Cavity Optimization for Unruh Effect at Small Accelerations

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One of the primary reasons behind the difficulty in observing the Unruh effect is that for achievable acceleration scales the finite temperature effects are significant only for the low frequency modes of the field. Since the density of field modes falls for small frequencies in free space, the field modes which are relevant for the thermal effects would be less in number to make an observably significant effect. In this Letter, we investigate the response of an Unruh-DeWitt detector coupled to a massless scalar field which is confined in a long cylindrical cavity. The density of field modes inside such a cavity shows a *resonance structure*, i.e., it rises abruptly for some specific cavity configurations. We show that an accelerating detector inside the cavity exhibits a nontrivial excitation and de-excitation rates for *small* accelerations around such resonance points. If the cavity parameters are adjusted to lie in a neighborhood of such resonance points, the (small) acceleration-induced emission rate can be made much larger than the already observable inertial emission rate. We comment on the possibilities of employing this detector-field-cavity system in the experimental realization of the Unruh effect, and argue that the necessity of extremely high acceleration can be traded off in favor of precision in cavity manufacturing for realizing noninertial field theoretic effects in laboratory settings.

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Introduction.—It is well known that the particle content of a quantum field is observer dependent [1], a fact manifested in numerous theoretical arenas, e.g., the Hawking radiation, cosmic fluctuations, and the Unruh effect [2–5]. In order to estimate the particle content and realize this theoretical idea, the Unruh-DeWitt detector (UDD) [5,6] is considered to be an operational device. The UDD is a two-level quantum system with the ground state $|E_0\rangle$ and the excited state $|E\rangle$, that is moving along a classical worldline $\tilde{x}(\tau)$, where τ is the proper time in the detector's frame of reference. The detector is coupled to a quantum field through the interaction Lagrangian $\mathcal{L}_{\rm int}[\phi(\tilde{x})] = \alpha m(\tau)\phi[\tilde{x}(\tau)]$, where α is a small coupling constant, and $m(\tau)$ is the detector's monopole moment [5,6] which also incorporates a switching function. In the first-order perturbation theory, the transition probability rate of the detector, assuming the scalar field $\hat{\phi}$ in its vacuum state $|0\rangle$, is given as $\dot{P}(\Delta E) = |\langle E|\hat{m}(0)|E_0\rangle|^2\times \dot{\mathcal{F}}(\Delta E)$, where $\dot{\mathcal{F}}(\Delta E) = \int_{-\infty}^{\infty} du\, e^{-i\Delta E u}\mathcal{W}(u,0)$ is called as the response rate of the detector, $\Delta E \equiv E - E_0$, and $\mathcal{W}(x,x') \equiv \langle 0|\hat{\phi}(x)\hat{\phi}(x')|0\rangle$ is the Wightman function of the field. The UDD probes the vacuum structure of the quantum field through $\mathcal{W}(x,x')$, and registers the excitation of the detector when it absorbs a field quanta. This detector-field system has been popularly employed in investigating the effects of quantum fields in noninertial frames, since it encompasses the essential aspects of an atom interacting with the electromagnetic field [7]. The response rate of a UDD moving in an inertial trajectory can be found to be vanishing, since the vacuum structure of the quantum field in inertial frames is invariant due to Poincaré symmetry [8]. However, since noninertial trajectories are not generated by Poincaré transformations, a UDD moving noninertially detects particles, a prime example being that for uniform acceleration a the detector shows a nonvanishing thermal response, known as the Unruh effect [5,6,8], i.e., $\dot{\mathcal{F}} = (\Delta E/2\pi)/(e^{2\pi\Delta E/a} - 1)$.

Despite being a fundamental prediction, experimental realization of the Unruh effect has not been made possible due to the demand of extremely high accelerations; for appreciable thermal effects one needs $a \geq 10^{21}$ m/s^2 [8]. For accelerations that are small compared with the energy gap ΔE of the detector, the response rate is exponentially suppressed, i.e., $\dot{\mathcal{F}} \approx (\Delta E/2\pi)e^{-2\pi\Delta E/a}$. This suppression basically originates from the fact that the temperature experienced by the accelerating detector is vanishingly small for achievable acceleration scales, since $T \sim \hbar a/k_B c$. Hence, for such small temperatures, the significant thermal contribution comes only from the low frequency modes, for which the density of field modes (the Bose-Einstein distribution) falls rapidly as $\rho(\omega) \sim \omega^2$ in free space, suppressing the response in turn, making experimental verification of the Unruh effect a nontrivial exercise of the current era.

