

Perfect Fluid Hydrodynamic Picture of Domain Wall Velocities at Strong Coupling

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We show that for a range of strongly coupled theories with a first order phase transition, the domain wall or bubble velocity can be expressed in a simple way in terms of a perfect fluid hydrodynamic formula, and thus in terms of the equation of state. We test the predictions for the domain wall velocities using the gauge/gravity duality.

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Introduction.—The study of the dynamics of the expansion of nucleated bubbles in a theory with a first order phase transition has recently gained renewed interest in view of gravitational wave detection experiments [1–5], as the collisions and coalescence of bubbles as well as the accompanying plasma in the early Universe may act as a source of potentially detectable gravitational waves.

In this context, a key parameter of interest is the velocity of the bubble wall. Despite the fact that there is a pressure difference between the two phases, any initial accelerated motion very soon stabilizes to a motion with a uniform velocity. The conventional explanation is that the net force is balanced by the friction of the metastable phase, which is, however, very challenging to compute (see Refs. [6–8] as well as Refs. [9–12] and references therein for recent work in this direction). From this perspective, the domain wall velocity is thus a consequence of nonequilibrium dynamics of the theory.

In this Letter we argue that for a range of strongly coupled theories which have a holographic dual, the physics of the uniform motion of the domain wall is much simpler and does not require discussing the nontrivial nonequilibrium or dissipative regime of the theory. Indeed, we show that the domain wall velocity may be reliably computed using just perfect fluid hydrodynamics and thus the equation of state. We test the predictions using holographic modeling of the process of bubble expansion both for nucleated bubbles of a stable phase within an supercooled medium and for an interface between two phases at different temperatures.

In order to be able to compare with numerical holographic simulations, all the domain walls and bubbles that we consider have a planar symmetry. The theories we model are a confining-deconfined system and a bottom-up holographic model with two deconfined phases. We also compare our predictions with some holographic results from Ref. [13].

The holographic frameworks.—A method of describing confinement in holographic settings is to start from a geometry dual to a $4 + 1$ dimensional quantum field theory and compactify one of the spatial dimensions on a circle [14–16]. Then the confining geometry takes the form of a cigar in the compactified and holographic directions. The circumference of the cigar is identified as the scale of confinement in the resulting $3 + 1$ dimensional field theory. This geometry is usually called the Witten model in the literature. In the case of $2 + 1$ dimensional field theory, the analogous confined geometry is the five-dimensional anti-de Sitter (AdS_5) soliton [17]. The deconfinement transition takes place between the soliton and the thermal AdS_5 geometry, i.e., a planar black hole (with one compactified spatial coordinate) [18]. Constructing the domain wall between the soliton and the black hole is challenging because the topologies of the two geometries differ, but have been found numerically by Aharony, Minwalla, and Wiseman (AMW) [19]. An analogous solution for the original Witten model has not been constructed so far; hence, we focus here on the lower dimensional version (which we will refer to as the Witten model in the following).

A direct time-dependent numerical relativity simulation of the Witten model is extremely difficult due to the different topology of the two phases [20]. Hence in the Witten model we will perform numerical evolution using a boundary field theory description introduced in Ref. [21].

Therein, we found that the energy-momentum tensor corresponding to the AMW domain wall solution can be

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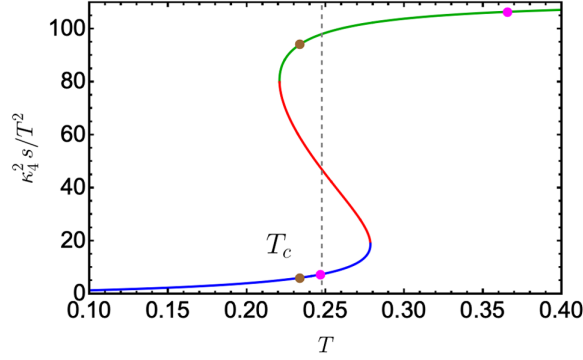


FIG. 1. Equation of state of the nonconformal model. Brown dots indicate sample phases in the nucleated bubble simulations, and pink dots indicate sample phases for the expanding interface between phases at different temperatures.

