Yangian Symmetry in Holographic Correlators

Konstantinos C. Rigatos
Department of Physics and Center for Theory of Quantum Matter, University of Colorado Boulder, 390 UCB, Colorado 80309, USA

Xinan Zhou (周稀楠)*
Kavli Institute for Theoretical Sciences, University of Chinese Academy of Sciences, Beijing 100190, China

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We point out that an infinite class of Witten diagrams is invariant under a Yangian symmetry. These diagrams are building blocks of holographic correlators and are related by a web of differential recursion relations. We show that Yangian invariance is equivalent to the consistency conditions of the recursion relations.

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Introduction.—Recently, there has been much progress in computing holographic correlators, which are the most basic observables for exploring and exploiting the AdS/CFT correspondence. For example, all four-point correlators of $\frac{1}{2}$-BPS operators with arbitrary Kaluza-Klein weights are known at tree level in all maximal supergravity theories [1–4] and super Yang-Mills theory (SYM) in AdS [5]. Examples of higher-point correlators have also been obtained in AdS_5 [6,7]. While these results are highly impressive, they are all obtained by using essentially the same kind of method, namely, the bootstrap approach which imposes superconformal symmetry and physical consistency conditions [8]. It is important to ask if there are other independent guiding principles which allow us to efficiently compute holographic correlators. Particularly, in the paradigmatic example of the AdS/CFT, the 4d $\mathcal{N}=4$ SYM theory, which is dual to IIB string theory in $\mathrm{AdS}_5\times S^5$, is known to be integrable in the planar limit. It is natural to wonder if integrability can play a role in the study of holographic correlators. Unfortunately, the standard integrability techniques are known to have difficulties in the supergravity regime [10]. As a result, a concrete relation between integrability and holographic correlators remains elusive. However, in this paper, we will provide hints for such a relation by pointing out that an infinite class of Witten diagrams in AdS enjoys a Yangian symmetry, which is a hallmark of integrability. While we consider only bosonic symmetry here, we hope that the analysis can be generalized to the supersymmetric case as well.

More precisely, we consider the contact Witten diagrams depicted in Fig. 1, which appear naturally in holographic models of boundary CFTs. The vertical co-dimension 1 surface is the holographic dual of the boundary. When all insertions are moved to the boundary, the diagrams are fully within the AdS_d subspace and reduce to the so-called D-functions in the AdS/CFT literature. These contact Witten diagrams are the building blocks of holographic correlators. As we will show, these diagrams can be identified with the following conformal Feynman integral in D-dimensional flat space

$$I_n=\int\frac{d^D x_0}{\prod_{j=1}^n (x_{j0}^2+m_j^2)^{\Delta_i}}, \tag{1}$$

where $x_{ij}^\mu=x_i^\mu-x_j^\mu$, $x_{ij}^2=x_{ij}^\mu x_{ij,\mu}$, and $\sum_{i=1}^n\Delta_i=D$. The perpendicular distances $x_{i,\perp}$ are identified with the masses m_i. These integrals, which generalize box diagrams, are remarkably invariant under the conformal Yangian algebra. The discovery of this property was motivated by special cases of such diagrams appearing in the so-called fishnet theories which are known to be integrable [16–19]. Integrability of (1) was first proven in the massless case, for integrals with $n=4, 6$. The proof was streamlined and

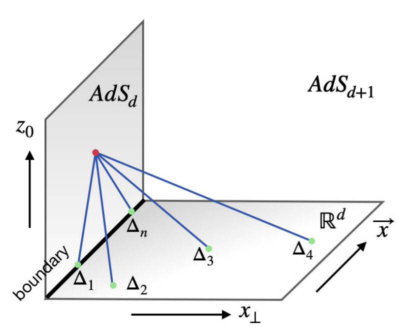

FIG. 1. A contact Witten diagram in Poincaré coordinates.

