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## Analysis of Ringdown Overtones in GW150914

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We analyze GW150914 postmerger data to understand if ringdown overtone detection claims are robust. We find no evidence in favor of an overtone in the data after the waveform peak. Around the peak, the Bayes factor does not indicate the presence of an overtone, while the support for a nonzero amplitude is sensitive to changes in the starting time much smaller than the overtone damping time. This suggests that claims of an overtone detection are noise dominated. We perform GW150914-like injections in neighboring segments of the real detector noise, and we show that noise can indeed induce artificial evidence for an overtone.

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Introduction.—Since the first detection of gravitational waves (GWs) from a binary black hole (BH) merger, GW150914 [\[1](#page-5-0)], the LIGO-Virgo-KAGRA (LVK) Collaboration [[2](#page-5-1)–[4](#page-5-2)] reported 90 events with a probability of astrophysical origin  $p_{astro} > 0.5$  during the first three observing runs [\[5](#page-5-3)–[8](#page-5-4)]. These GW signals, combined with those detected by independent groups [\[9](#page-5-5)–[13](#page-5-6)], have broadened our understanding of cosmology [\[14\]](#page-5-7), the astrophysics of compact objects [\[15](#page-5-8)], matter at supranuclear densities [\[16\]](#page-5-9), and general relativity (GR) in the strong-field regime [\[17\]](#page-5-10).

Among the numerous tests of GR proposed over the years, BH spectroscopy with the so-called "ringdown" relaxation phase following the merger presents unique opportunities to characterize the remnant as a Kerr BH. In linearized GR, the two GW polarizations  $h_{+x}$  can be decomposed as  $h_+ - ih_\times \equiv \sum_{\ell m} h_{\ell m}(t)_{-2} Y_{\ell m}(t,\phi)$ , where the (spin-weighted) spherical harmonics  ${}_{-2}Y_{\ell m}(t, \phi)$ depend on two angles that characterize the direction from the source to the observer. Each multipolar component is a superposition of damped exponentials known as quasinormal modes (QNMs):

<span id="page-0-0"></span>
$$
h_{\ell m}(t) \equiv \sum_{n} A_{\ell m n} e^{i[-\omega_{\ell m n}(t - t_{\ell m n}^{\text{start}}) + \phi_{\ell m n}]} e^{-(t - t_{\ell m n}^{\text{start}})/\tau_{\ell m n}}, \qquad (1)
$$

where we ignored spherical-spheroidal mode-mixing between different corotating  $\ell$  modes, and the contribution of counterrotating modes (a valid assumption for GW150914). In GR, the QNM frequencies  $\omega_{\ell mn}$  and damping times  $\tau_{\ell mn}$  depend only on the remnant BH's mass  $M_f$  and spin  $a_f$  [[18](#page-5-11)–[24\]](#page-5-12). The QNM amplitudes  $A_{\ell mn}$ and phases  $\phi_{\ell mn}$  were unknown before the first numerical BH merger simulations, and early work on BH spectroscopy [\[23\]](#page-5-13) had to rely on educated guesses [\[25\]](#page-5-14). We now know that radiation from a binary BH merger is dominated by the  $\ell = |m| = 2$  component, while higher multipoles are subdominant [[26](#page-5-15),[27](#page-5-16)]. For fixed  $(\ell, m)$ , the QNMs are sorted by the magnitude of  $\tau_{\ell mn}$ : the fundamental mode  $(n = 0)$  has the longest damping time, and the integer n labels the so-called "overtones."

It has long been known that including overtones improves the agreement between ringdown-only fits and the complete gravitational waveforms from perturbed BHs. This was first shown by direct integration of the perturbation equations sourced by infalling particles or collapsing matter [[28](#page-5-17)–[31](#page-5-18)] and then, more rigorously, using Green's function techniques [\[32](#page-5-19)–[36](#page-5-20)]. Overtones were shown to improve agreement with numerical simulations of collapse [\[37\]](#page-5-21), head-on collisions [[38](#page-5-22)], and quasicircular mergers [\[26\]](#page-5-15) leading to BH formation, and their omission leads to significant biases in mass and spin estimates [\[39,](#page-5-23)[40](#page-5-24)]. However, standard QNM tests often relied only on fundamental modes for two main reasons: overtones are shortlived and difficult to confidently identify in the data [[41](#page-5-25)], and it is unclear whether multiple overtones have physical meaning or they just happen to phenomenologically fit the nonlinear part of the merger signal [[26](#page-5-15),[27](#page-5-16)].

