$ coupling to the order parameter in the x–y plane, e.g., $-\Omega J^x$ (without loss of generality), is sufficient to mix the tower of states into a state exhibiting a net polarization $m = \langle J^x \rangle / N \neq 0$ [Fig. 1(b)]. The hallmark of spontaneous symmetry breaking (SSB) is then the persistence of a finite order parameter m in the limit $N \to \infty$, in which the field is also parametrically set to zero. Here we investigate paradigmatic XXZ models with nearest-neighbor (NN) interactions using finite-temperature and variational quantum Monte Carlo simulations, as well as of spin-wave theory. In the presence of SSB in the ground state, we show that the state polarized by the minimal field $\Omega \sim 1/N$ away from Dicke squeezing retains a strong asymmetry in the fluctuations of the collective spin components, exhibiting scalable (Wineland) spin squeezing with $\xi_R^2 \sim N^{-1/2}$. Such a state is shown to have minimal spin uncertainty, namely, squeezing is its optimal metrological resource, and it can be prepared adiabatically, starting from a coherent spin state stabilized at $\Omega \to \infty$ [Fig. 1(d)], and ramping down Ω to a value $\sim 1/N$ in a time scaling linearly with system size $\tau \sim N$. This finding opens the possibility to squeeze the collective spin of quantum simulators of U(1)-symmetric [or SU(2)-symmetric] qubit Hamiltonians, with potential applications to quantum sensors [16,33] and atomic clocks [34,35].

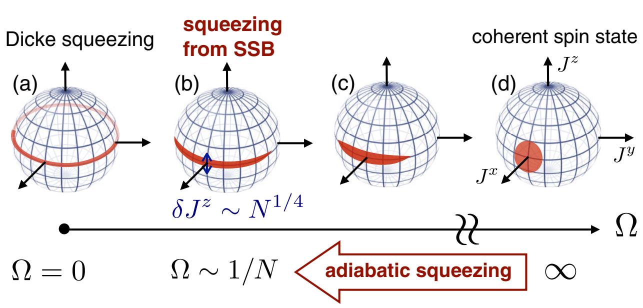

FIG. 1. Adiabatic squeezing from spontaneous symmetry breaking (SSB) in the XXZ model. Starting from a coherent spin state at $\Omega = \infty$, an adiabatic reduction of the field Ω coupling to the order parameter leads to the appearance of scalable spin squeezing when $\Omega \sim 1/N$, due to the scaling of the uncertainty on the J^z component, $\delta J^z = \sqrt{\mathrm{Var}(J^z)} \sim N^{1/4}$, and to the absence of scaling of the order parameter $m = \langle J^x \rangle / N$, as a consequence of SSB. The red areas depict the uncertainty regions of the collective spin on a sphere of radius $\sqrt{\langle \boldsymbol{J}^2 \rangle} \sim N$. As a consequence, the angular aperture of the uncertainty region along the z axis is $\delta\phi \approx \delta J^z / \sqrt{\langle \boldsymbol{J}^2 \rangle} \sim N^{-3/4}$, defining the sensitivity of the state to rotations around the y axis.

Model and methods.—We focus our attention on the $S = 1/2$ XXZ Hamiltonian:

$$\mathcal{H} = -\frac{1}{2} \sum_{ij} \mathcal{J}_{ij} (S_i^x S_j^x + S_i^y S_j^y - \Delta S_i^z S_j^z) - \Omega \sum_i \epsilon_i S_i^x, \quad (2)$$

where i, j are lattice sites on a d-dimensional hypercubic lattice of size $N = L^d$ with periodic boundary conditions. In the following we shall specialize our attention to nearest-neighbor interactions—$\mathcal{J}_{ij} = \mathcal{J}$ if i NN j [see Supplemental Material (SM) for an extension to longer-range interactions [36]]. Moreover, we choose $\mathcal{J} > 0$ and $-1 < \Delta \leq 1$, defining an XXZ model with ferromagnetic interactions in the plane and ferromagnetic or antiferromagnetic ones along the symmetry axis. Such a situation is realized, e.g., in bosonic Mott insulators [40–42]. Under these assumptions, the above model is known to break a continuous symmetry in the ground state in $d \geq 2$, and the field coupling to the order parameter is uniform, namely, $\epsilon_i = 1 \ \forall \ i$. But most of our results are completely general and apply to any model featuring SSB of a continuous symmetry—provided that the order parameter *does not* commute with the Hamiltonian (otherwise the ground state is simply a coherent spin state for all Ω, e.g., $\otimes_{i=1}^N |\rightarrow_x\rangle_i$ with $|\rightarrow_x\rangle_i$ the eigenstate of S_i^x with eigenvalue $+1/2$). In the case $\mathcal{J} < 0$ (antiferromagnetic interactions in the xy plane)—as realized in fermionic Mott insulators [40,43,44]—the order parameter is the staggered magnetization, e.g., $m = \langle J_{\mathrm{st}}^x \rangle / N = \langle \sum_i \epsilon_i S_i^x \rangle / N$; the field coupling to the order parameter must therefore be staggered, $\epsilon_i = (-1)^i$, and the relevant rotations are generated by $J_{\mathrm{st}}^y = \sum_i \epsilon_i S_i^y$. Yet, for NN interactions on a hypercubic lattice, the physics is equivalent to that of the bosonic insulators, as the two models are connected by a canonical transformation (rotation of π around the z axis for one of the two sublattices).

