Post-Newtonian Feasibility of the Closed String Massless Sector

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(Received 28 February 2022; revised 5 June 2022; accepted 8 July 2022; published 5 August 2022)

We perform post-Newtonian analysis of double field theory as a test of string theory in gravitational sector against observations. We identify the Eddington-Robertson-Schiff parameters β_{PPN}, γ_{PPN} with the charges of electric H flux and dilaton respectively, and further relate them to stress-energy tensor. We show $\beta_{\mathrm{PPN}}=1$ from weak energy condition and argue that the observation of $\gamma_{\mathrm{PPN}}\simeq 1$ signifies the ultrarelativistic equation of state in baryons, or the suppression of gluon condensate.

DOI: 10.1103/PhysRevLett.129.061603

Introduction.—What is the gravitational theory that string theory predicts? The conventional answer has been general relativity (GR) on account of a metric $g_{\mu\nu}$ appearing in closed string quantization. However, the metric is only one segment of the closed string massless sector that includes two additional fields, a two-form potential $B_{\mu\nu}$ and a scalar dilaton ϕ. Given the fact that the $\boldsymbol{O}(D,D)$ symmetry of T-duality [1–6] transforms the trio to one another, it may be not unreasonable to regard the whole sector as gravitational. This idea has come true in recent years. The $\boldsymbol{O}(D,D)$ manifest formulation of supergravity [7–12], dubbed double field theory (DFT), has matured into "stringy gravity," which is based on a beyond-Riemannian geometry [13–16] and has its own version of "Einstein field equations" carrying $\boldsymbol{O}(D,D)$ indices [17],

$$G_{AB}=T_{AB}. \tag{1}$$

As off-shell and on-shell conserved $\boldsymbol{O}(D,D)$ tensors, the left-hand side and right-hand side of the equality represent stringy curvature and matter respectively. Parametrizing the fundamental variables of DFT in terms of the trio $\{g,B,\phi\}$ (c.f. [18–25] for non-Riemannian alternatives), the above single formula is decomposed into

$$\begin{aligned} R_{\mu\nu}+2\nabla_{\mu}(\partial_{\nu}\phi)-\frac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma}&=K_{(\mu\nu)},\\ \frac{1}{2}e^{2\phi}\nabla^{\rho}(e^{-2\phi}H_{\rho\mu\nu})&=K_{[\mu\nu]},\\ R+4\Box\phi-4\partial_{\mu}\phi\partial^{\mu}\phi-\frac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}&=T_{(0)}. \end{aligned} \tag{2}$$

While $K_{(\mu\nu)}$, $K_{[\mu\nu]}$, and $T_{(0)}$ are *a priori* the components of the DFT stress-energy tensor T_{AB} in Eq. (1), each line of Eq. (2) may be identified—at least on shell—as the equation of motion of $g_{\mu\nu}$, $B_{\mu\nu}$ and a DFT-dilaton d absorbing $\sqrt{-g}$ as $d\equiv\phi-\frac{1}{2}\ln\sqrt{-g}$. The $\boldsymbol{O}(D,D)$ symmetry principle, once taken as the working hypothesis, fixes their coupling to matters completely. For example, when coupled to the standard model [26], spin-half fermions (diffeomorphically half-unit-weighted) respond to the H flux [15] and gauge bosons do so to the dilaton [27]:

$$\int d^4x\bar{\psi}\left[i\gamma^{\mu}\left(D_{\mu}+\frac{1}{24}H_{\mu\nu\rho}\gamma^{\nu\rho}\right)-m\right]\psi-\frac{1}{4}e^{-2d}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{3}$$

which gives [15,17] $T_{(0)}=\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ and

$$K_{\mu\nu}=\frac{-ie^{2d}}{4}\bar{\psi}\left(\gamma_{\mu}\vec{D}_{\nu}-\overleftarrow{D}_{\nu}\gamma_{\mu}+\frac{1}{4}H_{\nu\rho\sigma}\gamma_{\mu}{}^{\rho\sigma}\right)\psi+\frac{1}{2}\mathrm{Tr}(F_{\mu\rho}F_{\nu}{}^{\rho}). \tag{4}$$

