Carrollian Perspective on Celestial Holography

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We show that a 3D sourced conformal Carrollian field theory has the right kinematic properties to holographically describe gravity in 4D asymptotically flat spacetime. The external sources encode the leaks of gravitational radiation at null infinity. The Ward identities of this theory are shown to reproduce those of the 2D celestial CFT after relating Carrollian to celestial operators. This suggests a new set of interplays between gravity in asymptotically flat spacetime, sourced conformal Carrollian field theory and celestial CFT.

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Introduction.—The holographic principle [1,2] states that gravity in a given spacetime region can be encoded on a lower-dimensional boundary of that region. Extending this paradigm beyond the celebrated AdS/CFT correspondence [3–5] to the more realistic model of asymptotically flat spacetimes is part of an intensive ongoing research effort, referred to as "flat space holography" (see Refs. [6–12] for early works). The asymptotic symmetries preserving the boundary structure of asymptotically flat spacetimes form the Bondi–van der Burg–Metzner–Sachs (BMS) group [13–15], which is an infinite-dimensional enhancement of the Poincaré group with supertranslations.

Two different roads, referred here as "Carrollian" and "celestial" holographies, have emerged in order to describe quantum gravity in 4D asymptotically flat spacetime and might seem in apparent tension.

In the first picture, the dual theory is proposed to be a BMS field theory living on the 3D null boundary of the spacetime [16–23] or, equivalently [24,25], a conformal Carrollian field theory. While this 4D bulk/3D boundary theory point of view follows the familiar pattern of a codimension-one holographic duality, establishing such a flat space holographic dictionary presents new challenges that one is not used to encountering in AdS/CFT. Key differences include, on the one hand, the null nature of the conformal boundary and, on the other hand, the presence of radiative flux leaking through this boundary. However, this approach is suggested by a flat limit process in the bulk, which consists of taking the cosmological constant to zero. This implies an ultrarelativistic limit on the boundary theory contracting the conformal symmetries into BMS symmetries; see Refs. [26–33] for successful applications in 3D gravity and [29,34–41] for a fluid/gravity perspective.

In the second proposal, the holographic dual of gravity in 4D asymptotically flat spacetime is a two-dimensional "celestial conformal field theory" (CCFT) living on the conformal sphere at infinity. The celestial holography program is rooted on the observation that gravitational S-matrix elements written in a boost eigenstate basis take the form of conformal correlation functions [9,42,43]. Quantum field theory soft theorems can be elegantly encoded in CCFT in terms of celestial currents associated with asymptotic symmetry generators; see, e.g., Refs. [44–52] and [53] for more references. The obvious advantage of the celestial paradigm is that it provides a framework where one can readily make use of the plethora of powerful CFT techniques.

The main goals of this Letter are (i) to deepen the first picture by providing a precise proposal of a holographic description that properly takes into account leaks of gravitational radiation through the boundary and (ii) to initiate a dialog between these two different approaches to flat space holography.

In the next section, after a brief review of 4D asymptotically flat spacetimes, we argue that the coexistence of two proposals for flat space holography relies on the two complementary descriptions of the spacetime boundary. Following that, we propose that the dual field theory in the first picture is a conformal Carrollian field theory (CCarrFT) coupled with some external sources encoding the radiation leaking through the conformal boundary. We then show that the Ward identities of the sourced CCarrFT reproduce those of the CCFT after taking the appropriate integral transform with respect to the advanced or retarded time. This suggests a rich set of interplays, as depicted in

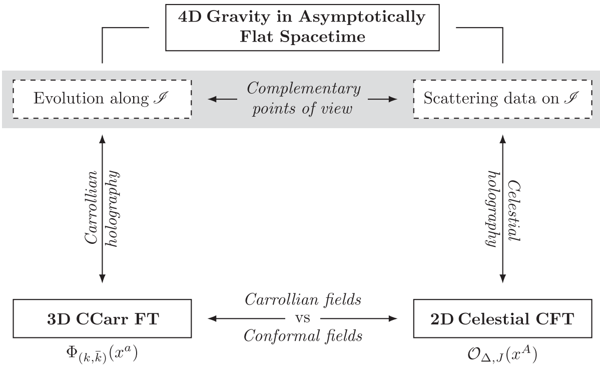

FIG. 1. Carrollian approach vs celestial approach to flat space holography.