We have studied the ground-state physics of the XXZ Hamiltonian in $d = 1$, 2, and 3 making use of numerically exact quantum Monte Carlo (QMC) simulations, based on the stochastic series expansion method [45], as well as of spin-wave theory, valid in the presence of spontaneous symmetry breaking ($d = 2$, 3). Moreover, we have investigated the (quasi)adiabatic dynamics of preparation of the ground state starting from a large Ω by making use of time-dependent variational Monte Carlo (tVMC) method, based on pair-product (or spin-Jastrow) wave functions

[32,46], as well as of time-dependent spin-wave theory (see SM for an extended discussion of the methods [36]).

Adiabatic squeezing from SSB.—In the following we shall only show results for the case $\Delta = 1$ (Heisenberg model)—analogous results for different Δ values are presented in the SM [36]. Figure 2 shows our QMC results for the $2d$ Heisenberg model calculated for different sizes $N = L^2$ at a temperature $T = \mathcal{J}/N$, chosen so as to effectively remove thermal effects at the energy scale of an applied field $\Omega \sim \mathcal{J}/N$. As we shall see, this choice is rather conservative, because the field opens in fact a gap in the spectrum scaling as $(\mathcal{J}\Omega)^{1/2}$. The QMC results are compared to linear spin-wave (LSW) theory—see SM for the details of the theory [36]—in the thermodynamic limit, which is expected to be very accurate at large Ω, and to quantitatively capture some selected features in the limit $\Omega \to 0$ [47]. The uniform magnetization $\langle J^x \rangle/N$, shown in Fig. 2(a), is indeed correctly predicted by LSW theory: the finite-size QMC data show that the LSW prediction is reproduced down to a field scaling as $\sim 1/N$, below which the finite-size gap between the states of the Anderson tower overcomes the field and the uniform magnetization is strongly suppressed. Hence, the SSB scenario, namely, the persistence of a finite magnetization down to $\Omega = 0$ when $N \to \infty$, is clearly shown. Concomitantly, the suppression of the symmetry-breaking field Ω leads to a strong suppression of the fluctuations of the U(1) symmetry generator J^z; LSW theory predicts that $\mathrm{Var}(J^z) \sim \Omega^{1/2}$ when $\Omega \to 0$, a prediction which appears to be consistent with our finite-size QMC results for, e.g., the $2d$ XX model (see Ref. [36]) for fields down to $\Omega \approx \mathcal{J}/N$, while the $2d$ Heisenberg model shows significant beyond-LSW corrections, which interestingly appear to lead to a further reduction of the variance, namely, to stronger squeezing [36]. The combination of these two results implies naturally that the ξ_R^2 parameter is smaller than unity for all values of Ω—in agreement with a recent theorem predicting ground-state squeezing in this model as soon as $\Omega < \infty$ [48]. Moreover ξ_R^2 scales as $\Omega^{1/2}$ (actually faster for the $2d$ Heisenberg model) down to fields $\sim \mathcal{J}/N$, namely, as $N^{-1/2}$ (or faster) for the lowest significant fields for each finite-size N. The evidence of scalable spin squeezing resulting from SSB—namely, from the absence of scaling (or persistence) of $\langle J^x \rangle/N$—is the main result of our work.