accurately described using a boundary field theory description involving hydrodynamic degrees of freedom as well as an additional scalar field γ which describes the interpolation between the two phases with $\gamma = 0$ corresponding to the deconfined phase and $\gamma = 1$ to the confined one. The Lagrangian of the model is

$$\mathcal{L} = [1 - \Gamma(\gamma)]p(T) + \Gamma(\gamma) - \frac{1}{2}c(T, \gamma)(\partial\gamma)^2 - d(T, \gamma)\gamma^2(1 - \gamma^2) \quad (1)$$

with $\Gamma(\gamma) = \gamma^2(3 - 2\gamma)$. The energy-momentum tensor is given by

$$T_{\mu\nu} = [1 - \Gamma(\gamma)]T_{\mu\nu}^{\text{confining}} + \Gamma(\gamma)T_{\mu\nu}^{\text{deconfined}} + T_{\mu\nu}^{\Sigma} \quad (2)$$

Explicit formulas for the energy-momentum tensors and the coefficients in Eq. (1) are given in the Supplemental Material [22]. See also Refs. [23–25] for different effective descriptions for first order phase transitions in holography.

As a complementary system we consider a bottom-up holographic gravity + scalar model in 4D bulk

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \quad (3)$$

with the potential

$$V(\Phi) = -6 \cosh\left(\frac{\Phi}{\sqrt{3}}\right) - 0.2\Phi^4 \quad (4)$$

exhibiting a first order phase transition which was studied in Refs. [26,27]. In this model both phases are deconfined and have a holographic black hole description. The equation of state is shown in Fig. 1. In this case we can perform full-fledged numerical relativity simulations which will allow us to confirm some qualitative features of the domain wall expansion deduced from the simplified model simulations. Since for this theory we can study the full holographic evolution, our treatment retains all possible nonequilibrium and dissipative features in the dynamics. We will refer to this model as the nonconformal holographic model.

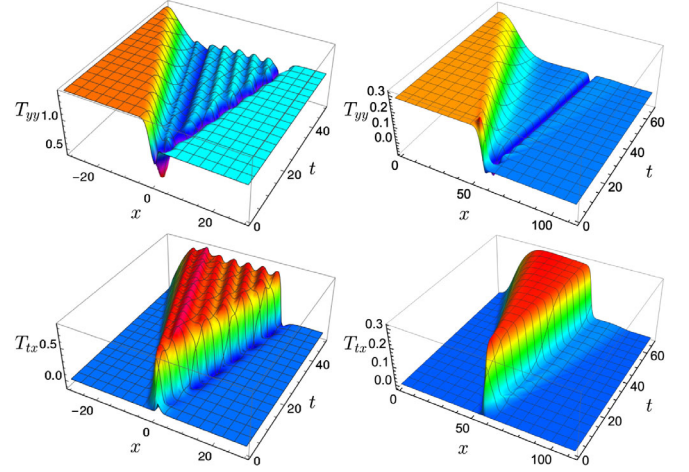


FIG. 2. Evolution of pressure T_{yy} (top row) and the momentum flow T_{tx} (bottom row) in the Witten model (left) and nonconformal model (right). The high entropy (deconfined) phase is on the left with the domain wall moving to the right (clearly seen as a dip of T_{yy}).

Qualitative structure of domain wall motion at strong coupling.—Let us first study the evolution of the interface between two phase domains: deconfined phase at $T > T_c$ and the confined phase. The pressure of the relevant phases can be directly read off from the T_{yy} component of the energy-momentum tensor (where y is the coordinate along the domain wall). In Fig. 2(top left) we show the time evolution of T_{yy} in the simplified model [Eqs. (1) and (2)].

Since the confined phase in the Witten model does not depend on the temperature, one could interpret this configuration as either a system at $T > T_c$ or as an interface between a high entropy phase at $T > T_c$ and the low entropy phase at $T = T_c$. In Fig. 2(top right) we show a plot of a counterpart of the latter configuration in the nonconformal holographic model.