Fig. 1. In the final section, we summarize the results and discuss some implications of this Letter.

Asymptotically flat spacetimes.—In this section, we review the analysis of four-dimensional asymptotically flat spacetimes at null infinity, denoted $\mathscr{I}^+$, and its relation with Carrollian geometry.

In (retarded) Bondi coordinates (u, r, x^A), $x^A = (z, \bar{z})$ [13–15], the solution space of four-dimensional asymptotically flat metrics reads as [54,55]

$$\begin{aligned} ds^2 = &\left[\frac{2M}{r} + \mathcal{O}(r^{-2})\right] du^2 - 2[1 + \mathcal{O}(r^{-2})]dudr \\ &+ [r^2 \mathring{q}_{AB} + rC_{AB} + \mathcal{O}(r^{-1})]dx^A dx^B \\ &+ \left[\frac{1}{2}\partial_B C^B_A + \frac{2}{3r}\left(N_A + \frac{1}{4}C^B_A \partial_C C^C_B\right) + \mathcal{O}(r^{-2})\right] dudx^A, \end{aligned} \tag{1}$$

where the asymptotic shear $C_{AB}(u, z, \bar{z})$ is a two-dimensional symmetric trace-free tensor. For simplicity, we chose the transverse boundary metric to be the flat metric, namely $\mathring{q}_{AB} dx^A dx^B = 2dzd\bar{z}$. The Bondi mass $M(u, z, \bar{z})$ and angular momentum aspects $N_A(u, z, \bar{z})$ in Eq. (1) satisfy the time evolution or constraint equations

$$\begin{aligned} \partial_u M &= -\frac{1}{8} N_{AB} N^{AB} + \frac{1}{4} \partial_A \partial_B N^{AB}, \\ \partial_u N_A &= \partial_A M + \frac{1}{16} \partial_A (N_{BC} C^{BC}) - \frac{1}{4} N^{BC} \partial_A C_{BC} \\ &\quad - \frac{1}{4} \partial_B (C^{BC} N_{AC} - N^{BC} C_{AC}) \\ &\quad - \frac{1}{4} \partial_B \partial^B \partial^C C_{AC} + \frac{1}{4} \partial_B \partial_A \partial_C C^{BC}, \end{aligned} \tag{2}$$

with $N_{AB} = \partial_u C_{AB}$ the Bondi news tensor encoding the gravitational radiation.

The diffeomorphisms preserving the solution space displayed above are generated by vector fields $\xi = \xi^u \partial_u + \xi^z \partial + \xi^{\bar{z}} \bar{\partial} + \xi^r \partial_r$ whose leading order components read as

$$\begin{aligned} \xi^u &= \mathcal{T} + u\alpha, \qquad \alpha = \frac{1}{2}(\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}), \\ \xi^z &= \mathcal{Y} + \mathcal{O}(r^{-1}), \qquad \xi^{\bar{z}} = \bar{\mathcal{Y}} + \mathcal{O}(r^{-1}), \\ \xi^r &= -r\alpha + \mathcal{O}(r^0), \end{aligned} \tag{3}$$

where $\mathcal{T} = \mathcal{T}(z, \bar{z})$ is the supertranslation parameter and $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation. Using a modified Lie bracket [55], the asymptotic Killing vectors, Eq. (3), satisfy the (extended) BMS algebra.

The infinitesimal transformation of the asymptotic shear C_{AB} under BMS symmetries can be split into hard and soft pieces $\delta_\xi C_{zz} = \delta^H_\xi C_{zz} + \delta^S_\xi C_{zz}$, which are respectively homogeneous and inhomogeneous in C_{zz} [56,57]. This reads explicitly as

$$\begin{aligned} \delta^H_\xi C_{zz} &= \left[(\mathcal{T} + u\alpha)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} - \frac{1}{2}\bar{\partial}\bar{\mathcal{Y}}\right] C_{zz}, \\ \delta^S_\xi C_{zz} &= -2\partial^2 \mathcal{T} - u\partial^3 \mathcal{Y}, \end{aligned} \tag{4}$$

together with the complex conjugate relations for $C_{\bar{z}\bar{z}}$.