Recently, Ref. [\[42\]](#page-5-26) showed that including overtones up to  $n = 7$  in the ringdown model improves the agreement with numerical relativity simulations for all times beyond the time  $t_{\text{peak}}$  where  $|h_+^2 + h_\times^2|$  has a maximum, claiming

that this observation "implies that the spacetime is well described as a linearly perturbed BH with a fixed mass and spin as early as the peak." Their study's insistence on an intrinsically linear physical description spurred a sequence of additional investigations, both on the modeling and on the observational side [\[36,](#page-5-20)[43](#page-5-27)–[52\]](#page-6-0). If higher overtones can indeed be measured by starting at the peak, the larger ringdown signal-to-noise ratio (SNR) would open the door to more precise tests of GR. This theoretical argument motivated a reanalysis of GW150914. Reference [\[53\]](#page-6-1) fitted the postpeak waveform with a QNM superposition including overtones, and claimed evidence for "at least one overtone [...] with  $3.6\sigma$  confidence." The claim seems at odds with Ref. [[45](#page-5-28)] and with the subsequent LVK analysis [\[17\]](#page-5-10), both reporting weak evidence (with a  $log_{10}$ -Bayes factor of only ∼0.6) in favor of the "overtone model" including both  $n = 0$  and  $n = 1$  (henceforth Kerr<sub>221</sub>) relative to the model including only  $n = 0$  (henceforth  $Kerr_{220}$ ).

In this Letter we ask whether overtone detection claims in GW150914 data are robust. We use geometrical units  $G = c = 1$ , restoring physical units when needed, and we always quote redshifted BH masses as measured in a geocentric reference frame.

*Methods.*—The  $\ell = |m| = 2$  multipole is largely dominant in GW150914 [\[17,](#page-5-10)[54](#page-6-2)], so we can ignore higher multipoles and mode-mixing contributions in the general waveform model [\(1\).](#page-0-0) The system does not show evidence for antialigned progenitor spins (and more generally, for any nonzero spin), so counterrotating modes can be safely ignored [[17](#page-5-10)[,55](#page-6-3)]. We make several assumptions to match as closely as possible the analysis of Ref. [\[53\]](#page-6-1). First, we include only one or two QNMs ( $n = 0, 1$ ) and we assume that all overtones start at the same time  $t_{\ell mn}^{\text{start}} = t_{\text{start}}$ . We fix  $(i, \phi) = (\pi, 0)$  rad, since in our model these parameters are strongly degenerate with the free overtone amplitudes and phases, respectively. Since there is no evidence for misaligned spins in GW150914, we also assume that the waveform amplitudes satisfy  $h_{\ell m} = h_{\ell - m}^*$ , a good approximation when the progenitor spins are nearly aligned with the orbital angular momentum of the binary. The strain measured by GW detectors is  $h_D(t) = F_+h_+ + F_\times h_\times$ , where the detector pattern functions  $F_{+x}(\alpha, \delta, \psi)$  depend on the right ascension, declination, and polarization angles  $\alpha$ ,  $\delta$ , and  $\psi$  [[56](#page-6-4)]. Following Ref. [[53](#page-6-1)] we set  $(\alpha, \delta, \psi)$  =  $(1.95, -1.27, 0.82)$  rad. We fix  $t_{\text{start}}^{\text{H1}}$  in the Hanford detector and compute the starting time in the Livingston detector using a fixed time delay determined from the sky position parameters listed above. We assume flat priors on all free parameters in the ranges  $M_f \in [20, 200]$   $M_\odot$ ,  $a_f \in [0, 0.99], A_{22n} \in [0, 5 \times 10^{-20}], \text{ and } \phi_{22n} \in [0, 2\pi].$ 