In response, efforts have been made to enhance the detector response for maximum achievable accelerations (in the foreseeable future) using techniques such as optical cavities [9], ultraintense lasers [10,11], and Penning traps

[12]. Techniques involving capturing the finite temperature effects of an accelerating system, such as monitoring thermal quivering [13], decay of accelerated protons [14], and radiation emission in Bose-Einstein condensate [15,16] are also proposed. Other than these, there are attempts using geometric phases [17], and properly selected Fock states [18] to enhance the effects of noninertial motion. Despite these nontrivial attempts, the efforts are still far from the experimental realization of the Unruh effect (however, see Ref. [19] for a recent claim).

In this Letter, we focus on the low acceleration properties of the UDD inside an optimized cavity. To observe the Unruh effect for small accelerations, it is important to characterize scenarios where the density of field modes is increased appreciably, and the correlators of the quantum field are modified nontrivially, so that the detector responds in a distinct manner.

The response rate of an UDD moving along a given trajectory $\tilde{x}(\tau)$ can be written in a more general manner as

$$\dot{\mathcal{F}}(\Delta E) \propto \int_0^\infty d\omega_k\, \rho(\omega_k)\mathcal{I}(\Delta E, \omega_k)\mathcal{J}(\omega_k, \eta^i), \quad (1)$$

where $\rho(\omega_k)$ is the density of field modes. The quantity $\mathcal{I}$ depends on the trajectory of the detector through field correlations, and determines the field modes which stimulate the detector. For example, in the case of an inertial detector $\mathcal{I}(\Delta E, \omega_k)$ is proportional to $\delta(\Delta E + \omega_k)$, i.e., only modes with energy $\omega_k = -\Delta E$ can contribute to the response rate of the detector, leading to a null response. The function $\mathcal{J}$ depends on the frequency of the field modes ω_k, and the coordinates η^i that are held fixed on the trajectory of the detector. Therefore, the response rate of the detector can be enhanced in the following ways. (i) One can increase the density of field modes $\rho(\omega_k)$ at small ω_k, say, by changing the boundary conditions, leading to nontrivial changes in the correlators, an aspect missed in the single mode analysis that is usually employed [9,17,20–23]. Even for the near resonant frequency modes, the response rate for a single mode [23] is suppressed compared with the full-mode analysis (see Supplemental Material [24]). The analysis in this Letter justifiably makes use of the complete set of modes, and not a few modes that are near the resonant cavity frequency, which gives an additional enhancement channel *even at small accelerations*. (ii) One can choose the trajectory of the detector appropriately. Even for fixed boundary conditions, different noninertial trajectories associate different quantum fluctuations to a given inertial field vacuum [25], leading to a change in $\mathcal{I}(\Delta E, \omega_k)$ which the detector is sensitive to. (iii) One can choose mechanisms, e.g., the stimulated emission, which are extremely sensitive to both the boundary conditions and the change in field correlations.

Making use of these, we demonstrate that for a uniformly accelerated UDD in a *long* cylindrical cavity, the acceleration-induced emission rate can be significantly enhanced, even dominating the inertial spontaneous emission, for low accelerations.

Uniformly accelerating detector in cavity: Role of resonance points.—We consider an UDD inside a cylindrical cavity of radius R. The length of the cylindrical cavity is assumed to be much larger than any scale associated with the detector. The scalar field $\phi(x)$ is assumed to satisfy the Dirichlet boundary condition, i.e., $\phi[\rho = R, \theta, z] = 0$ in the cylindrical polar coordinates. The Wightman function corresponding to the scalar field inside the cavity can be expressed as

$$\mathcal{W}(x, x') = \frac{1}{(2\pi R)^2} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \frac{J_m(\xi_{mn}\rho/R) J_m(\xi_{mn}\rho'/R)}{J^2_{|m|+1}(\xi_{mn})} \times \int_{-\infty}^{\infty} \frac{dk_z}{\omega_k} e^{-i\omega_k(t-t'-i\epsilon)} e^{im(\theta-\theta')} e^{ik_z(z-z')}, \quad (2)$$

where ξ_{mn} denotes the nth zero of the Bessel function $J_m(z)$, and $\omega_k^2 = k_z^2 + (\xi_{mn}/R)^2$ (see Supplemental Material [24]).