extended to the massive case in [20,21], where it was shown that all such integrals are Yangian invariant. Since contact Witten diagrams are essentially I_n, it follows that they are Yangian invariant as well. On the other hand, we will show that the contact Witten diagrams satisfy an intricate web of differential recursion relations shifting the weights Δ_i. For example, there are differential operators $\mathbb{O}_{ij}$ which shift Δ_i and Δ_j by 1

$$\mathbb{O}_{ij} W \propto W|_{\Delta_{i,j}\to\Delta_{i,j}+1}, \tag{2}$$

For these relations to be consistent, the action of $\mathbb{O}_{ij}\mathbb{O}_{kl}$ must be equal to that of $\mathbb{O}_{ik}\mathbb{O}_{jl}$ as they lead to the same contact Witten diagram. This imposes nontrivial constraints on W. Remarkably, we find that the full set of consistency conditions is precisely the Yangian invariance condition.

Yangian generators.—The Feynman integrals (1) are invariant under the conformal group $\mathrm{SO}(D,2)$ which is generated by $\mathrm{J}^a = \sum_{j=1}^n \mathrm{J}_j^a$. Here J_j^a are single-site generators acting on x_j

$$\begin{aligned}
&\mathrm{P}_j^{\hat\mu} = -i\partial_{x_j}^{\hat\mu}, \qquad \mathrm{L}_j^{\hat\mu\hat\nu} = i x_j^{\hat\mu}\partial_{x_j}^{\hat\nu} - i x_j^{\hat\nu}\partial_{x_j}^{\hat\mu},\\
&\mathrm{D}_j = -i(x_{j,\mu}\partial_{x_j}^\mu + m_j\partial_{m_j} + \Delta_j),\\
&\mathrm{K}_j^{\hat\mu} = -2i x_j^{\hat\mu}(x_{j,\nu}\partial_{x_j}^\nu + m_j\partial_{m_j} + \Delta_j) + i(x_j^2 + m_j^2)\partial_{x_j}^{\hat\mu},
\end{aligned} \tag{3}$$

and μ runs from 1 to D. The index $\hat\mu$ runs from 1 to $D+1$, but only $\hat\mu = 1,\ldots,D$ correspond to the symmetries of I_n. Note that with $\hat\mu = 1,\ldots,D+1$, (3) are the $\mathrm{SO}(D+1,2)$ conformal generators in $D+1$ dimensions where the $(D+1)$th dimension is $x^{D+1} = m$. Conformal symmetry is partially broken along this dimension to $\mathrm{SO}(D,2)$. The symmetry breaking is exactly the same as inserting a boundary at $x^{D+1} = 0$.

The massive Yangian is generated by the above level-zero generators and the following level-one generators [22]

$$\hat{\mathrm{J}}^a = \frac{1}{2} f^a{}_{bc}\sum_{j<k}^n \mathrm{J}_j^c\mathrm{J}_k^b + \sum_{j=1}^n s_j\mathrm{J}_j^a, \tag{4}$$

where $f^a{}_{bc}$ are the structure constants and s_i are the evaluation parameters. The integrals I_n are annihilated by the level-one generators, and consequently the entire Yangian. Moreover, I_n are invariant under level-zero generators and the level-one generators are in the adjoint representation of the level-zero algebra. It is therefore sufficient to require that I_n is annihilated by the level-one momentum operators $\hat{\mathrm{P}}^\mu$. Furthermore, because (1) also has permutation symmetry, invariance under $\hat{\mathrm{J}}^a$ is equivalent to invariance under any two-site operators [21]

$$\hat{\mathrm{J}}_{jk}^a = \frac{1}{2} f^a{}_{bc}\mathrm{J}_j^c\mathrm{J}_k^b + \frac{\Delta_k}{2}\mathrm{J}_j^a - \frac{\Delta_j}{2}\mathrm{J}_k^a. \tag{5}$$

In terms of $\hat{\mathrm{J}}_{jk}^a$, $\hat{\mathrm{J}}^a$ can be written as $\hat{\mathrm{J}}^a = \sum_{k>j=1}^n \hat{\mathrm{J}}_{jk}^a$. The momentum operator is given by

$$\hat{\mathrm{P}}_{jk}^\mu = \frac{i}{2}[\mathrm{P}_j^\mu\mathrm{D}_k + \mathrm{P}_{j,\nu}\mathrm{L}_k^{\mu\nu} - i\Delta_k\mathrm{P}_j^\mu - (j\leftrightarrow k)]. \tag{6}$$