We analyze the ringdown signal using the Bayesian parameter estimation PYRING [\[54,](#page-6-2)[57\]](#page-6-5), employed by the LVK collaboration to perform ringdown-only tests of GR. The PYRING package relies on the nested sampling algorithm cpnest [\[58\]](#page-6-6) (for additional details needed to reproduce our analysis, see the Software section), that allows us to compare alternative hypotheses by computing their relative Bayes factors. We use 4096 live points and 4096 maximum Markov chain (MC) steps, which typically result in ∼20 000 independent samples at the end of each of our runs. We have tested the robustness of our results to sampling configurations by repeating the runs close to the peak time using 10 000 live points and MC steps, together with four different random seeds in the instantiations of the nested sampling. All the obtained results are consistently recovered under these changes of settings. The autocorrelation function (ACF) of the background noise was chosen to be as close as possible to the settings of Ref. [\[53\]](#page-6-1). The ACF was computed using a stretch of 64 s of data starting at 1 126 257 417 s of GPS time (see the Software section for more details). We have verified that ACFs estimated using different data stretches close to the event do not significantly impact our conclusions, in agreement with the hypothesis of wide-sense stationarity of the noise. The data are appropriately cropped to avoid contamination from earlier stages of the coalescence [[59](#page-6-7)], beginning from the starting time of the analysis and up to a duration of 0.1 s. We analyze publicly available data from GWOSC [\[60\]](#page-6-8) with a sampling rate of 16384 Hz (the maximum resolution available). This rate, larger than the rate of 2048 Hz used in Ref. [\[53\]](#page-6-1), was chosen to minimize the impact of the time discretization. Repeating the analysis using a rate of 4096 Hz left our conclusions unaltered. When investigating the consequences of slightly changing the analysis settings, we found that the choice of  $t_{\text{start}}$  (which has be set equal to  $t_{\text{peak}}$  according to the theoretical arguments in [[42](#page-5-26)]) has by far the largest impact. The effect of varying  $\psi$ , *i* is milder, and it will be discussed in a forthcoming paper [[61](#page-6-9)], together with the impact of dropping the symmetry assumption on the amplitudes  $h_{\ell m}$ . Reference [\[53\]](#page-6-1) assumed  $t_{\text{start}}^{\text{H1}} = t_{\text{peak}}^{\text{H1}} = 1126259462.423$  s. However the value of  $t_{\text{peak}}^{\text{H1}}$  must be estimated from the data, and as such it is uncertain. Fixing it to a specific value can induce systematic biases. We quantify this uncertainty by reconstructing  $t_{\text{peak}}^{\text{H1}}$  using the posterior distributions of the parameters of GW150914 [[62](#page-6-10)] obtained with the IMR waveform model SEOBNRv4 [[63](#page-6-11)] (see Supplemental Material [[64](#page-6-12)] for details). We check that the reconstruction is robust against waveform systematics by using also the IMRPhenomPv2 waveform model [[67](#page-6-13)–[69](#page-6-14)]. In the Hanford detector, the resulting posterior distribution has median  $\bar{t}^{\text{H1}}_{\text{peak}} =$ 1 126 259 462.423 23 s and standard deviation  $\sigma =$ 0.000 59 s. We will vary  $t_{\text{start}}^{\text{H1}}$  within the  $\pm 2\sigma$  interval of its posterior distribution.

Mass and spin estimates.—In Fig. [1](#page-2-0) we show the mass and spin of the GW150914 BH remnant estimated using the Kerr<sub>220</sub> (blue), Kerr<sub>221</sub> (red), and full IMR model [[70](#page-6-15)] (dashed black) for ten selected values of  $\Delta t_{\text{start}}^{\text{H1}} \equiv t_{\text{start}}^{\text{H1}} - \bar{t}_{\text{peak}}^{\text{H1}}$ . For  $\Delta t_{\text{start}}^{\text{H1}} / M \ge -1.45$ , the IMR posterior overlaps with both the  $\text{Kerr}_{220}$  and  $\text{Kerr}_{221}$  models