Another significant feature of the low-Ω state of XXZ models exhibiting SSB is that of being a state of *minimal uncertainty* for the collective spin; namely, the collective spin components saturate the Heisenberg-Robertson inequality, $\mathrm{Var}(J^y)\mathrm{Var}(J^z) \geq \langle J^x \rangle^2/4$. To discuss this aspect and its metrological implications, it is useful to introduce the quantum Fisher information (QFI) density [11] for the J^y component, defined as $f(J^y) = (2/N) \times \sum_{nm}((p_n - p_m)^2/(p_n + p_m))|\langle m|J^y|n\rangle|^2$, where $|n\rangle$ and p_n are the eigenstates and corresponding eigenvalues of the density matrix ρ. When the state in question is rotated around the y axis by the transformation $U_y(\phi) = e^{-i\phi J^y}$, the QFI density expresses the minimal uncertainty on the angle ϕ, $\delta\phi \geq (f_Q N)^{-1/2}$; namely, $f_Q \neq 1$ implies a deviation of this uncertainty with respect to the standard quantum limit. The latter property, combined with the fact that $4\mathrm{Var}(J^y)/N$ is an upper bound to the QFI density, leads to the inequality chain:

$$\xi_R^{-2} = \frac{\langle J^x \rangle^2}{N\mathrm{Var}(J^z)} \leq f_Q(J^y) \leq \frac{4\mathrm{Var}(J^y)}{N}. \tag{3}$$

If a state has minimal uncertainty, namely, $\mathrm{Var}(J^y)\mathrm{Var}(J^z) \simeq \langle J^x \rangle^2/4$, the above inequality chain collapses to an equality, namely, $\xi_R^{-2} \approx f_Q(J^y) \approx 4\mathrm{Var}(J^y)/N$. This collapse is clearly exhibited by our numerical data for all values of $\Omega > 0$ and all systems sizes; see Fig. 2(d). In particular, among all the macroscopic observables built as a sum of local observables, J^y is arguably the one with the largest QFI density [36], so that the estimation of the rotation angle ϕ is the optimal phase-estimation protocol for the low-Ω states. The fact that $\xi_R^{-2} \approx f_Q$ implies that the measurement of the

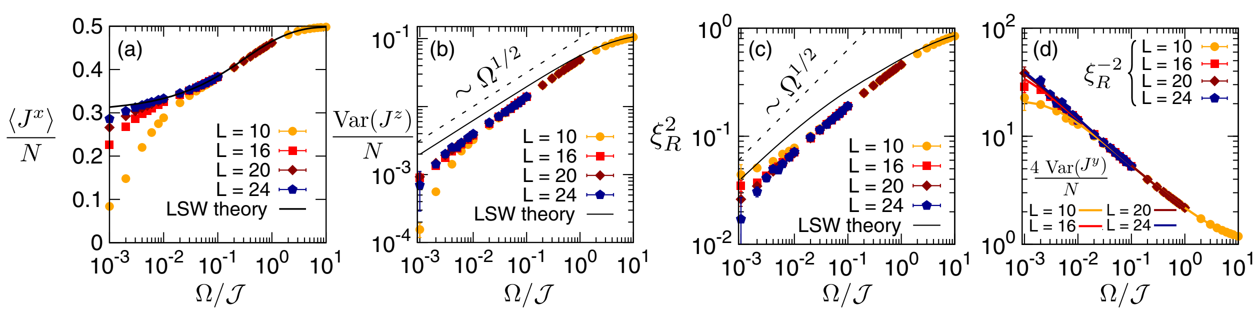

FIG. 2. Adiabatic squeezing from SSB in the $2d$ Heisenberg model. (a) Field-induced magnetization $\langle J^x \rangle/N$ for various lattice sizes $N = L^2$. (b) Variance of the collective spin component J^z. (c) Resulting spin-squeezing parameter ξ_R^2. (d) Comparison between ξ_R^{-2} and $4\mathrm{Var}(J^y)/N$. In all panels the solid black line indicates the prediction of linear spin-wave (LSW) theory, and the dotted line in (b) and (c) shows the $\Omega^{1/2}$ scaling as a reference (multiplied by an arbitrary prefactor). Here and in Fig. 3, the error bars denote one standard deviation of the statistical fluctuations in QMC.

rotation of the average collective spin (corresponding to Ramsey interferometry) is the *optimal measurement* for this protocol, leading to a phase-estimation error $\delta\phi = \xi_R/\sqrt{N} \sim N^{-3/4}$.

Finally, we would like to stress that the above results are not at all limited to the $2d$ Heisenberg model, but they are valid for all the $S = 1/2$ XXZ models spontaneously breaking a U(1) [or SU(2)] symmetry in the thermodynamic limit (see Ref. [36] for further examples). Figure 3(a) shows the field dependence of the spin-squeezing parameters ξ_R^2 for the Heisenberg model in $d = 1$, 2, and 3. We observe that the scaling of ξ_R^2 as $\Omega^{1/2}$ is clearly exhibited in $d = 3$. On the other hand, for $d = 1$ (Heisenberg chain) SSB is not realized because of the critical strength of quantum fluctuations [49]: as a consequence, $\langle J^x \rangle$ vanishes when $\Omega \to 0$, leading to the breakdown of the mechanism that underpins scalable spin squeezing in higher dimensions. Figure 3(a) shows that $\langle J^x \rangle^2$, vanishing as $\Omega^{1/2}$, leads to a squeezing parameter ξ_R^2 that goes to a constant as $\Omega \to 0$. Similar results for the XX model ($\Delta = 0$) are shown in the SM [36].