Having stated the above, to the best of our knowledge, the gravitational coupling in quantum field theories has never been tested experimentally. All the observations are based on the geodesic motion (and deviation) of point particles, including photons. The $\boldsymbol{O}(D,D)$ symmetry, then, enforces the particles to couple to $g_{\mu\nu}$ only in the usual manner, $-m_* \int d\tau \sqrt{-g_{\mu\nu}\dot{y}^\mu \dot{y}^\nu}$, which gives $T_{(0)} = 0$ and

$$K^{\mu\nu}(x) = \frac{1}{2} m_* \int d\tau\, \dot{y}^\mu \dot{y}^\nu e^{2d} \delta^{(4)}[x - y(\tau)]. \quad (5)$$

That is to say [28], the strong equivalence principle is saved in the string frame. While the sources can be stringy, the probes are still pointlike up to now.

Yet, given the augmented stress-energy tensor, the gravitational physics of DFT should be richer than that of GR. To develop some physical intuition on the governing equations [Eq. (2)], we may take the following three steps: (i) The second formula dictates how $K_{[\mu\nu]}$ shapes the H flux. (ii) Subtracting the third from the trace of the first, one solves for the dilaton

$$\Box(e^{-2\phi}) = \left(K_\mu{}^\mu - T_{(0)} + \frac{1}{6} H_{\lambda\mu\nu} H^{\lambda\mu\nu}\right) e^{-2\phi}. \quad (6)$$

(iii) The metric is lastly determined by the first formula.

It is the purpose of the present Letter to address the feasibility of the closed string massless sector, or $D = 4$ DFT as a "stringy gravity" alternative to GR, within the parametrized post-Newtonian (PPN) formalism [29–31]. Keeping astrophysical systems in mind rather than the entire Universe, we shall neglect any large-scale contribution to $T_{(0)}$ such as a (noncritical string) cosmological constant [32–35]. We postulate the spacetime to be asymptotically flat and static. The Eddington-Robertson-Schiff parameters $\beta_{\rm PPN}$, $\gamma_{\rm PPN}$ are then defined in an isotropic coordinate system,

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2MG_{\rm N}}{r} + \frac{2\beta_{\rm PPN}(MG_{\rm N})^2}{r^2} + \cdots\right) dt^2 \\ &+ \left(1 + \frac{2\gamma_{\rm PPN} MG_{\rm N}}{r} + \cdots\right) dx^i\, dx^j\, \delta_{ij}, \end{aligned} \quad (7)$$

where $r = \sqrt{x^i x^j \delta_{ij}}$ is the isotropic radius and $G_{\rm N}$ is the Newtonian constant of gravitation (setting "$\alpha_{\rm PPN} \equiv 1$").

The current stringent observational bounds for the solar gravity are $\gamma_{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (Shapiro time delay by Cassini spacecraft) [31,36] and adopting this value $\beta_{\rm PPN} - 1 = (-4.1 \pm 7.8) \times 10^{-5}$ or $(0.4 \pm 2.4) \times 10^{-4}$ (perihelion shifts of Mercury or Mars) [31,37]. Other observations include $4\beta_{\rm PPN} - \gamma_{\rm PPN} - 3 = (4.44 \pm 4.5) \times 10^{-4}$ for the Earth gravity [38] and $\gamma_{\rm PPN} = 0.98 \pm 0.07$ on galactic size scales [39]. These all agree with the GR prediction $\beta_{\rm PPN} = \gamma_{\rm PPN} = 1$ and rule out various alternative gravities [40–42]. For example, a class of scalar-tensor theories, notably Brans-Dicke, gives

$$\gamma_{\rm PPN} = \frac{1 + \omega_{\rm BD}}{2 + \omega_{\rm BD}}, \quad (8)$$