Using (3), we can write it explicitly as

$$\begin{aligned}
\hat{\mathrm{P}}_{jk}^\mu = \frac{i}{2}[&X^{\nu\mu\rho}\partial_{x_j,\rho}\partial_{x_k,\nu} + (2\Delta_j + m_j\partial_{m_j})\partial_{x_k}^\mu\\
&- (2\Delta_k + m_k\partial_{m_k})\partial_{x_j}^\mu],
\end{aligned} \tag{7}$$

where

$$X^{\nu\mu\rho} = x_{jk}^\nu\eta^{\mu\rho} + x_{jk}^\rho\eta^{\mu\nu} - x_{jk}^\mu\eta^{\nu\rho}. \tag{8}$$

In addition to the above operators $\hat{\mathrm{J}}_{jk}^a$, it was observed in [20] that I_n are also annihilated by an extra set of bilocal operators $\hat{\mathrm{J}}_{\mathrm{extra},jk}^a$. For example,

$$\begin{aligned}
\hat{\mathrm{P}}_{jk,\mathrm{extra}}^\mu &= \frac{i}{2}[\mathrm{P}_{j,D+1}\mathrm{L}_k^{\mu,D+1} - (j\leftrightarrow k)]\\
&= \frac{i}{2}[\partial_{m_j}x_k^\mu\partial_{m_k} - \partial_{m_j}m_k\partial_{x_k}^\mu - (j\leftrightarrow k)].
\end{aligned} \tag{9}$$

Here, we have written down a mass m_i for each site i. The massless (or partially massless) case is obtained by just setting the masses to zero.

Witten diagrams.—The contact Witten diagram in Fig. 1 is defined as an integral over AdS_d

$$W = \int \frac{dz_0 d^{d-1}\vec z}{z_0^d}\prod_{i=1}^n G_{B\partial}^{\Delta_i}(z,\vec x_i,m_i), \tag{10}$$

where $G_{B\partial}^{\Delta_i}$ are the bulk-to-boundary propagators

$$G_{B\partial}^{\Delta_i}(z,\vec x_i,m_i) = \left(\frac{z_0}{z_0^2 + (\vec z - \vec x_i)^2 + m_i^2}\right)^{\Delta_i}. \tag{11}$$

These diagrams arise in holographic models of boundary CFTs or interface CFTs where the defect is a probe brane [24–28]. They are generated by contact vertices which are localized on the AdS_d subspace. When all masses are zero, W reduces to the D function $D_{\Delta_1,\ldots,\Delta_n}$ in AdS_d. Note that unlike the Feynman integral I_n, there is no constraint relating Δ_i and d. The conformal invariance of W is inherited from the isometry of AdS. These contact diagrams have been systematically studied in [28] and we will use its results to establish the equivalence between W and I_n.

A particularly useful representation of W is [28]

$$W = C_n \int_0^\infty \prod_{i=1}^n dt_i t_i^{\Delta_i - 1} e^{-\sum_{i<j} t_i t_j P_{ij} - (\sum_{i=1}^n t_i m_i)^2}, \quad (12)$$

which is obtained by using the Schwinger parametrization and integrating out the AdS coordinates. Here, $C_n = \pi^{((d-1)/2)}\Gamma[((\sum_{i=1}^n \Delta_i - d + 1)/2)] \prod_{i=1}^n \Gamma^{-1}[\Delta_i]$ is a coefficient and we have defined

$$P_{ij} = x_{ij}^2 + (m_i - m_j)^2. \quad (13)$$

An important consequence of (12) is that contact Witten diagrams are *dimension independent* after factoring out a numerical coefficient

$$\tilde{W} = C_n^{-1} W. \quad (14)$$

On the other hand, if we integrate out only the radial coordinate z_0, we find

$$\tilde{W} = \frac{\pi^{\frac{1-d}{2}}}{2} \int d^{d-1}\vec{z} \int_0^\infty \prod_{i=1}^n dt_i t_i^{\Delta_i - 1} \times \left(\sum_{i=1}^n t_i\right)^{\frac{d-1-\sum_{i=1}^n \Delta_i}{2}} e^{-\sum_{i=1}^n t_i((\vec{z}-\vec{x}_i)^2 + m_i^2)}. \quad (15)$$