For an UDD on a uniformly accelerating trajectory, i.e., $\tilde{x}(\tau) = [t, \rho, \theta, z] = (a^{-1}\sinh a\tau, \rho_0, \theta_0, a^{-1}\cosh a\tau)$, where ρ_0 and θ_0 are constants, and a denotes proper acceleration of the detector,the response rate can be found to be

$$\dot{\mathcal{F}}(\Delta E) = \frac{1}{2\pi}\int_0^\infty d\omega_k \underbrace{\frac{8}{a^2 e^{\pi\Delta E/a}} \frac{K_{2i\Delta E/a}(2\omega_k/a)}{(2\omega_k/a)}}_{\mathcal{I}(\Delta E, \omega_k)} \times \underbrace{\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty} \frac{(\omega_k/\pi R^2)}{J^2_{|m|+1}(\xi_{mn})} \frac{\Theta(\omega_k - \xi_{mn}/R)}{\sqrt{\omega_k^2 - (\xi_{mn}/R)^2}}}_{\rho(\omega_k)} \times \underbrace{J_m^2(\xi_{mn}\rho_0/R)}_{\mathcal{J}(\rho_0/R)}, \quad (3)$$

where $K_\nu(z)$ is the modified Bessel function of second kind, and $\Theta(x)$ is the Heaviside theta function. One can see that the density of field modes $\rho(\omega_k)$ has some special features. Firstly, as expected it is independent of the detector parameters—a or ΔE. Secondly, we can see that $\rho(\omega_k)$ rises abruptly whenever $\omega_k^2 \to (\xi_{mn}/R)^2$, called *cavity resonance points*, implying the existence of field modes inside the cavity that have very large support in terms of density of states. How such modes contribute to the response rate of the detector is controlled by $\mathcal{I}(\Delta E, \omega_k)$. In order to study that, we further evaluate the previous expression to

$$\dot{\mathcal{F}}(\Delta E) = \frac{e^{-\pi\Delta E/a}}{\pi^2 R^2 a} \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty} \frac{J_m^2(\xi_{mn}\rho_0/R)}{J^2_{|m|+1}(\xi_{mn})} \times K^2_{i\Delta E/a}(\xi_{mn}/Ra). \quad (4)$$

In the $R \to \infty$ limit, the density of field modes reduces to $\rho(\omega_k) \propto \omega_k^2$, which is the standard density of field modes in free space, provided one makes the following replacements: $2\pi \sum_{n=1}^{\infty} \to R \int_0^{\infty} dq$ and $\xi_{mn}/R \to q$ and the response rate [Eq. (4)] reproduces a thermal form. In the limit $a \to 0$, the function $\mathcal{I}(\Delta E, \omega_k)$ is proportional to $\delta(\Delta E + \omega_k)$, as expected (see Supplemental Material [24]). Thus, in the inertial case there are not any modes which contribute to the detector response, including those at the resonance points. However, for the case of noninertial detector, the function $\mathcal{I}(\Delta E, \omega_k)$ allows for the modes around $\omega_k \sim \xi_{mn}/R$ to contribute, with some weightage, leading to a nonzero response.

In order to quantify the effects of cavity in enhancing the response rate of the accelerating detector inside the cavity, when compared to the response rate of an accelerating detector in free space $\dot{\mathcal{F}}_{\mathcal{M}}$, we define a quantity $\mathcal{E} \equiv \dot{\mathcal{F}}/\dot{\mathcal{F}}_{\mathcal{M}}$, called *enhancement* in the response rate of the detector. In the small acceleration limit, i.e., $a \ll \Delta E$, we make use of the asymptotic expansion of $K_{i\alpha}(\alpha z)$ for large values of α [26], with $\alpha \in \mathbb{R}$ and $|\arg z| < \pi$, to approximate (see Supplemental Material [24])