and thus the observations compel the coupling constant $\omega_{\rm BD}$ unnaturally huge. Truncating $K_{[\mu\nu]}$, $T_{(0)}$, and the H flux consistently, the leftover components of Eq. (2) coincide with the Brans-Dicke field equations for $\omega_{\rm BD} = -1$. This has been often regarded as a "smoking gun" to necessarily eliminate the string dilaton ϕ. Below, analyzing Eq. (2) with care, we show that spherically symmetric static objects in stringy gravity (henceforth collectively "stars" though applicable to the Earth) have $\beta_{\rm PPN} = 1$ and make $\gamma_{\rm PPN}$ depend on equation-of-state parameters. As a byproduct, we shall point out that the well-known relation [Eq. (8)] is an artifact of restricting to "pressureless" stars. It can be generalized to $\gamma_{\rm PPN} = 3p/\rho$ for $\omega_{\rm BD} = -1$, and thus becomes phenomenologically viable with ultrarelativistic pressure.

Post-Newtonian double field theory.—Following the three steps above—(i), (ii), and (iii)—it is possible to solve the linearized version of Eq. (2), using the Green function method with retarded time $t' = t - |\mathbf{x} - \mathbf{x}'|$ [43],

$$\begin{aligned} H_{\lambda\mu\nu}(x) &\simeq -\frac{1}{2\pi} \int d^3x' \frac{3\partial'_{[\lambda} K_{\mu\nu]}(x')}{|\mathbf{x} - \mathbf{x}'|}, \\ \phi(x) &\simeq \frac{1}{8\pi} \int d^3x' \frac{[K_\mu{}^\mu - T_{(0)} + \frac{1}{6} H_{\lambda\mu\nu} H^{\lambda\mu\nu}](x')}{|\mathbf{x} - \mathbf{x}'|}, \\ g_{\mu\nu}(x) &\simeq \eta_{\mu\nu} + \frac{1}{2\pi} \int d^3x' \frac{[K_{(\mu\nu)} + \frac{1}{4} H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma}](x')}{|\mathbf{x} - \mathbf{x}'|}. \end{aligned} \quad (9)$$

Mapping this to the isotropic coordinate system, we can read off $MG_{\rm N}$ and $\gamma_{\rm PPN}$ but not $\beta_{\rm PPN}$, which would require higher order analysis. Instead, we turn to a known $D = 4$ three-parameter exact solution to Eq. (2) [44]. It has vanishing sources ($T_{AB} = 0$) and can be identified as the most general, asymptotically flat, static, and spherically symmetric "vacuum" geometry ($G_{AB} = 0$) [28]. For our purposes, we perform a radial coordinate transformation, $r_{\rm there} = r + [(a^2 + b^2)/16r] + [(\alpha_{\rm there} - \beta_{\rm there})/2]$ from [28], and put the geometry decisively in the isotropic (as well as often spherical) coordinate system,

$$\begin{aligned} e^{2\phi} &= \gamma_+ \left(\frac{4r - \sqrt{a^2 + b^2}}{4r + \sqrt{a^2 + b^2}}\right)^{\frac{2b}{\sqrt{a^2+b^2}}} + \gamma_- \left(\frac{4r + \sqrt{a^2 + b^2}}{4r - \sqrt{a^2 + b^2}}\right)^{\frac{2b}{\sqrt{a^2+b^2}}}, \\ H_{(3)} &= h \sin\vartheta dt \wedge d\vartheta \wedge d\varphi = h dt \wedge \left(\frac{\epsilon_{ijk} x^i dx^j \wedge dx^k}{2r^3}\right), \\ ds^2 &= g_{tt}(r) dt^2 + g_{rr}(r)(dr^2 + r^2 d\Omega^2), \end{aligned} \quad (10)$$

where $d\Omega^2 = d\vartheta^2 + \sin^2\vartheta d\varphi^2$, and

$$g_{tt}(r) = -e^{2\phi(r)}\left(\frac{4r-\sqrt{a^2+b^2}}{4r+\sqrt{a^2+b^2}}\right)^{\frac{2a}{\sqrt{a^2+b^2}}},$$
$$g_{rr}(r) = e^{2\phi(r)}\left(\frac{4r+\sqrt{a^2+b^2}}{4r-\sqrt{a^2+b^2}}\right)^{\frac{2a}{\sqrt{a^2+b^2}}}\left(1-\frac{a^2+b^2}{16r^2}\right)^2.$$