The conventional picture explaining a constant domain wall velocity despite the imbalance of pressures on both sides of the domain wall is that the net force coming from the difference of pressures is balanced by friction induced by the phase in front of the moving domain wall [8]. This makes the determination of the domain wall velocity a very challenging problem.

The holographic simulations shown in Fig. 2 show, however, a quite different picture which arises at strong coupling. Firstly, the pressures on both sides of the domain wall are in fact very close to each other. Therefore, the domain wall motion with constant velocity is in fact very natural here and does not need any balancing friction force. Secondly, the pressure difference between the phases is supported on a hydrodynamic wave moving in the high entropy phase *away* from the domain wall [28].

Plotting the T^{tx} component of the energy-momentum tensor in the bottom of Fig. 2, we see that the high entropy phase to the left of the domain wall is not static and has a

flow velocity in the same direction as the motion of the domain wall. In fact, as we will argue shortly, for the studied holographic systems the flow velocity is very close to the domain wall velocity.

Perfect fluid description.—From the numerical simulations discussed above, we observe that the late time configuration consists of a domain wall moving with a constant velocity with balanced pressures on both sides and a hydrodynamic wave moving in the opposite direction supporting the pressure difference. We can therefore try to glue together the two ingredients.

In the Witten model, there exists an exact holographic solution for a moving domain wall, which is the static AMW domain wall geometry boosted to the domain wall velocity. Since in the AMW domain wall solution, the fluid in the deconfined phase is at rest, this implies that in the boosted solution, the velocity of the fluid would be equal to the velocity of the domain wall. This is a crucial relation as it translates the difficult problem of computing the domain wall velocity into computing the fluid velocity in the high entropy deconfined phase.

In addition, the pressure of the boosted solution would be equal to the pressure at the first order phase transition $p = p_c \equiv p(T_c)$.

For the consistency of this picture it is important to check that the (boosted) AMW domain wall is the only static (moving) domain wall solution in our hydrodynamic description of the Witten model. We studied numerically in the rest frame of the wall such solutions of Eq. (1), where the fluid has nonzero velocity on at least one side of the wall, and only found interpolating solutions when the velocity vanishes on both sides, i.e., the standard domain wall solution in the rest frame.

So we are left with finding a hydrodynamic solution interpolating between a static plasma with $p = p_A > p_c$ and a moving plasma with $p = p_c$ and velocity v . We would like to express v as a function of the pressure difference. It is illuminating to first consider the problem in the linearized approximation around some reference point:

$$p = p_{\text{ref}} + \delta p \quad u^\mu = (\cosh \alpha, \sinh \alpha, 0) \quad (5)$$

where both δp and α are small. Then the solution for the perfect fluid hydrodynamics for a left-moving wave is

$$\delta p = f(x + c_s t) \quad \alpha = -\frac{f(x + c_s t)}{(\varepsilon_{\text{ref}} + p_{\text{ref}})c_s} + \text{const} \quad (6)$$

Implementing the above boundary conditions for the pressures and velocities we obtain

$$v_{\text{domain wall}} = v_{\text{fluid}} = \tanh \frac{\Delta p}{(\varepsilon_{\text{ref}} + p_{\text{ref}})c_s}. \quad (7)$$

It turns out, however, that nonlinear hydrodynamic effects are important. The generalization is called *a simple wave* [29] (see also Supplemental Material [22]) and the counterpart of Eq. (7) is

$$v_{\text{domain wall}} = \tanh \int_{p_c}^{p_A} \frac{dp}{(\varepsilon + p)c_s} \equiv \tanh \int_{T_c}^{T_A} \frac{dT}{Tc_s}. \quad (8)$$

Note that the formula is expressed completely in terms of equation of state data.

We have checked numerically that both the velocity of the moving plasma and the velocity of the domain wall in the Witten model agree well with Eq. (8) at small pressure differences:

$$v_{\text{domain wall}} \approx v_{\text{fluid}} \approx \sqrt{3} \frac{\Delta T}{T} \quad (9)$$

within the precision of less than half a percent.