$$\mathcal{E}(\Delta E) \approx \frac{4\pi}{(R\Delta E)^2} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})} \times \begin{cases} \frac{(\beta_{mn}^{<}\Delta E/a)^{1/3}}{[1-(\frac{\xi_{mn}}{R\Delta E})^2]^{1/2}} \mathrm{Ai}^2[-(\beta_{mn}^{<}\Delta E/a)^{2/3}]; & \frac{\xi_{mn}}{R\Delta E} < 1 \\ \frac{1}{3^{4/3}\Gamma^2(2/3)} (\Delta E/2a)^{1/3}; & \frac{\xi_{mn}}{R\Delta E} = 1 \\ \frac{(\beta_{mn}^{>}\Delta E/a)^{1/3}}{[(\frac{\xi_{mn}}{R\Delta E})^2 - 1]^{1/2}} \mathrm{Ai}^2[(\beta_{mn}^{>}\Delta E/a)^{2/3}]; & \frac{\xi_{mn}}{R\Delta E} > 1 \end{cases}, \tag{5}$$

where $\mathrm{Ai}(z)$ is known as the Airy function [26], and

$$\beta_{mn}^{<} \equiv \frac{3}{2}\left[\mathrm{sech}^{-1}\left(\frac{\xi_{mn}}{R\Delta E}\right) - \sqrt{1 - \left(\frac{\xi_{mn}}{R\Delta E}\right)^2}\right], \tag{6}$$

$$\beta_{mn}^{>} \equiv \frac{3}{2}\left[\sqrt{\left(\frac{\xi_{mn}}{R\Delta E}\right)^2 - 1} - \sec^{-1}\left(\frac{\xi_{mn}}{R\Delta E}\right)\right]. \tag{7}$$

It is evident from Eq. (5) that in the small acceleration limit the enhancement $\mathcal{E}$ receives a large amplification, proportional to $(\Delta E/a)^{1/3}$, at the resonance points, i.e., $R\Delta E = \xi_{mn}$. Thus at small accelerations, if one chooses the radius of the cylindrical cavity such that it coincides with one of the resonance points, e.g., $R\Delta E = \xi_{01} = 2.405$, the enhancement in response rate $\mathcal{E}$ shows very large amplifications (see Fig. 1).

Though the enhancement in detector response diverges at the resonance points as $(\Delta E/a)^{1/3}$ in the limit $a/\Delta E \to 0$, the actual response rate of the detector inside the cavity is still small due to the exponential suppression of the free space response rate

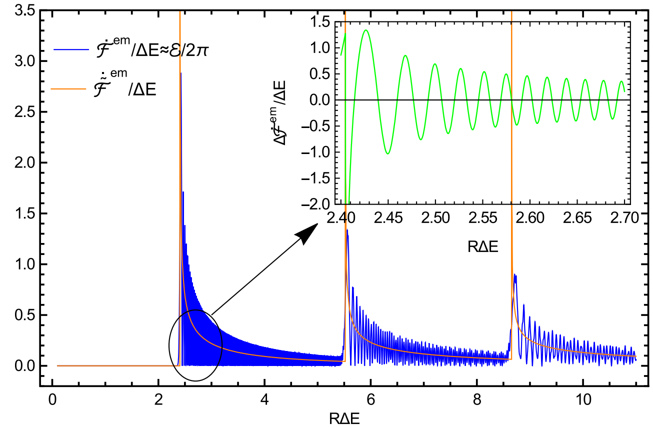

FIG. 1. The emission rates for the accelerating detector $\dot{\mathcal{F}}^{\mathrm{em}}$ (which is also proportional to the enhancement factor $\mathcal{E}$ at small accelerations) and the inertial detector $\dot{\bar{\mathcal{F}}}^{\mathrm{em}}$ with respect to $R\Delta E$, with $\rho_0 = 0$, and $a/\Delta E = 10^{-3}$. Inset: the discrete plot for the difference in emission rates of the accelerating and the inertial detectors $\Delta\dot{\mathcal{F}}^{\mathrm{em}}$ around the first resonance point ξ_{01}. The range of $R\Delta E$ and its step size are chosen such that the contribution exactly at the resonance point ξ_{01} is avoided.

for small accelerations, i.e., $\lim_{a/\Delta E\to 0}\dot{\mathcal{F}} = \lim_{a/\Delta E \to 0}\dot{\mathcal{F}}_{\mathcal{M}} \times \mathcal{E} \approx (\Delta E/2\pi)e^{-2\pi\Delta E/a} \times \lim_{a/\Delta E\to 0}\mathcal{E}$. It has been argued in Ref. [9] that the exponential suppression in the response rate inside a cavity can be regulated considerably by introducing nonadiabatic switching of the detector. Now, if the size of the cylindrical cavity is optimized at one of the resonance points in addition to the usage of an appropriate switching function, or state selection, as proposed in Ref. [9], the response rate of the detector can potentially be enhanced exponentially. This line of study, however, will be pursued elsewhere.