Quasiadiabatic ramps.—The preparation of the ground state at low fields requires the initialization of the system in a coherent spin state aligned with the Ω field with $\Omega \gg \mathcal{J}$, and the subsequent gradual reduction of the field along an adiabatic down-ramp—a protocol analogous to that of adiabatic quantum computing [50]. The adiabatic theorem mandates that the duration τ of an adiabatic ramp that prepares the system in the ground state at a final field Ω_f should be $\tau\mathcal{J} \gtrsim (\Delta E_{\min}/\mathcal{J})^{-2}$, where $\Delta E_{\min} = \min_{\Omega\in[\Omega_f,\infty]}[E_1(\Omega) - E_0(\Omega)]$ is the minimal gap between the Ω-dependent ground-state energy (E_0) and the energy of the first excited state (E_1) over the field range of the ramp. This gap can be calculated by LSW theory [36]—in good agreement with exact diagonalization on small system sizes [36]—and for the Heisenberg model ($\Delta = 1$) and $\Omega_f \ll \mathcal{J}$ it is shown to be $\Delta E_{\min}/\mathcal{J} \approx (z\Omega_f/\mathcal{J})^{1/2}$, where $z = 2d$ is the coordination number. This result implies that the adiabatic preparation of the ground state for the minimal field $\Omega_f \sim 1/N$ at size N takes a time $\tau/\mathcal{J} \gtrsim (z\Omega_f/J)^{-1} \sim N$.

We complement the above general prediction from LSW theory with realistic calculations of quasiadiabatic ramps based on tVMC—which show remarkable agreement with independent calculations based on time-dependent LSW [36], mutually corroborating their quantitative validity. We start the state evolution from the ground state at a large initial field value $\Omega_i = 10\mathcal{J}$—obtained by minimization of the variational energy of the spin-Jastrow ansatz [36]—and then we ramp the field down to Ω_f with the schedule $\Omega(t) = \Omega_i + F(t/\tau)(\Omega_f - \Omega_i)$, where $F(x) = \frac{1}{2}\times e^{-1/x+2}\theta(1/2 - x) + [1 - \frac{1}{2}e^{-1/(1-x)+2}]\theta(x - 1/2)$ for $t \in [0, \tau]$, while $\Omega(t) = \Omega_f$ for $t > \tau$. The function $F(t)$ (chosen heuristically) has the property of having vanishing derivatives at all orders at the two extremes of the $[0, \tau]$ interval, so that it is continuous along with all of its derivatives when it is extended to $t < 0$ and $t > \tau$ by constant functions. Figures 3(b) and 3(c) show the tVMC results for the evolution of the ξ_R^2 parameter in the $2d$ Heisenberg model ($L = 12$) with two different final fields ($\Omega_f/\mathcal{J} = 10^{-1}$ and 10^{-2}), and various ramp durations. Our main observation is that, even when the ramp fails to keep the system in its ground state down to Ω_f, the squeezing parameter exceeds the adiabatic value only for $t \lesssim \tau$, while it systematically evolves to lower values at immediately later times, and then oscillates around the adiabatic value. Therefore, failure to follow a perfectly adiabatic ramp (which in Fig. 3 is observed for all considered ramp