Using the d independence of $\tilde{W}$, we can conveniently set $d = D + 1$. Then (15) is nothing but the conformal integral I_n after using the Schwinger parametrization

$$\tilde{W} = \frac{\pi^{-\frac{\sum_{i=1}^n \Delta_i}{2}} \prod_{i=1}^n \Gamma[\Delta_i]}{2} I_n. \quad (16)$$

Since the integrals I_n are invariant under the Yangian [20,21,29], the contact Witten diagrams W are Yangian invariant as well.

Recursions and consistency conditions.—The representation (12) also makes the recursion relations of Witten diagrams manifest. Let us denote

$$\mathbb{O}_{ij} = \frac{\partial}{\partial P_{ij}}\bigg|_{P,m}, \qquad N_i = \frac{\partial}{\partial m_i}\bigg|_{P,m} \quad (17)$$

as the partial derivatives, where P_{ij}, m_i are regarded as the independent variables. Then N_i is related to ∂_{m_i} in (3), where x_i^μ and m_i are regarded as the independent variables, by

$$m_i \partial_{m_i} = \mathbb{D}_i + 2\sum_{j\neq i} m_i^2 \mathbb{O}_{ij}, \quad (18)$$

and we have defined

$$\mathbb{D}_i = m_i N_i - 2\sum_{j\neq i} m_i m_j \mathbb{O}_{ij}. \quad (19)$$

From the integral representation (12), it is obvious that we have the following *differential recursion relations*

$$\mathbb{O}_{ij} W = \frac{2\Delta_i \Delta_j}{d - 1 - \sum_i \Delta_i} W\bigg|_{\Delta_{i,j}\to\Delta_{i,j}+1}, \quad (20)$$

$$\mathbb{D}_i W = \frac{4m_i^2 \Delta_i(\Delta_i + 1)}{d - 1 - \sum_i \Delta_i} W\bigg|_{\Delta_i\to\Delta_i+2}, \quad (21)$$

which shift the conformal dimensions. These relations generalize the well-known weight-shifting relations of D functions [30]. However, the relations (20) and (21) must give the same answer when reaching the same point in weight space following different paths. This gives rise to the following *consistency conditions*

$$(\mathbb{O}_{ij}\mathbb{O}_{kl} - \mathbb{O}_{ik}\mathbb{O}_{jl}) W = 0, \qquad i, l \neq j, k, \quad (22)$$

$$\mathbb{D}_i \mathbb{O}_{kl} W = 2m_i^2 \mathbb{O}_{ik}\mathbb{O}_{il} W, \qquad i, j, k \text{ all different}, \quad (23)$$

$$\mathbb{D}_j \mathbb{D}_k W = 4m_j^2 m_k^2 \mathbb{O}_{jk}\mathbb{O}_{jk} W, \qquad j \neq k. \quad (24)$$

Note that these conditions are also satisfied by I_n because they are identical to W up to overall coefficients.

Let us also mention that the conformal invariance of Witten diagrams implies the following relations

$$(m_i N_i + P_{ij}\mathbb{O}_{ij}) W = -\Delta_i W. \quad (25)$$

These conditions can be easily derived in the embedding space formalism and they follow from requiring W to scale correctly when independently rescaling the embedding vector of each operator [28]. The details can be found in the Supplemental Material [31]. It is conceivable that the conditions (22), (23), (24) should also have a symmetry origin. Since only conformal symmetry is involved in this setup, the natural guess, as will be verified, is the conformal Yangian.