In this Letter we couple the enhancement in response rate $\mathcal{E}$ at the resonance points, due to the change in density of field modes $\rho(\omega_k)$, to another scheme which is extremely sensitive to the change in field correlators, namely the stimulated emission. Since stimulated emission is sensitive to the number of particles present, and a uniformly accelerating detector perceives the Minkowski vacuum as a state with particles, one could expect that a uniformly accelerating detector can undergo stimulated emission. The higher the number of particles in the Minkowski vacuum the detector perceives, the higher is its emission rate. The emission profile for a rotating detector was utilized in Ref. [27] to propose measurable detection of noninertial quantum field theoretic effects. In Ref. [28] the emission from a rotating muonic hydrogen atom in the so-called *Trojan states* is shown to be extremely enhanced. Thus, modifying the density of field modes $\rho(\omega_k)$ would further strengthen such effects which we analyze next.

Acceleration-assisted enhanced emission in cavity: Role of $\mathcal{I}(-\Delta E, \omega_k)$.—The response rate corresponding to the emission from the UDD can simply be obtained as

$\dot{\mathcal{F}}^{\rm em}(\Delta E) = \dot{\mathcal{F}}(-\Delta E)$. One can show that the principle of detailed balance is satisfied for the detector-field system inside the cavity, i.e., $\dot{\mathcal{F}}^{\rm em}/\dot{\mathcal{F}} = e^{2\pi\Delta E/a}$, leading to a thermal distribution of population in equilibrium for a collection of such detectors $n_g/n_e = e^{-2\pi\Delta E/a}$, where n_g and n_e denote the number of detectors in the ground and the excited states respectively.

Since only the function $\mathcal{I}$ in Eq. (3) is sensitive to $\Delta E \to -\Delta E$, the emission rate in the cylindrical cavity can be written as

$$\dot{\mathcal{F}}^{\rm em}(\Delta E) = \frac{1}{\pi R^2}\int_0^\infty d\omega_k \mathcal{I}(-\Delta E, \omega_k) \times \sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\frac{\Theta(\omega_k - \xi_{mn}/R)}{\sqrt{\omega_k^2 - (\xi_{mn}/R)^2}}. \tag{8}$$

Note that in the $a \to 0$ limit $\mathcal{I}(-\Delta E, \omega_k) \to \delta(-\Delta E + \omega_k)$, so for an inertial detector only the modes with energy $\omega_k = \Delta E$ are responsible for the emission of the detector. Since the density of field modes diverges for modes with energy $\omega_k = \xi_{mn}/R$, the emission rate becomes divergent if $R\Delta E = \xi_{mn}$, so an inertially moving excited detector emits *instantaneously* inside such a cavity. On the other hand, for uniformly accelerating detector $\mathcal{I}(-\Delta E, \omega_k) \propto a^{-1}e^{\pi\Delta E/a}K_{-2i\Delta E/a}(2\omega_k/a)$, there is a distribution of modes which determines the emission rate. Some resulting salient features are as follows.

Firstly, since $\delta(-\Delta E + \omega_k)$ in the expression for inertial emission rate in Eq. (8) is replaced by a smooth function $\mathcal{I}(-\Delta E, \omega_k)$, the emission rate of the accelerating detector inside a cavity which is optimized at its resonant configuration $R\Delta E = \xi_{mn}$ is large, but finite. Thus, if the cavity is tuned to be at one of its resonance points, while the inertial detector de-excites in no time, the de-excitation of the accelerating detector takes a finite amount of time, the delay marking the noninertial effect.

Secondly, due to the change in $\mathcal{I}(-\Delta E, \omega_k)$, caused by the accelerated motion, the emission rate of the detector in a cavity, optimized *slightly away* from the resonance points, is larger than that of an inertial detector (see Fig. 1). This is due to the fact that the Delta function (inertial detector) shows a sharper fall off away from the resonance points as compared with the smoother function $\mathcal{I}(-\Delta E, \omega_k)$ of the accelerated detector.