<span id="page-2-0"></span>

FIG. 1. Mass and spin of the remnant BH for GW150914. Each panel corresponds to a different value of  $\Delta t_{\text{start}}^{\text{H1}} = t_{\text{start}}^{\text{H1}} - \bar{t}_{\text{peak}}^{\text{H1}}$ , quoted in units of M. All  $\Delta t_{\text{start}}^{\text{H1}}$  values used in panels with dark (light) gold backgrounds are consistent with the median of the  $t_{\text{peak}}^{\text{H1}}$  distribution at  $1\sigma$  ( $2\sigma$ ). In each panel, dashed black, solid red, and solid blue contours correspond to 90% credible level in the BH parameters measured using the full IMR [[70](#page-6-15)],  $Kerr_{221}$ , and  $Kerr_{220}$  models, respectively.

at 90% credibility, although the  $Kerr_{221}$  reconstruction peaks closer to the IMR estimate. The  $Kerr_{221}$  model agrees much better than  $Kerr_{220}$  with the IMR posterior especially when we start fitting before the peak  $(\Delta t_{\text{start}}^{H1}/M \le -2.17)$ , where such a fit is not well motivated by the overtone model (see Fig. 1 of [\[42\]](#page-5-26)). The starting time used in Ref. [[53](#page-6-1)] corresponds to  $\Delta t_{\text{start}}^{\text{H1}}/M = -0.72$  in Fig. [1](#page-2-0). Note that the  $(M_f, a_f)$  measurements obtained with the  $Kerr_{221}$  model overlap with the GR prediction even when  $\Delta t_{\text{start}}^{\text{H1}}/M = -3.62$ , outside of the  $2\sigma$  confidence interval on the peak location. This is likely due to a combination of two effects: (i) since  $\omega_{221} < \omega_{220}$ , any overtone model naturally includes a low-frequency component, thus improving the fit to the low-frequency, premerger part of the signal; and (ii) the  $Kerr_{221}$  model has a larger number of parameters than the  $Kerr_{220}$  model, thus at low signal-to-noise ratios it can still fit the signal with the values of  $(M_f, a_f)$  determined by the late-time ringdown behavior.

Bayes factors.—To quantify the evidence for the presence of an overtone in GW150914, we compare the hypotheses that the data can be described by the  $Kerr_{221}$  vs  $Kerr_{220}$ models and compute the resulting Bayes factor  $\mathcal{B}_{220}^{221}$ . In the top panel of Fig. [2](#page-2-1) we show  $log_{10} B_{220}^{221}$  (red crosses)

<span id="page-2-1"></span>

FIG. 2. Top: log-Bayes factor ( $\log_{10} B_{220}^{221}$ ) between the Kerr<sub>221</sub> and Kerr<sub>220</sub> hypotheses as a function of  $\Delta t_{\text{start}}^{\text{H1}} = t_{\text{start}}^{\text{H1}} - \bar{t}_{\text{peak}}^{\text{H1}}$ . For the GW150914 signal (red crosses),  $\bar{t}_{\text{peak}}^{H1}$  is the median of the posterior distribution from the full IMR analysis; dark (light) gold bands correspond to the  $1\sigma(2\sigma)$  uncertainties on the median. For the GW150914-like injections (black),  $t_{\rm peak}^{\rm H1}$  is computed from the simulation, and so it is known exactly. Black dots correspond to a GW150914-like injection in zero noise. The blue dots (and related "error bars") are computed by repeating the analysis at each  $t_{\text{start}}^{\text{H1}}$  under different realizations of the real detector noise close to the GW150914 trigger. Bottom: amplitude of the overtone  $A_1$  measured for different  $t_{\text{start}}^{\text{H1}}$ . The red (black) curves correspond to the measurement obtained from the GW150914 signal (GW150914-like injection in zero noise). The blue curves are the overtone amplitudes measured on the GW150914-like injection in real noise.