From a geometric perspective, BMS symmetries are the conformal symmetries of a Carrollian structure on $\mathscr{I}^+$ with coordinates $x^a = (u, z, \bar{z})$ [24,25] (see also Refs. [58–66]). This Carrollian structure is given by a degenerate metric q_{ab} and a vector field n^a in the kernel of q_{ab}, i.e., $q_{ab} n^b = 0$. From Eq. (1), it reads explicitly $q_{ab} dx^a dx^b = 0du^2 + 2dzd\bar{z}$ and $n^a \partial_a = \partial_u$. The conformal Carrollian symmetries are generated by vector fields $\bar{\xi} = \bar{\xi}^a \partial_a$ on $\mathscr{I}^+$ satisfying

$$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}, \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a. \tag{5}$$

The solution $\bar{\xi}$ of Eq. (5) is precisely given by the restriction to $\mathscr{I}^+$ of the asymptotic Killing vectors, Eq. (3), i.e.,

$$\bar{\xi} = (\mathcal{T} + u\alpha)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}. \tag{6}$$

The standard Lie bracket on $\mathscr{I}^+$ of these vector fields reproduces the BMS algebra.

At each cut $\mathcal{S} \equiv \{u = \text{constant}\}$ of $\mathscr{I}^+$, the charges associated with the BMS symmetries, Eq. (3), are given by [56,57,67–73]

$$\begin{aligned} Q_\xi &= \kappa \int_{\mathcal{S}} dzd\bar{z}(4\mathcal{T}M + 2\mathcal{Y}^A \bar{N}_A), \\ \bar{N}_A &= N_A - u\partial_A M + \frac{1}{4} C^B_A \partial_C C^C_B + \frac{3}{32} \partial_A (C^C_B C^B_C) \\ &\quad + \frac{u}{4} \partial^B \partial_B \partial_C C^C_A - \frac{u}{4} \partial^B \partial_A \partial_C C^C_B, \end{aligned} \tag{7}$$

where $\mathcal{Y}^A = (\mathcal{Y}, \bar{\mathcal{Y}})$ and $\kappa = (16\pi G)^{-1}$. These charges differ from the prescriptions considered, e.g., in [74–77]; we refer to [78] for a recent discussion on the relation between the various proposals. Using the appropriate bracket [76] (see also Refs. [69,79–84]), these charges satisfy an algebra from which flux-balance laws can be deduced. The charges, Eq. (7), are conserved in absence of outgoing radiation, namely when $N_{AB} = 0$.

Notice that the analysis of asymptotically flat spacetimes can be performed at $\mathscr{I}^-$ in an analogous manner by working in advanced Bondi coordinates (v, r, x^A).

From the analysis of 4D asymptotically flat spacetimes, two complementary pictures emerge, which correspond to the two possible roads to flat space holography, as discussed in the introduction (see Fig. 1). In the first picture, $\mathscr{I}^+$ ($\mathscr{I}^-$) is seen as a boundary along which there is retarded (advanced) time evolution. This is suited to describe the dynamics of the system through flux-balance equations such as the famous Bondi mass loss formula originally derived in [13–15]. In other terms, Eq. (2) are interpreted as evolution equations, suggesting a 4D bulk/3D boundary holographic correspondence, where the dual field theory lives at null infinity and obeys Carrollian physics. Since the charges are generically not conserved due to the outgoing (ingoing) radiation going through null infinity, it is tempting to think of the dual theory as coupled to some external sources responsible for the dissipation. We elaborate on this proposal below.

In the second picture, $\mathscr{I}^+$ ($\mathscr{I}^-$) is seen as a portion of a Cauchy surface in the asymptotic future (past). This point of view is well adapted to describe the scattering problem in asymptotically flat spacetime between $\mathscr{I}^-$ and $\mathscr{I}^+$ and provides information about the state of the system at early and late times. The equivalence between BMS Ward identities and soft theorems was established in this picture [56,57,67]. Equation (2) are now seen as constraint equations in the Hamiltonian framework. Scattering amplitudes in the bulk can be rewritten as correlation functions on the celestial sphere obeying some Ward identities encoding the information on soft theorems. This suggests a 4D bulk/2D boundary correspondence, with a 2D CCFT as holographic dual. We come back on this proposal below, where we relate it to the Carrollian framework.