An analogous discussion for the nonconformal model with two deconfined phases leads to a couple of subtleties. In this case, both phases of the theory have a hydrodynamic description, and we are faced with a choice which phase (if any) to use for the hydrodynamic formula [Eq. (8)]. Moreover, an examination of our numerical holographic simulations indicates that the stationary moving domain wall will no longer be just a boosted version of the static solution as fluid velocities on both sides of the domain wall are not equal.

To understand this issue, it is convenient to pass to the domain wall rest frame and use the standard relations following from energy-momentum conservation which link the hydrodynamic parameters of the two fluids on both sides of the domain wall [7,30]:

$$\frac{v_H}{v_L} = \frac{\varepsilon_L + p_H}{\varepsilon_H + p_L} \quad v_H v_L = \frac{p_H - p_L}{\varepsilon_H - \varepsilon_L} \quad (10)$$

where the subscripts H and L denote the high and low entropy phase respectively. Let us evaluate v_H from the first equation

$$v_H = \frac{\varepsilon_L + p_H}{\varepsilon_H + p_L} v_L < \frac{\varepsilon_L + p_H}{\varepsilon_H + p_L} \quad (11)$$

where the inequality follows from $v_{L,H} < 1$. The last ratio can be evaluated from the equation of state. Assuming that the pressures $p_L \sim p_H$ [as seen in Fig. 2(top right)], the ratio is approximately equal to the ratio of entropies of the two phases (using $Ts = \varepsilon + p$). In the case of our model this is a small number, similarly for any confinement-deconfinement system due to the scaling with N_c . Since v_H is small in the rest frame of the domain wall, this means that in the laboratory frame, the fluid velocity of the high entropy phase should be close to the domain wall velocity. Hence the conclusion is that the formula [Eq. (8)] should be evaluated in the high entropy phase of the system.

In Fig. 3, we compare the formula [Eq. (8)] to the domain wall velocities obtained from the holographic simulations of the nonconformal model [Eq. (3)]. We find a very good agreement for a significant range of pressure differences with deviations occurring only for a quite large velocity.

Nucleated bubbles.—Up until now we have discussed the motion of a domain wall separating two (potentially infinite)

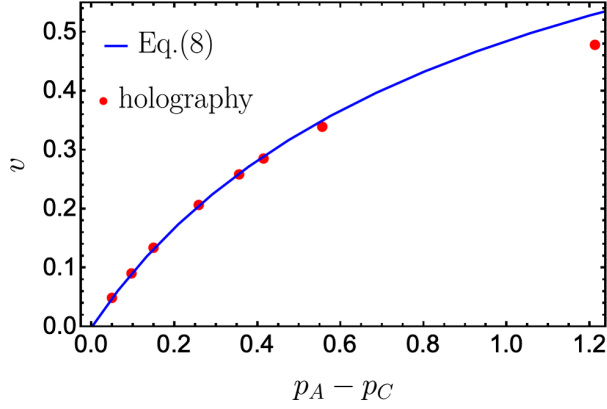


FIG. 3. Domain wall velocities obtained from holographic simulations in the nonconformal model as a function of pressure difference compared with formula (8) evaluated in the high entropy phase. The high entropy phase is at a temperature $T_A > T_c$. The low entropy phase is at T_c .

phases at different temperatures. We will now move on to the physically more relevant case of the expansion of a nucleated bubble. As the initial conditions we take a small bubble of the stable low entropy phase in a supercooled medium of the high entropy phase at the same temperature (cf. Fig. 1).

A sample velocity profile extracted from the holographic simulation is shown in Fig. 4. We find that the fluid velocity extracted from T_{tt} , T_{tx} , and T_{yy} assuming a perfect fluid form of $T_{\mu\nu}$ describes quite well the remaining T_{xx} component outside of the domain wall. This indicates that dissipative effects are not really important for the flow apart from the initial transient region.

Since the high entropy supercooled phase has smaller pressure than the stable phase, the hydrodynamic simple wave is now in front of the domain wall and moving in the same direction. This is in contrast to the case of the domain wall between high entropy phase with $T > T_c$ and the low entropy phase with $T = T_c$, which we discussed above.