Yangian constraints as consistency conditions.—We now show that the Yangian invariance conditions

$$\hat{\mathrm{P}}_{jk}^\mu W = 0, \quad (26)$$

$$\hat{\mathrm{P}}_{jk,\mathrm{extra}}^\mu W = 0, \quad (27)$$

are equivalent to the consistency conditions of the recursion relations (22), (23), (24). Instead of working with cross ratios, which spoils manifest permutation symmetry, we work with the variables P_{ij} and m_i. Then, using

$$\partial^\mu_{x_j} = 2\sum_{i\neq j} x^\mu_{ji}\mathbb{O}_{ij},$$
$$\partial^\rho_{x_j}\partial^\nu_{x_k} = 4\sum_{i\neq k}\sum_{l\neq j} x^\rho_{jl}x^\nu_{ki}\mathbb{O}_{jl}\mathbb{O}_{ki} - 2\eta^{\rho\nu}\mathbb{O}_{jk}, \tag{28}$$

and (18) we can take all derivatives with respect to P_{ij} and m_i. We will find that the action of the operators can be written in the form

$$-2i\hat{\mathrm{P}}^\mu_{jk}W = \sum_{a<b}\mathrm{T}^\mu_{ab}E_{ab},$$
$$-2i\hat{\mathrm{P}}^\mu_{jk,\mathrm{extra}}W = \sum_{a<b}\mathrm{T}^\mu_{ab}E_{ab,\mathrm{extra}}, \tag{29}$$

where $\mathrm{T}^\mu_{ab} = (x^\mu_{ab}/P_{ab})$. The coefficients E_{ab}, $E_{ab,\mathrm{extra}}$ have the same scaling dimensions as W. It was shown in [21] that T^μ_{ab} are linearly independent with respect to coefficients which are functions of cross ratios [32]. Yangian invariance then requires that all coefficient functions E_{ab}, $E_{ab,\mathrm{extra}}$ must vanish separately. The upshot is that these conditions boil down to the three basic relations (22), (23) and (24).

The massless case.—For simplicity, let us first demonstrate the equivalence for the massless case, i.e., $m_i = 0$, which is relevant for D functions in pure AdS. Note that $\hat{\mathrm{P}}^\mu_{jk,\mathrm{extra}}$ vanishes in this case so we have only (26) with m_i set to zero. From (7) it is not difficult to see that almost all terms are already in the form of (29), except for those coming from the contraction with $X^{\nu\mu\rho}$. To proceed, we note the following useful identity

$$X^{\nu\mu\rho}x^\rho_{jl}x^\nu_{ki} = \frac{1}{2}(\mathrm{T}^\mu_{jk}P_{jk}P_{li} - \mathrm{T}^\mu_{ji}P_{ji}P_{kl} - \mathrm{T}^\mu_{jl}P_{jl}P_{ki} + \mathrm{T}^\mu_{ki}P_{ki}P_{jl} + \mathrm{T}^\mu_{kl}P_{kl}P_{ij} - \mathrm{T}^\mu_{il}P_{il}P_{jk}). \tag{30}$$

We then find all the coefficient functions are given by

$$E_{il} = -2P_{il}P_{jk}(\mathbb{O}_{jl}\mathbb{O}_{ik} - \mathbb{O}_{ji}\mathbb{O}_{kl})W, \tag{31}$$

$$E_{ki} = 2\Bigg\{\sum_{l\neq j,k}P_{ki}P_{jl}\mathbb{O}_{jl}\mathbb{O}_{ki} + 2P_{ki}P_{jk}\mathbb{O}_{jk}\mathbb{O}_{ki} + \sum_{l\neq j,k}P_{ki}P_{jl}\mathbb{O}_{ji}\mathbb{O}_{kl} + 2\Delta_j P_{ki}\mathbb{O}_{ki}\Bigg\}W, \tag{32}$$

$$E_{jl} = -E_{ki}|_{j\leftrightarrow k, i\leftrightarrow l}, \tag{33}$$

$$E_{jk} = 2\Bigg\{\sum_{i,l\neq j,k}P_{jk}P_{il}\mathbb{O}_{jl}\mathbb{O}_{ki} - 2P^2_{jk}\mathbb{O}_{jk}\mathbb{O}_{jk} - (2-D+2\Delta_j+2\Delta_k)P_{jk}\mathbb{O}_{jk}\Bigg\}W, \tag{34}$$