Sourced conformal Carrollian field theory.—In this section, we write the Ward identities for a sourced quantum field theory. We then specify this result for a sourced CCarrFT and argue that it holographically encodes the asymptotic dynamics of gravity.

Let us start with a theory of fields Φ^i on a n-dimensional manifold $\mathscr{M}$ with coordinates x^a and admitting a well-defined variational principle. We assume that the theory exhibits some global symmetries $\delta_K \Phi^i = K^i[\Phi]$ with associated conserved Noether currents j^a_K. If one couples the theory to external sources σ, which are nondynamical fields, the infinitesimal transformations $\delta_K \Phi^i = K^i[\Phi]$ are generically no longer symmetries due to the presence of the sources (see, e.g., Refs. [79,85,86]). This translates into the fact that the Noether currents are no longer conserved, namely

$$\partial_a j^a_K(x) = F_K(x), \tag{8}$$

where the local flux $F_K(x)$, which generically depends on Φ and σ, vanishes when $\sigma = 0$. At the quantum level, taking the presence of external sources into account in the standard derivation of the Ward identities [87], we obtain

$$\partial_a \langle j^a_K(x) X\rangle + \frac{\hbar}{i}\sum_{i=1}^{N} \delta^{(n)}(x - x_i)\delta_{K^i}\langle X\rangle = \langle F_K(x) X\rangle, \tag{9}$$

where $X \equiv \Phi^{i_1}(x_1)\ldots\Phi^{i_N}(x_N)$ denotes the collection of quantized fields and $\delta_{K^i}\langle X\rangle \equiv \langle \Phi^{i_1}(x_1)\ldots K^i[\Phi(x_i)]\ldots\Phi^{i_N}(x_N)\rangle$. This generalizes the local version of the infinitesimal Ward identities in presence of external sources. In particular, with no field insertion in the correlators, it implies

$$\partial_a \langle j^a_K(x)\rangle = \langle F_K(x)\rangle, \tag{10}$$

which reproduces the classical flux-balance law, Eq. (8). Integrating Eq. (9) over the manifold $\mathscr{M}$ with boundary $\partial\mathscr{M}$, we get the integrated version of the infinitesimal Ward identities with external sources:

$$\sum_{i=1}^{N} \delta_{K^i}\langle X\rangle = \frac{i}{\hbar}\left\langle \left(\int_{\mathscr{M}} \boldsymbol{F}_K - \int_{\partial\mathscr{M}} \boldsymbol{j}_K\right) X\right\rangle, \tag{11}$$

where bold letters denote the forms associated with the objects defined above, e.g., $\boldsymbol{F}_K = F_K(d^n x)$ and $\boldsymbol{j}_K = j^a_K(d^{n-1}x)_a$. The standard result of the invariance of the correlators under symmetry transformations is recovered by turning off the sources and assuming that the Noether currents vanish at the boundary.

We now apply these general results to the case of a 3D CCarrFT coupled to some external sources.

BMS symmetries are to 3D CCarrFT what Virasoro symmetries are to 2D CFT, while the 4D Poincaré group is on the same footing as the Möbius group $SL(2,\mathbb{C})$. “Global” subgroups inside BMS and Virasoro groups are not unique and correspond to Poincaré and Möbius subgroups, respectively. Accordingly, conformal Carrollian primary fields [37,38,61] will be taken to transform infinitesimally as

$$\delta_\xi \Phi_{(k,\bar{k})} = [(\mathcal{T} + u\alpha)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\partial\mathcal{Y} + \bar{k}\bar{\partial}\bar{\mathcal{Y}}]\Phi_{(k,\bar{k})} \tag{12}$$

under full conformal Carrollian symmetries, Eq. (6), while quasiprimary fields are only required to transform properly under the global subgroup. Here, the Carrollian weights $(k, \bar{k})$ are some integers or half-integers.