As the velocity profile extracted from the holographic simulations indicates that the phase inside the bubble is at

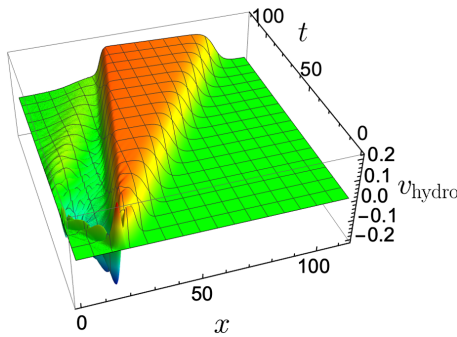


FIG. 4. Perfect fluid velocities extracted from the energy momentum tensor for the evolution of a nucleated bubble in an supercooled medium.

rest (this is the so-called deflagration case), we can improve in this case our formula [Eq. (8)]. Indeed using the notation from Eq. (10), we see that $v_L = -v_{\text{domain wall}}$; hence computing v_H from Eq. (10) and passing back to the laboratory frame we have

$$v_{\text{domain wall}} = \frac{1}{1 - \frac{\epsilon_L + p_H}{\epsilon_H + p_L}} v_{\text{fluid}} \quad (12)$$

where v_{fluid} is given by the hydrodynamic formula in the high entropy phase. Hence we set

$$v_{\text{domain wall}} = \frac{1}{1 - \frac{\epsilon_L + p_H}{\epsilon_H + p_L}} \tanh \int_{p_A}^{p_C} \frac{dp}{(\epsilon + p)c_s} \quad (13)$$

where p_C is the pressure inside the nucleated bubble, while p_A is the pressure of the supercooled environment. The formula [Eq. (13)] is compared with the numerical results from holography in Fig. 5.

An interesting phenomenological formula for the nucleated bubble domain wall velocity in a holographic theory was proposed in Ref. [13]:

$$v = \text{const} \cdot \frac{\Delta p}{\epsilon_A} \quad (14)$$

where the proportionality constant was fitted to be equal to 1.95 for a bottom-up scalar + gravity model (with 5D bulk)—see Fig. 7 in Ref. [13]. Our formula [Eq. (13)] does not lead to an exactly linear dependence, but we can evaluate the slope coefficient. Indeed, for small pressure differences we are close to T_c , and hence we can evaluate Eq. (13) with $p_L = p_H = p_c$. Keeping in mind that in this limit $\epsilon_H \simeq \epsilon_A$ we get

$$v_{\text{domain wall}}^{\text{linearized}} = \underbrace{\frac{\epsilon_H}{\epsilon_H - \epsilon_L} \frac{1}{c_s}}_{\text{const}} \Big|_{T=T_c} \cdot \frac{\Delta p}{\epsilon_A}. \quad (15)$$

For the theory considered in Ref. [13], the constant can be evaluated to be 2.044. In Fig. 6 (left) we compare the two linear coefficients to the data from Ref. [13]. We see that for

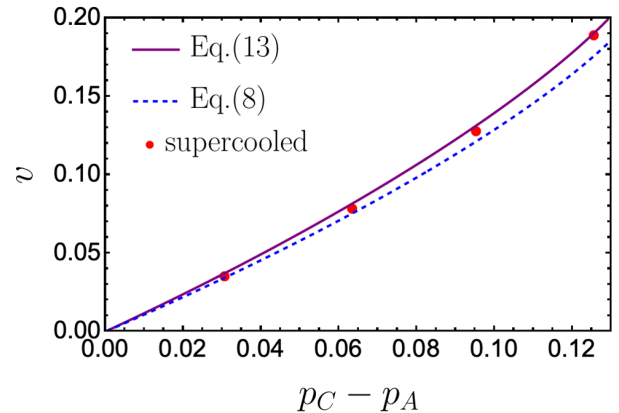


FIG. 5. Nucleated bubble domain wall velocities obtained from holographic simulations in the nonconformal model as a function of pressure difference compared with formula (13) evaluated in the high entropy phase.