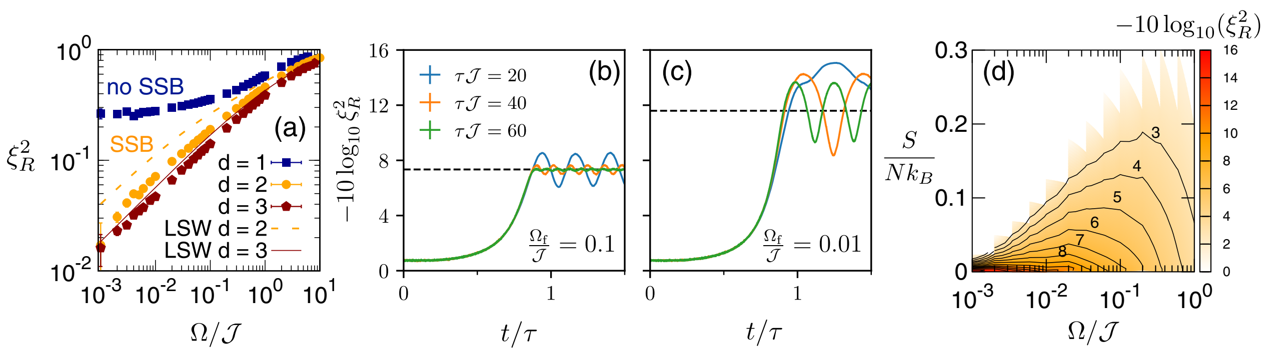

FIG. 3. Quasiadiabatic preparation of the low-field state. (a) Comparison between the field dependence of the squeezing parameter ξ_R^2 for the ground state of the Heisenberg model in $d = 1$, 2, and 3. For each value of Ω, we use a system size N such that $\Omega \geq \mathcal{J}/N$, at a temperature $T/\mathcal{J} = 1/N$ removing thermal effects. The dashed and solid lines show the prediction of LSW theory. (b),(c) tVMC results for the evolution of the spin-squeezing parameter in the $2d$ Heisenberg model ($L = 12$) along two field ramps starting from $\Omega_i/\mathcal{J} = 10$ and ending at (b) $\Omega_f/\mathcal{J} = 10^{-1}$ and (c) 10^{-2}; see text for the ramp protocol. Each panel shows three different ramps for different ramp durations τ. The dashed lines show the ground-state spin-squeezing parameters—obtained by variational minimization of the energy with the spin-Jastrow ansatz. (d) Spin-squeezing parameter versus applied field and entropy per spin in the $2d$ Heisenberg model, $L = 24$.

durations when $\Omega_f = 10^{-2}\mathcal{J}$) does not *per se* imply a degradation of the amount of squeezing that can be produced in the system.

A final comment concerns the possibly of imperfect preparation of the initial state of the quasiadiabatic ramp: this would generically entail the presence of finite entropy in the initial state, persisting then in the evolved one. Figure 3(d) tests the robustness of squeezing to the presence of finite entropy in the case of the equilibrium state of the $2d$ Heisenberg model. Not surprisingly, a finite entropy imposes a limit to the achievable squeezing; yet adiabatic spin squeezing can be obtained up to spin entropies $S/N \lesssim 0.3k_B$.

Conclusions.—In this work we have demonstrated a fundamental mechanism for the equilibrium preparation of many-qubit entangled states featuring scalable spin squeezing, based on the adiabatic preparation of low-field magnetized ground states for Hamiltonians breaking a continuous [U(1) or SU(2)] symmetry in the thermodynamic limit. At variance with the existing schemes for spin squeezing using collective-spin interactions [20–23,25,26], here we offer a specific protocol for the production of scalable spin squeezing using *short-range* qubit Hamiltonians with continuous symmetry, whose implementation is common to nearly all quantum simulation platforms. Our results are immediately relevant for Mott insulators of bosonic ultracold atoms in optical lattices, realizing the XXZ model with SU(2) symmetry or U(1) symmetry (easy-plane anisotropy)—see, e.g., the two relevant cases of ^{7}Li [41] and ^{87}Rb [42])—and to Mott insulators of fermionic atoms, realizing the Heisenberg antiferromagnet [43,44]. In the bosonic case the Ω field coupled to the order parameter is a uniform, coherent Rabi coupling between two internal states; while in the fermionic case the field coupling to the order parameter must be *staggered*, and it can be potentially created by Stark shifting a sublattice of a square or cubic lattice by a superlattice, therefore creating a Rabi-frequency difference between the two sublattices. This scheme opens the possibility to squeeze the spin state of optical-lattice clocks in the Mott insulating regime (e.g., based on ^{87}Sr [34,35] in the fermionic case and on ^{174}Yb in the bosonic case [51,52]; see SM for further discussion [36]). Our protocol (with a uniform Rabi field Ω) is also relevant for superconducting circuits realizing, e.g., the $2d$ XX Hamiltonian [53]; for Rydberg atoms with resonant interactions [54], realizing the dipolar XX model $\Delta = 0$, $\mathcal{J}_{ij} \sim |r_i - r_j|^\alpha$ with $\alpha = 3$; as well as for trapped ions, realizing the XX model with long-range interactions ($0 < \alpha < 3$) [55]. Our findings pave the way for the controlled adiabatic preparation of scalable spin-squeezed states, with the double bonus of a solid entanglement certification via the measurement of the collective spin and of the possibility to accelerate the size scaling of phase-estimation error compared to separable states.

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