Noether currents associated with the conformal Carrollian symmetries, Eq. (6), are taken to be of the form

$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b, \tag{13}$$

where $\mathcal{C}^a{}_b$ is the analog of the stress-energy tensor for a CCarrFT encoding the Carrollian momenta [33,37,38,89–95] as follows:

$$\mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_B \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}. \tag{14}$$

The Noether currents, Eq. (13), associated with the global subalgebra of the conformal Carrollian algebra verify the flux-balance law, Eq. (8), provided $\mathcal{C}^a{}_b$ satisfies the following:

$$\begin{aligned} \text{Translations } (\partial_a) &\Rightarrow \partial_a \mathcal{C}^a{}_b = F_b, \\ \text{Rotation } (-z\partial + \bar{z}\,\bar{\partial}) &\Rightarrow \mathcal{C}^z{}_z = \mathcal{C}^{\bar{z}}{}_{\bar{z}}, \\ \text{Boosts } (x^A \partial_u) &\Rightarrow \mathcal{C}^A{}_u = 0, \\ \text{Dilatation } (x^a \partial_a) &\Rightarrow \mathcal{C}^a{}_a = 0, \end{aligned} \tag{15}$$

where we assumed that the flux is linear in the parameters $\bar{\xi}^a$ and can be written as $F_{\bar{\xi}} = F_a \bar{\xi}^a$ in the right-hand side of Eq. (8), which is sufficient for the purpose of this Letter. Notice that the term $\mathcal{C}^a{}_b \partial_a \bar{\xi}^b$ does not contribute to the left-hand side of Eq. (8) as a consequence of Eqs. (6) and (15). The Carrollian special conformal transformations $K_0 = -2z\bar{z}\partial_u$, $K_1 = 2u\bar{z}\partial_u + 2\bar{z}^2\bar{\partial}$, and $K_2 = 2uz\partial_u + 2z^2\partial$ do not impose further constraints. Furthermore, the above global conformal Carrollian symmetries are enough to completely constrain $\mathcal{C}^a{}_b$, i.e., Eq. (8) is automatically satisfied by the supertranslation (and superrotation) currents provided Eq. (15) holds. In terms of the Carrollian momenta, the constraints, Eq. (15), imply

$$\begin{aligned} &\partial_u \mathcal{M} = F_u, \qquad \mathcal{B}^z = 0, \\ &\partial_u \mathcal{N}_z - \frac{1}{2}\partial\mathcal{M} + \bar{\partial}\mathcal{A}^{\bar{z}}{}_z = F_z, \qquad 2\mathcal{A}^z{}_z + \mathcal{M} = 0, \end{aligned} \tag{16}$$

together with the complex conjugate relations.

At the quantum level, one can write the infinitesimal Ward identities, Eq. (9), for the specific case of a CCarrFT. Assuming that the operators inserted in the correlators are (quasi)conformal Carrollian primary fields, we obtain

$$\begin{aligned} &\partial_u \langle \mathcal{M} X \rangle + \frac{\hbar}{i}\sum_i \delta^{(3)}(x - x_i)\partial_{u_i}\langle X \rangle = \langle F_u X \rangle, \\ &\partial_u \langle \mathcal{N}_z X \rangle - \frac{1}{2}\partial\langle \mathcal{M} X \rangle + \bar{\partial}\langle \mathcal{A}^{\bar{z}}{}_z X \rangle \\ &\quad + \frac{\hbar}{i}\sum_i [\delta^{(3)}(x - x_i)\partial_i \langle X \rangle - \partial(\delta^{(3)}(x - x_i) k_i \langle X \rangle)] = \langle F_z X \rangle, \\ &\langle \mathcal{B}^z X \rangle = 0, \\ &\left\langle \left(\mathcal{A}^z{}_z + \frac{1}{2}\mathcal{M}\right) X \right\rangle + \frac{\hbar}{i}\sum_i \delta^{(3)}(x - x_i) k_i \langle X \rangle = 0, \end{aligned} \tag{17}$$

together with the complex conjugate relations. With no field insertion in the correlators, the expectation value of the operators reproduce the classical relations, Eq. (16). At any stage of this analysis, one can turn off the sources by setting $F_a = 0$ to obtain the Ward identities of an honest 3D CCarrFT.