where $i, l \neq j,\ k$ [33]. From $E_{il} = 0$, we reproduce the consistency condition (22). Since (23) and (24) are identically zero on both sides in the massless limit, the remaining conditions must not produce nontrivial constraints. To show $E_{ki} = 0$, we first use (22) to write E_{ki} as

$$E_{ki} = 4\Bigg\{\sum_{l\neq j,k}P_{ki}P_{jl}\mathbb{O}_{jl}\mathbb{O}_{ki} + P_{ki}P_{jk}\mathbb{O}_{jk}\mathbb{O}_{ki} + \Delta_j P_{ki}\mathbb{O}_{ki}\Bigg\}W. \tag{35}$$

Then using the massless limit of (25) we find that E_{ki} vanishes. Symmetry implies that $E_{jl} = 0$ as well. To see $E_{jk} = 0$, we use permutation symmetry and (22) to write

$$\sum_{i,l\neq j,k}P_{jk}P_{il}\mathbb{O}_{jl}\mathbb{O}_{ki}W = \sum_{i,l\neq j,k}P_{jk}P_{il}\mathbb{O}_{jk}\mathbb{O}_{il}W.$$

From (25) we also have

$$\sum_{i,l\neq j,k}P_{il}\mathbb{O}_{il}W = (-D + 2\Delta_j + 2\Delta_k + 2P_{jk}\mathbb{O}_{jk})W,$$

where we have used $D = \sum_{i=1}^n \Delta_i$. It is then clear that E_{jk} also vanishes.

The massive case.—Having proven the equivalence in the massless limit, let us now move on to the general case. We first focus on the condition (26) where the proof is similar to the massless case above. To cast the action of $\hat{\mathrm{P}}^\mu_{jk}$ in the form of (29), let us use the following massive version of (30)

$$X^{\nu\mu\rho}x^\rho_{jl}x^\nu_{ki} = \frac{1}{2}(\mathrm{T}^\mu_{jk}P_{jk}P_{li} - \mathrm{T}^\mu_{ji}P_{ji}P_{kl} - \mathrm{T}^\mu_{jl}P_{jl}P_{ki} + \mathrm{T}^\mu_{ki}P_{ki}P_{jl} + \mathrm{T}^\mu_{kl}P_{kl}P_{ij} - \mathrm{T}^\mu_{il}P_{il}P_{jk}) + x^\mu_{jl}m_k(m_k - m_i) - x^\mu_{ki}m_j(m_j - m_l) + x^\mu_{ij}m_k m_l + x^\mu_{jk}m_i m_l + x^\mu_{kl}m_i m_j + x^\mu_{li}m_j m_k.$$

We find the coefficient functions are

$$P^{-1}_{il}E_{il} = -2(P_{jk} + 2m_j m_k)(\mathbb{O}_{jl}\mathbb{O}_{ki} - \mathbb{O}_{ji}\mathbb{O}_{kl})W, \tag{36}$$

$$P^{-1}_{ki}E_{ki} = 2\Bigg\{(2\Delta_j + m_j N_j)\mathbb{O}_{ki} + 2m_j m_k\mathbb{O}_{jk}\mathbb{O}_{ki} + \sum_{l\neq k}(P_{jl} + 2m_l m_j)\mathbb{O}_{ji}\mathbb{O}_{kl} + \sum_{l\neq j}P_{jl}\mathbb{O}_{jl}\mathbb{O}_{ki} + P_{jk}\mathbb{O}_{jk}\mathbb{O}_{ki}\Bigg\}W, \tag{37}$$

$$P^{-1}_{jl}E_{jl} = -P^{-1}_{ki}E_{ki}|_{j\leftrightarrow k, i\leftrightarrow l}, \tag{38}$$