for selected values of  $\Delta t_{\text{start}}^{\text{H1}}$ . In the bottom panel we show the posterior of the overtone amplitude  $A_1 \equiv A_{221}$  for the Kerr<sub>221</sub> model (red curves). When  $\Delta t_{\text{start}}^{\text{H1}}/M \ge -1.45$ , there is no evidence for the overtone in the data ( $log_{10} B_{220}^{221} < 0$ ), and the posterior distributions in the bottom panel have significant support for  $A_1 = 0$ , hence the Kerr<sub>220</sub> model is favored with respect to  $Kerr_{221}$ . We observe significant Bayesian evidence for the presence of the overtone  $(\log_{10} \beta_{220}^{221} > 2)$  only for  $\Delta t_{\text{start}}^{H1}/M \le -4.34$ , i.e., well outside of the nominal region of validity of the  $Kerr_{221}$  model. For  $\Delta t_{\text{start}}^{\text{H1}}/M = -0.72$ , which corresponds to the  $t_{\text{peak}}^{\text{H1}}$  value used in Ref. [[53](#page-6-1)], we find that  $log_{10} B_{220}^{221} = -0.60$ , while the amplitude has large support for zero. At the peak time  $A_1$  is maximum away from zero, but there is still some support for zero amplitude. This may lead us to conclude that the overtone is measurable in this ringdown signal. However, both the Bayes factor and  $A_1$  *decrease* for values of  $\Delta t_{\text{start}}^{\text{H1}}$ located immediately before and after  $\Delta t_{\text{start}}^{\text{H1}}/M = 0$ . Now, the decay time for the overtone in question is  $\tau_{221} \approx 1.3 \text{ ms} \approx 4M$ . If the overtone were measurable, we would expect to find evidence for its presence when changing  $t_{\text{start}}^{\text{H1}}$  by only ~0.24 ms ≈ 0.72*M*. Since this is not the case, we must consider the hypothesis that the (weak) evidence in favor of an overtone for  $\Delta t_{\text{start}}^{\text{H1}}/M = 0$  could be driven by a noise fluctuation.

We test this hypothesis by using a synthetic signal ("injection," in LVK jargon) obtained from a numerical solution of the Einstein equations consistent with the GW150914 signal [\[71\]](#page-6-16) (see Supplemental Material [\[64\]](#page-6-12) for details). In this case,  $t_{\text{peak}}^{\text{H1}}$  is known exactly. We analyze the signal using different values of  $t_{\text{start}}^{\text{H1}}$ , such that  $\Delta t_{\text{start}}^{\text{H1}}$  is consistent with the values used for the real signal. For each selected  $\Delta t_{\text{start}}^{\text{H1}}$ , we first perform the analysis described above in the case of the real signal, but we now set the noise realization to zero ("zero-noise" injection). The resulting parameter distributions will thus have an uncertainty consistent with the actual signal, while eliminating a possible shift of the posterior median due to noise fluctuations coincident with the signal. The values of  $log_{10} B_{220}^{221}$ and  $A_1$  obtained from this zero-noise injection are shown as black dots and black curves in the upper and lower panels of Fig. [2.](#page-2-1) When  $\Delta t_{\text{start}}^{\text{H1}}/M = 0$  there is no evidence for an overtone  $(\log_{10} \mathcal{B}_{220}^{221} = -0.21 < 0)$  and  $A_1$  has a large support for zero. For the zero-noise injection, the Bayes factor is greater than unity only when  $\Delta t_{\text{start}}^{\text{H1}}/M \le -1.45$ , and it generally increases for lower values of  $\Delta t_{\text{start}}^{\text{H1}}$ , similarly to what happens for the real signal. The inferred amplitude of the overtone is consistent with the behavior observed for the Bayes factor, increasing for large negative values of  $\Delta t_{\text{start}}^{\text{H1}}/M$ .