Therefore, in comparison to the inertial detector, acceleration of the detector causes a *delay* in its emission *at the* resonance points of the cavity, but exhibits substantial enhancement in the emission rate *slightly away* from the resonance points. Further, in the low acceleration limit the enhancement $\mathcal{E}$ can be related to the emission response rate of the detector $\dot{\mathcal{F}}^{\rm em}$ as $\lim_{a/\Delta E\to 0}\dot{\mathcal{F}}^{\rm em} \approx (\Delta E/2\pi)\times \lim_{a/\Delta E\to 0}\mathcal{E}$. As the enhancement in the response rate $\mathcal{E}$ of the detector exhibits a sharp amplification at the resonance points for small accelerations, one could estimate the amount of noninertial contribution in the emission rate of the detector at the resonance points of the cavity. In order to further quantify, we subtract the emission rate of an inertial detector $\dot{\bar{\mathcal{F}}}^{\rm em}$ from the noninertial one, i.e., $\Delta\dot{\mathcal{F}}^{\rm em} \equiv \dot{\mathcal{F}}^{\rm em} - \dot{\bar{\mathcal{F}}}^{\rm em}$, obtaining the purely noninertial contribution in the emission rate slightly away from any resonance point as

$$\Delta\dot{\mathcal{F}}^{\rm em} \approx \frac{2\Delta E}{(R\Delta E)^2}\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{J_m^2(\xi_{mn}\rho_0/R)}{J_{|m|+1}^2(\xi_{mn})}\begin{cases}\frac{1}{[1-(\frac{\xi_{mn}}{R\Delta E})^2]^{1/2}}\{(\beta_{mn}^{<}\Delta E/a)^{1/3}{\rm Ai}^2[-(\beta_{mn}^{<}\Delta E/a)^{2/3}] - \frac{1}{2\pi}\}; & \frac{\xi_{mn}}{R\Delta E} < 1. \\ \frac{(\beta_{mn}^{>}\Delta E/a)^{1/3}}{[(\frac{\xi_{mn}}{R\Delta E})^2-1]^{1/2}}{\rm Ai}^2[(\beta_{mn}^{>}\Delta E/a)^{2/3}]; & \frac{\xi_{mn}}{R\Delta E} > 1\end{cases} \tag{9}$$

Since $\Delta\dot{\mathcal{F}}^{\rm em} > 0$ amounts to a dominating noninertial emission, we see (Fig. 1) that the emission rate of the accelerating detector can be much higher than that of the inertial detector, if the cavity is designed to be *slightly away from one of its resonance points*, i.e., $Q_R \equiv 1 - (\xi_{mn}/R\Delta E)^2$ is a small (nonzero) number. Since the inertial response diverges at the resonance points, very close to the resonance points $\Delta\dot{\mathcal{F}}^{\rm em}$ is a large negative number (see the inset of Fig. 1). However, once one starts moving away from the resonance, both inertial and noninertial emission rates start decaying with the later decaying much more slowly in comparison to the inertial delta function. As a consequence, closer to the resonance point there is a region where the noninertial response dominates significantly (see Fig. 2). *Hence, the highly enhanced emission rate of the UDD in a slightly off-resonant cavity will clearly be a distinguishable direct realization of the Unruh effect.* Thus, the requirement of high acceleration for observing the Unruh effect can be compensated for a precise cavity design, i.e., one with small Q_R.

Precision in cavity design.—Since the nonzero acceleration of the detector allows a width of $R\Delta E$ about any resonance point (see the inset of Fig. 1) where the noninertial component dominates, we explore the noninertial component of the emission rate when we go off resonant by an infinitesimal amount ϵ, i.e.,