$$P_{jk}^{-1}E_{jk}=2\Big\{\sum_{i,l\neq j,k}P_{il}\mathbb{O}_{jl}\mathbb{O}_{ki}-2P_{jk}\mathbb{O}_{jk}\mathbb{O}_{jk}+(D-2-2\Delta_j-2\Delta_k-m_jN_j-m_kN_k)\mathbb{O}_{jk}+2\sum_{i\neq k}\sum_{l\neq j}m_im_l\mathbb{O}_{jl}\mathbb{O}_{ki}-2m_jm_k\mathbb{O}_{jk}\mathbb{O}_{jk}\Big\}W, \quad (39)$$

with $i, l \neq j, k$. Requiring (36) to vanish, we recover the first consistency condition (22). Following similar manipulations as in the massless case, which are detailed in the Supplemental Material [31], we find from $E_{ki}=0$ the second consistency condition (23). However, the condition from the coefficient (39) yields no further constraint. In fact, we find that $E_{jk}=0$ follows from (22) and (23). To derive the last consistency condition (24), we must examine the extra contraint (27). The explicit operator action reads

$$-2im_jm_k\hat{\mathrm{P}}^{\mu}_{jk,\mathrm{extra}}W=-x^{\mu}_{jk}m_j\partial_{m_j}m_k\partial_{m_k}W-2\sum_{l\neq k}x^{\mu}_{kl}\mathbb{O}_{kl}m_k^2m_j\partial_{m_j}W+2\sum_{i\neq j}x^{\mu}_{ji}\mathbb{O}_{ji}m_j^2m_k\partial_{m_k}W, \quad (40)$$

where $m_j\partial_{m_j}$ should be expressed in terms of $\mathbb{D}_j$ and $\mathbb{O}_{ij}$ using (18). Naively, the form of (40) seems to be in contradiction with (29). However, this expression can be greatly simplified upon using (22), (23) and the conformal invariance condition (25) (see Supplemental Material [31]). We find that all $E_{ab,\mathrm{extra}}$ vanish except for $E_{jk,\mathrm{extra}}$

$$E_{jk,\mathrm{extra}}=-P_{jk}(\mathbb{D}_j\mathbb{D}_k-4m_j^2m_k^2\mathbb{O}_{jk}\mathbb{O}_{jk})W, \quad (41)$$

which gives the last condition (24).

Discussions and outlook.—In this Letter, we established a new connection between integrability and holography by reinterpreting Yangian invariant Feynman integrals as Witten diagrams in AdS. We also provided an interesting reformulation of the Yangian constraints as the consistency conditions of weight-shifting relations satisfied by Witten diagrams. These conditions are obtained explicitly as (22), (23), (24), and are valid for arbitrary n-point functions. Compared to the original Yangian invariance constraints (26) and (27), these conditions no longer contain redundancies and are much simpler to exploit (e.g., to explicitly compute I_n as power series [20,21]). The remarkable simplicity of these conditions might also provide further insight into their underlying structures and hopefully open a door to applying the full power of integrability methods to holographic correlators.

There are plenty of future directions worth exploring. First, we only focused on contact Witten diagrams which correspond to one-loop Feynman integrals. It would be interesting to study Yangian symmetry in exchange Witten diagrams. Certain two-loop Feynman integrals are also known to be Yangian invariant [20,21,29] and coincide with exchange Witten diagrams when conformal dimensions satisfy special conditions [34,35]. However, the general story is still unclear at the moment. Second, another exciting research avenue is to extend the analysis to include supersymmetry. The superconformal Yangian constraints should be highly nontrivial and will presumably select "superspace D-functions" with quantized dimensions as their solutions. It would be extremely interesting to see if these superconformal Yangian constraints can be used as an alternative method to rederive the general results of holographic four-point correlators of IIB supergravity in $\mathrm{AdS}_5\times S^5$ [1,2]. Finally, Witten diagrams also play an important role in the analytic functional approach to the conformal bootstrap where they serve as generating functions for the analytic functionals [36–42]. It would be interesting to explore the consequence of Yangian symmetry in that context.

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*Corresponding author.
xinan.zhou@ucas.ac.cn

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