We now argue that a quantum CCarrFT coupled with external sources would be a right candidate to describe holographically gravity in 4D asymptotically flat spacetimes reviewed above (few explicit examples of quantum CCarrFT are known; see, e.g., Refs. [22,96–99]). We propose the following correspondence between Carrollian momenta and gravitational data at $\mathscr{I}^+$:

$$\begin{aligned} &\langle \mathcal{M} \rangle = 4\kappa M, \\ &\langle \mathcal{N}_A \rangle = 2\kappa \bigg(N_A + \frac{1}{4} C^B_A \partial_C C^C_B + \frac{3}{32}\partial_A (C^C_B C^B_C) \\ &\qquad + \frac{u}{4}\partial^B \partial_B \partial_C C^C_A - \frac{u}{4}\partial^B \partial_A \partial_C C^C_B \bigg), \\ &\langle \mathcal{A}^A{}_B \rangle + \frac{1}{2}\delta^A{}_B \langle \mathcal{M} \rangle = 0. \end{aligned} \tag{18}$$

The factors are fixed by demanding that the gravitational charges, Eq. (7), correspond to the Noether currents, Eq. (13), of the CCarrFT integrated on a section $u =$ constant. The correspondence, Eq. (18), is reminiscent of the AdS/CFT dictionary where the holographic stress-energy tensor of the CFT is identified with some subleading order in the expansion of the bulk metric [100,101]. It would be interesting to push this analogy further and see if the Carrollian momenta can be obtained by varying a bulk partition function with respect to the boundary sources at null infinity (see, e.g., Refs. [30,32] for discussions along these lines in 3D gravity).

The Bondi news tensor N_{AB} is a free datum at $\mathscr{I}^+$ that encodes the outgoing gravitational radiation. It is responsible for the nonconservation of the BMS charges, Eq. (7), at null infinity. It is therefore suggestive to holographically identify the external sources σ_{AB} at the boundary with the Bondi news tensor N_{AB} as

$$\sigma_{AB} = N_{AB}. \tag{19}$$

The external sources are responsible for the dissipation in the CCarrFT through the fluxes

$$\begin{aligned} F_u &= -\kappa[\sigma^{zz}\sigma_{zz} - 2(\bar{\partial}^2\sigma_{zz} + \partial^2\sigma_{\bar{z}\bar{z}})], \\ F_z &= \frac{\kappa}{2}[\partial(\sigma^{zz}\Phi_{zz}) + 2\Phi_{zz}\partial\sigma^{zz} + u\partial(\bar{\partial}^2\sigma_{zz} - \partial^2\sigma_{\bar{z}\bar{z}})]. \end{aligned} \tag{20}$$

Here, Φ_{zz} denotes the operator associated with a perturbation in the asymptotic shear: $\langle \Phi_{AB} \rangle = C_{AB}$. Comparing Eq. (4) with Eq. (12), one deduces that Φ_{zz} is a quasi-conformal Carrollian primary field of weights $(\frac{3}{2}, -\frac{1}{2})$. It constitutes a particular example of a correlator insertion. Notice that, from the boundary perspective, the momentum

$\Pi_{AB} = \partial_u \Phi_{AB}$ conjugated to Φ_{AB} should be distinguished from the sources σ_{AB}. They are identified only through vacuum expectation value as $\langle \Pi_{AB} \rangle = \sigma_{AB}$.

Taking the identifications, Eqs. (18) and (20), into account, one can then check explicitly that the time evolution equations in the sourced Ward identities, Eq. (17), reproduce the gravitational retarded time evolution equations, Eq. (2), when there is no insertion in the correlators.

Let us emphasize that a similar identification can be performed with the solution space in advanced Bondi coordinates (v, r, x^A) at $\mathscr{I}^-$. It is therefore natural to assume that the dual sourced CCarrFT is living on $\hat{\mathscr{I}} = \mathscr{I}^- \sqcup \mathscr{I}^+$, where the two manifolds $\mathscr{I}^-$ and $\mathscr{I}^+$ are glued together by identifying antipodally $\mathscr{I}^+_-$ with $\mathscr{I}^-_+$. Geometrically, the gluing 2-sphere on $\hat{\mathscr{I}}$ is distinguished by the vanishing of the vector field n^a defining the Carrollian structure. Indeed, $\mathscr{I}^+_-$ and $\mathscr{I}^-_+$ are stable under supertranslation. The gluing is consistent with the antipodal matching conditions proposed in [56,57,67] and confirmed in [102–107] by an analysis at spacelike infinity. In particular, the Carrollian data are identified with the solution space of the retarded (advanced) Bondi gauge at $\mathscr{I}^+$ ($\mathscr{I}^-$), with a continuous interpolation in the gluing region thanks to the antipodal matching. The conformal Carrollian symmetries act on the whole $\hat{\mathscr{I}}$ and correspond to the diagonal BMS symmetries identified in [56]. They are generated by Eq. (5) on $\mathscr{I}^+$ and the analog antipodally matched symmetries on $\mathscr{I}^-$.