To assess the impact of the detector noise on the measurement of  $\log_{10} \mathcal{B}_{220}^{221}$  and  $A_1$ , for each  $\Delta t_{\text{start}}^{\text{H1}}$  we repeat the above analysis superposing the simulated signal to ten different segments of the real detector noise close to

the time of coalescence of GW150914 (see Supplemental Material [\[64\]](#page-6-12)). The resulting Bayes factors are reported as blue dots and related "error bars" on  $log_{10} B_{220}^{221}$ : for each time  $\Delta t_{\text{start}}^{\text{H1}}$ , each dot corresponds to a specific noise realization, while the upper (lower) boundary of the error bar corresponds to the largest (smallest)  $log_{10} B_{220}^{221}$  obtained from these injections. The blue curves in the lower panel are the posterior distributions of  $A_1$  corresponding to the different noise realizations. These distributions (to be compared with the zero-noise black curves) quantify the impact of noise fluctuations on amplitude measurements. For  $\Delta t_{\text{start}}^{\text{H1}}/M = 0$  and neighboring points, the negative values of  $log_{10} B_{220}^{221}$  measured in the real signal are consistent with the negative values measured in the synthetic signal, if we account for the detector noise. The posterior distributions of  $A_1$  show that a "favorable" realization of the detector noise can lead to a measurement of  $A_1$  that peaks away from zero (blue curves)—similarly to the actual signal (red curve)—although  $A_1$  is consistent with zero in the case of the zero-noise injection (black curve). We conclude that the mild support for an overtone observed in the amplitude posterior (although never confirmed by the Bayesian evidence) is driven by the detector noise.

Discussion.—We have performed a Bayesian data analysis of the GW150914 ringdown signal to understand if ringdown overtone detection claims are robust. We found no Bayesian evidence in favor of an overtone, nor a significant overtone amplitude measurement in GW150914 data after the waveform peak, where the inclusion of overtones in the ringdown model is expected to improve the agreement with numerical relativity simulations [[40](#page-5-24)[,42\]](#page-5-26). There is mild support for a nonzero overtone amplitude in the data at the peak, but such support for  $A_1 = 0$  is sensitive to changes in the starting time smaller than the overtone damping time. Most importantly, the Bayes factors never favor the detection of an overtone when varying the starting time within the  $1\sigma$  credible region of the peak time reconstruction. This suggests that the detection is noise dominated. We verified this hypothesis by performing GW150914-like injections in different segments of the real detector noise. These results differ from Ref. [[59](#page-6-7)], where the impact of the real detector noise and peak time uncertainty were not considered.

For both real and synthetic signals, the evidence for the overtone and the uncertainty on the evidence (as measured by the blue "error bars") generally increase for large negative values of  $\Delta t_{\text{start}}^{\text{H1}}$ . The overtone model is not expected to be valid in this region, but the larger number of degrees of freedom in the model can pick up a larger portion of the low-frequency, premerger signal power. At the same time, the evidence uncertainty grows dramatically—spanning up to 4 orders of magnitude for the earliest times shown in

Fig. [2](#page-2-1)—because the poorly constrained model can easily pick up noise fluctuations.

Our results reveal an intrinsic instability of the inference based on such a model. The instability may happen even in the absence of noise, because the mass and spin of the remnant extracted from numerical simulations vary significantly close to the peak of the radiation [[27](#page-5-16),[40](#page-5-24),[72](#page-6-17)], and thus the assumption of a linear superposition of QNMs starting at the peak can lead to conceptual issues [\[43,](#page-5-27)[73](#page-6-18)]. As reported in Table I of Ref. [\[42\]](#page-5-26), the amplitude of the fundamental mode is stable up to a few parts in  $10<sup>3</sup>$  under the addition of overtones, but higher overtones have much less stable amplitudes:  $A_{221}$  varies by 8%, while  $A_{223}$  varies by more than 200%. This is inconsistent with our understanding of ringdown in the linearized regime, where (by definition) the QNM amplitudes should be constant [\[41](#page-5-25)[,44](#page-5-29),[74](#page-6-19),[75](#page-6-20)]. This phenomenon was also found in Ref. [[76](#page-6-21)] over the full nonprecessing parameter space. In the absence of fitting errors for the overtone amplitudes, it is difficult to quantify how much of this variation can be ascribed to the current accuracy of numerical BH merger simulations, rather than being due to a time-evolving background. This instability might also explain the incompatibility of the measurement  $A_{221}/A_{220} \leq 2$  reported in [\[53](#page-6-1)[,59\]](#page-6-7), compared to the predicted value  $A_{221}/A_{220} \sim 4$ reported in Table I of [\[42\]](#page-5-26).