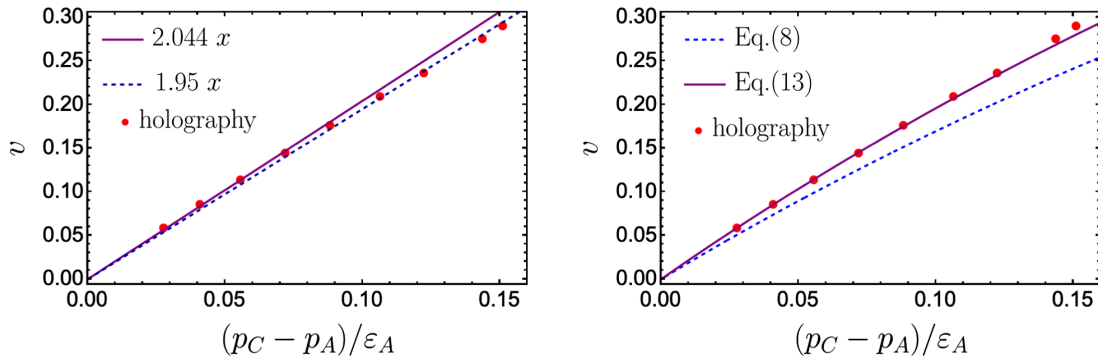


FIG. 6. Data from Fig. 7 of Ref. [13] compared with the linear formula [Eq. (15)] and the original fit from Ref. [13] (left), and with our nonlinear formula [Eq. (13)] (right).

small velocities, the larger coefficient seems to work better. In Fig. 6(right), we compare the same data with our full nonlinear formula [Eq. (13)].

Another interesting physical aspect of the nucleated bubble expansion is the energy balance and the fate of the latent heat which gets released when the bubble of the stable vacuum expands, which we discuss in the Supplemental Material [22].

Discussion.—In this Letter, we showed that for various strongly coupled theories with a holographic dual, the domain wall velocities may be expressed just in terms of the properties of the equation of state, employing perfect fluid hydrodynamics. The resulting predictions agree quite well with direct holographic simulations of a moving interface between $T_A > T_c$ and T_c (see Fig. 3) and nucleated bubbles (see Figs. 5 and 6).

A-posteriori, since the final formulas [Eqs. (8) and (13)] are expressed in terms of the equation of state, holography is not really needed for the evaluation of the domain wall velocity. For us holography was, however, crucial in order to check the validity of our formulas. We expect that the key requirements for the applicability of our predictions is the existence of a hydrodynamic description with a low viscosity as well as a relatively large ratio of entropies of the two phases.

This Letter leads to many open questions related to the velocities and other properties of domain walls: (i) Our results relate to a variety of physical systems, including confinement and deconfinement, deconfined phase and an supercooled one, and conformal and nonconformal theories, related to holographic top-down and bottom-up scenarios and to gauge theories in $2 + 1$ and $3 + 1$ dimensions. To what extent does this simple dynamics of the domain wall apply to other systems of first order phase transition? (ii) We expect that it should be possible to generalize the results to systems with circular or spherical bubbles (cf. holographic simulations in Ref. [31]). (iii) We addressed pure gauge theories. Incorporating flavored quark degrees of freedom, which in the holographic picture means adding probe flavor branes, is of utmost importance. Some interesting work in

this direction has been recently done in Refs. [23,32]. (iv) The transition from a deconfined to a confined phase is achieved via hadronization, whose holographic description remains elusive. It is not clear how this affects the dynamics of the domain wall. (v) There might be certain possible applications of our results to heavy ion collisions and quark gluon plasma, potentially at nonzero density. It would also be very interesting to investigate possible applications for the early Universe.

Finally, as the final formulas [Eqs. (8) and (13)] have a simple form and work quite well in strongly coupled holographic settings, we hope that they may be used as points of reference for domain wall velocities in various contexts, akin to the holographic shear viscosity to entropy ratio [33].

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