Relation with celestial holography.—In this section, we show that the Ward identities of the sourced CCarrFT reproduce the BMS Ward identities of the celestial CFT after performing the right integral transformations. This constitutes a central argument to relate the two approaches of flat space holography; see Fig. 1.

Specifying the integrated version of the sourced Ward identities, Eq. (11), to the conformal Carrollian symmetries of the theory living on $\hat{\mathscr{I}}$ suggested in the previous section, we obtain

$$\delta_{\bar{\xi}} \langle X \rangle = \frac{i}{\hbar} \left\langle \left(\int_{\hat{\mathscr{I}}} \boldsymbol{F}_{\bar{\xi}} - \int_{\mathscr{I}^+_+} \boldsymbol{j}_{\bar{\xi}} + \int_{\mathscr{I}^-_-} \boldsymbol{j}_{\bar{\xi}} \right) X \right\rangle, \quad (21)$$

where $X \equiv \Phi^{\text{out}}_{(k_1,\bar{k}_1)}(x_1) \ldots \Phi^{\text{out}}_{(k_m,\bar{k}_m)}(x_m) \Phi^{\text{in}}_{(k_1,\bar{k}_1)}(x_1) \ldots \times \Phi^{\text{in}}_{(k_n,\bar{k}_n)}(x_n)$, $\Phi^{\text{out}}_{(k_i,\bar{k}_i)}(x_i)$, and $\Phi^{\text{in}}_{(k_j,\bar{k}_j)}(x_j)$ denoting insertions at $\mathscr{I}^+$ and at $\mathscr{I}^-$, respectively. To simplify the discussion, let us assume that we are describing a scattering of massless particles, so that the current $\boldsymbol{j}_{\bar{\xi}}$ vanishes at $\mathscr{I}^+_+$ and $\mathscr{I}^-_-$. In addition, we require that the integrated flux on $\mathscr{I}^+$ is equal to minus the integrated flux on $\mathscr{I}^-$ at the level of the operators, namely

$$\int_{\mathscr{I}^-} \boldsymbol{F}_{\bar{\xi}} = - \int_{\mathscr{I}^+} \boldsymbol{F}_{\bar{\xi}}. \quad (22)$$

This constraint on the sources is compatible with the classical bulk requirement that the integrated ingoing flux is equal to the integrated outgoing flux for the massless scattering considered. Taking these assumptions into account, the integrated Ward identities, Eq. (21), imply

$$\delta_{\bar{\xi}} \langle X \rangle = 0. \quad (23)$$

This is the statement that the correlators are conformal Carroll invariant. The consequences of this relation have been studied, e.g., in [22,108,109].

To relate Eq. (23) with the CCFT Ward identities, we use similar technical steps than those advocated in [56,57,67] to relate BMS Ward identities and soft graviton theorems in the bulk. Let us first specify Eq. (23) for the supertranslation symmetries to recover the corresponding Ward identities of the CCFT. We split the variation into hard and soft parts $\delta_{\mathcal{T}} \langle X \rangle = \delta^H_{\mathcal{T}} \langle X \rangle + \delta^S_{\mathcal{T}} \langle X \rangle$ and rewrite the soft part of the transformation as the soft charge insertion by using the quantum commutator

$$[\Pi_{zz}(u,z,\bar{z}), \Phi_{\bar{w}\bar{w}}(u',w,\bar{w})] = \frac{i\hbar}{\kappa} \delta(u-u') \delta^{(2)}(z-w), \quad (24)$$

which ensures the compatibility with the Poisson bracket on the radiative phase space [110,111]. Then we specify the relation for $\mathcal{T}(z,\bar{z}) = 1/(z-w)$ and introduce the supertranslation current $P(z,\bar{z})$ [56] through

$$P(z,\bar{z}) = \frac{1}{4G} \left(\int du + \int dv \right) \bar{\partial} \Pi_{zz}. \quad (25)$$