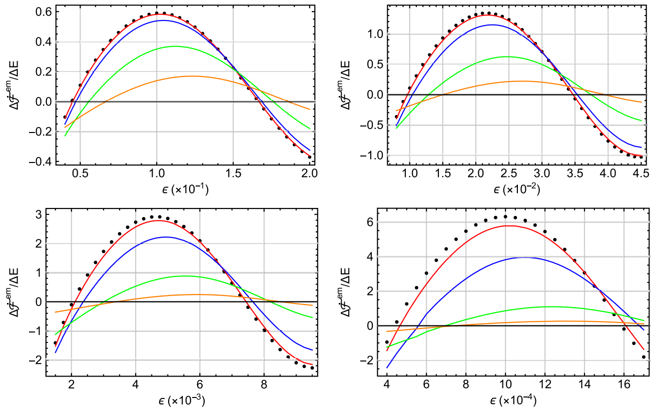

FIG. 2. The noninertial contribution to the emission rate $\Delta\dot{\mathcal{F}}^{\rm em}$ around the first resonance point, i.e., $R\Delta E = \xi_{01} + \epsilon$, with respect to ϵ for various values of $a/\Delta E$ (plots in the top row represent $a/\Delta E = 10^{-2}, 10^{-3}$, while for bottom row $a/\Delta E = 10^{-4}, 10^{-5}$). The black (dotted), red, blue, green, and orange curves correspond to $\beta = 0, 10^{-4}, 10^{-3}, 10^{-2}$, and 10^{-1} respectively.

$R\Delta E = \xi_{mn} + \epsilon$. As can be seen in Fig. 2, even for a smaller value of acceleration ($a/\Delta E \sim 10^{-3}$), with increased precision [$\epsilon \sim (1.5 - 3) \times 10^{-2}$] in cavity design, the emission rate of the detector is substantially enhanced.

Moreover, in a realistic experimental setup the cylindrical cavity would not be ideal, and the associated $\rho(\omega_k)$ may not really be diverging at the resonance points, as discussed above. Nevertheless, using a reasonably regularized cavity expressed by $\rho(\omega_k) \propto 1/[\beta/R + \sqrt{\omega_k^2 - (\xi_{mn}/R)^2}]$, with the regularization parameter $\beta \ll 1$, it can be demonstrated that the dominance of noninertial contribution near the resonance points is qualitatively independent of the occurrence of divergence in $\rho(\omega_k)$ (see Fig. 2). Further, such near-resonance features remain present for a realistic cavity with leakage of modes as well, which typically provides a Lorentzian broadening for the inertial detectors. In the cylindrical cavity this leakage only modifies the function $\mathcal{I}(\Delta E, \omega_k)$, leaving the structure of density of modes $\rho(\omega_k)$, which harbors the resonance, intact. For achievable quality factors [29,30] ($\sim 10^{4-6}$), the modification in the emission for the case of Rindler motion is only marginal, i.e., $< 1\%$ (see the Supplemental Material [24]), suggesting the robustness of the scheme.

Conclusions.—To summarize, for small accelerations $a/\Delta E \to 0$, the enhancement $\mathcal{E}(\Delta E)$ in the response rate of the accelerating detector inside a long cylindrical cavity diverges as $(\Delta E/a)^{1/3}$ at the resonance points of the cavity, i.e., $\xi_{mn}/R\Delta E = 1$. Such resonant configurations of cavity can be utilized very fruitfully for observing the Unruh effect at small accelerations if one couples it with stimulated emission. Since the emission rate of the inertial detector has a sharp fall off away from the resonant frequencies, unlike the accelerating case, to study the noninertial emission rate of the accelerating detector it is advisable to design a cylindrical cavity to be in a close neighborhood of a resonance point, i.e., $R\Delta E = \xi_{mn} + \epsilon$. In such a cavity, even with small enough acceleration, the noninertial emission rate can be made much larger than the inertial emission rate and observable. Similar suppression (dominance) of resonant (nonresonant) effects due to the accelerated motion is also observed in recent works [27,31].

The calculations presented in this Letter can easily be generalized for other fields, e.g., for a UDD with $\Delta E \sim$ MHz (e.g., hydrogen atom making a $2p \to 2s$ transition [27]) inside an optical cavity. The required dimensions of the cavity for such atoms could be ($\ell \sim aT^2 \gg a/\Delta E^2 \sim$ cm and $R \sim \xi_{mn}/\Delta E \sim$ cm). For such dimensions even a marginal acceleration $a \sim 10^9$ ms^{-2}, which can easily be obtained for instance by setting up a thermal gradient [32] of $\Delta T = (ma/k_B)\Delta x \sim 1$ K across the cavity of $\Delta x \sim$ cm, leads to a significant emission enhancement. Further, multiple noninteracting accelerated particles, e.g., a beam of UDDs, can be sent inside the cavity, and an integrated enhanced effect can be observed to further strengthen the signal [33].

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