A physical parametrization of the overtone amplitudes as a function of the progenitors parameters, similar to the one proposed in Refs. [[41](#page-5-25),[75](#page-6-20)] for the fundamental modes, may alleviate this problem. However, parametrizations of nonspinning binary BH mergers find that such a "global" fit is not robust under variations of the starting time: see, e.g., Figs. 3 and 4 of [\[44\]](#page-5-29). Overfitting issues are particularly difficult to address. For example, the accuracy of overtone models constructed using GR QNMs can be matched (or even surpassed) by adding "unphysical" low-frequency components corresponding to non-GR values of the frequency and damping time [[43](#page-5-27),[47](#page-5-30)]. Similar "pseudo-QNMs" were introduced in the context of effective-onebody models [[77](#page-6-22)–[79\]](#page-6-23).

Our results for the Bayes factors are consistent with previous work. The large number of free parameters in the overtone model introduces an Occam penalty that must be balanced by large SNRs [\[45\]](#page-5-28). Even when modeling the overtone amplitudes as functions of the properties of the remnant progenitors, measuring several overtone frequencies may still be impractical: Fisher matrix estimates [\[44\]](#page-5-29) suggest that it will be easier to obtain evidence for multiple modes using higher angular harmonics rather than overtones. These results are in contrast with the predictions of [\[59\]](#page-6-7), which employed a different detection criterion. In future work we plan to investigate strategies for a robust modeling and measurement of higher overtones, and to revisit the BH spectroscopy horizon estimates of Refs. [[66](#page-6-24),[80](#page-6-25)].

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Software.—LIGO-Virgo data are interfaced through GWpy [[81](#page-6-26)]. Projections onto detectors are computed through LALSuite [\[82\]](#page-6-27). The ACFs are computed using the function get acf of the ringdown package [[59](#page-6-7)]. The pyRing package is publicly available at [[83](#page-6-28)]. We use the cpnest version 0.11.3 and the pyRing commit 2b96c569ff663bb71dabe6dae5f4177b79854340 on the master branch. To allow for reproducibility, we release the configuration file employed for our analysis at the reference time, see Ref. [[84](#page-6-29)]. The other results on observational data can be reproduced by changing the starting time by the amount specified in Fig. [2,](#page-2-1) while we give the details needed to reproduce the injections in Supplemental Material [[64](#page-6-12)]. This study made use of the opensoftware python packages: corner, cython, h5py, matplotlib, numpy, scipy, seaborn [[85](#page-6-30)–[91\]](#page-6-31).

Note added.—Recently, some of the authors of [\[53\]](#page-6-1) revisited their original analysis, extending it to multiple times around the peak [[65](#page-6-32)]. In Supplemental Material [\[64\]](#page-6-12) we present a comparison with their publicly available data. Small differences between the two analyses (i.e., a different sampling algorithm, data sampling rate, and autocorrelation function estimation method) lead to moderately different overtone amplitudes, but we observe broad agreement with our main results. In particular, both sets of posteriors show significant railing against zero within the peak time uncertainty. This comparison does not point to any fundamental discrepancy between the two investigations, and our conclusions are unaltered. A third independent reanalysis [\[92\]](#page-6-33) made use of a standard frequency domain approach employed for most of the LVK parameter estimation runs, hence relying on extensively tested algorithms for sampling and estimation of the noise properties. The authors confirm our main conclusions. They report a "modest"  $(1.8\sigma)$ significance for the detection of an overtone, whereas Ref. [\[53\]](#page-6-1) claimed "3.6 $\sigma$  confidence." Perhaps more remarkably, the authors of Ref. [[92](#page-6-33)] find a negative Bayes factor in favor of an overtone when marginalizing over all of the relevant uncertainty in the peak strain time. Their work confirms that current detection claims depend on subtle data analysis details (such as, e.g., frequency-domain vs time-domain estimation of the noise properties), which should not have any impact on a robust detection.

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