Furthermore, we obtain the CCFT operators $\mathcal{O}^{\text{out}}_{\Delta,J}(z,\bar{z})$ and $\mathcal{O}^{\text{in}}_{\Delta,J}(z,\bar{z})$ of conformal dimension Δ and spin J from the conformal Carrollian operators $\Phi^{\text{out}}_{(k,\bar{k})}(u,z,\bar{z})$ and $\Phi^{\text{in}}_{(k,\bar{k})}(v,z,\bar{z})$ of Eq. (12) through the integral transform

$$\mathcal{O}^{\text{out}}_{\Delta,J}(z,\bar{z}) = i^\Delta \Gamma[\Delta] \int_{-\infty}^{+\infty} du u^{-\Delta} \Phi^{\text{out}}_{(k,\bar{k})}(u,z,\bar{z}),$$
$$\mathcal{O}^{\text{in}}_{\Delta,J}(z,\bar{z}) = i^\Delta \Gamma[\Delta] \int_{-\infty}^{+\infty} dv v^{-\Delta} \Phi^{\text{in}}_{(k,\bar{k})}(v,z,\bar{z}). \quad (26)$$

The above integral is the composition of a Fourier transform (from position to momentum space) and a Mellin transform that maps energy to boost eigenstates [42,43]. It trades the time dependence of the Carrollian operators for the conformal dimension of the CCFT operators. Importantly, Carrollian weights are related to the 2D spin via

$$k = \frac{1}{2}(1+J), \qquad \bar{k} = \frac{1}{2}(1-J), \quad (27)$$

which can be seen to be consistent with the radiative falloffs in the conformal compactification. Taking into account the aforementioned steps, we recover the CCFT Ward identity for supertranslations [45,56,57] ($N = m + n$)

$$\left\langle P(z,\bar{z}) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i) \right\rangle + \hbar \sum_{q=1}^{N} \frac{1}{z - z_q} \langle \ldots \mathcal{O}_{\Delta_q+1,J_q}(z_q,\bar{z}_q) \ldots \rangle = 0, \qquad (28)$$

which is the celestial encoding of the leading soft graviton theorem.

Now, one can specify Eq. (23) for superrotations and follow the same series of steps. Choosing $\mathcal{Y}(z) = 1/(z - w)$ and defining the 2D stress tensor as

$$T(z) = -\frac{i}{8\pi G} \int \frac{dw d\bar{w}}{z - w} \left(\int du u + \int dv v \right) \partial^3 \Pi_{\bar{w}\bar{w}}, \qquad (29)$$

we recover the 2D CFT Ward identities [46,112–114] after performing Eq. (26):

$$\left\langle T(z) \prod_{i=1}^{N} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i) \right\rangle + \hbar \sum_{q=1}^{N} \left[\frac{\partial_q}{z - z_q} + \frac{h_q}{(z - z_q)^2} \right] \times \left\langle \prod_{i=1}^{N} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i) \right\rangle = 0, \qquad (30)$$

where $h_q = \frac{1}{2}(\Delta_q + J_q)$ (similar results hold for the antiholomorphic stress tensor). This result encodes the information on the subleading soft graviton theorem in the bulk.

Discussion.—Here, we have argued that the null nature of $\mathscr{I}$ leads to two complementary roads to flat space holography (see Fig. 1): Carrollian vs Celestial. We have presented a holographic description in the first picture in terms of a codimension-one CCarrFT coupled with external sources encoding gravitational radiation at null infinity. The Ward identities have been shown to reproduce those of the celestial CFT, by relating celestial primary operators to conformal Carrollian fields living at $\mathscr{I}$. This provides a bridge between two different approaches to the ambitious program of finding a holographic description of quantum gravity for realistic spacetimes.

This Letter raises new questions for the future. For instance, it would be interesting (i) to deduce the low-point correlation functions in the CCarrFT from the Ward identities and match them with those of the CCFT, (ii) to understand if the sourced CCarrFT can be obtained in the flat limit of the AdS/CFT correspondence (building on the works [71,115,116]), and (iii) to write a concrete proposal for the sourced CCarrFT reproducing the features described in this Letter. We leave this research program for future investigations.

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