The three "free" parameters are $\{a, b, h | b^2 \geq h^2\}$, which set $\gamma_\pm = \frac{1}{2}(1 \pm \sqrt{1-h^2/b^2})$.

The geometry [Eq. (10)] can be identified as the outer geometry of a (stringy) compact star. Crucially, from the full Einstein equations [Eq. (2)], it becomes possible to ascribe the three parameters to the star's stress-energy tensor or $\{K_t{}^t, K_r{}^r, K_\vartheta{}^\vartheta = K_\varphi{}^\varphi, K_{tr}, K_{[\vartheta\varphi]}\}$ [17], such as

$$a = \frac{1}{4\pi}\int d^3x\, e^{-2d}\left(K_i{}^i - K_t{}^t - T_{(0)} + \frac{1}{6}H_{ijk}H^{ijk}\right). \tag{11}$$

Inside the star, while the electric H flux is persistently of the rigid form [Eq. (10)], the magnetic flux can be nontrivial,

$$H^{r\vartheta\varphi} = -2e^{2d}\int_r^{r_\star} dr'\, e^{-2d}K^{[\vartheta\varphi]}, \tag{12}$$

where the upper limit of the integral $r_\star$ denotes the star's finite radius. As implied by the first of Eq. (2) for the isotropic metric, the electric and the magnetic fluxes should meet

$$H_{t\vartheta\varphi}H_r{}^{\vartheta\varphi} = -2K_{(tr)}, \tag{13}$$

while the second of Eq. (2) gives $K_{[tr]} = 0$.

Now, from the $1/r$ expansion of the outer geometry [Eq. (10)], we are able to identify the Newtonian mass [28],

$$MG_{\rm N} = \frac{1}{2}(a + b\sqrt{1-h^2/b^2}), \tag{14}$$

and the two post-Newtonian parameters,

$$(\beta_{\rm PPN}-1)(MG_{\rm N})^2 = \frac{h^2}{4},$$
$$(\gamma_{\rm PPN}-1)MG_{\rm N} = -b\sqrt{1-\frac{h^2}{b^2}}. \tag{15}$$

Since the inverse relations hold,

$$h = \pm 2\sqrt{\beta_{\rm PPN}-1}MG_{\rm N}, \qquad a = (\gamma_{\rm PPN}+1)MG_{\rm N},$$
$$b = (1-\gamma_{\rm PPN})\sqrt{1+4\frac{\beta_{\rm PPN}-1}{(\gamma_{\rm PPN}-1)^2}}MG_{\rm N},$$

the outer geometry [Eq. (10)] is also completely determinable by the triple of $\{MG_{\rm N}, \beta_{\rm PPN}, \gamma_{\rm PPN}\}$, such as

$$\phi \simeq \frac{(\gamma_{\rm PPN}-1)MG_{\rm N}}{2r} + \frac{(\beta_{\rm PPN}-1)(MG_{\rm N})^2}{r^2},$$
$$H_{(3)} = \pm\sqrt{\beta_{\rm PPN}-1}MG_{\rm N}dt \wedge \left(\frac{\epsilon_{ijk}x^i dx^j \wedge dx^k}{r^3}\right),$$
$$g_{rr} \simeq 1 + \frac{2\gamma_{\rm PPN}MG_{\rm N}}{r} + \frac{(6\beta_{\rm PPN}+7\gamma_{\rm PPN}^2-7)(MG_{\rm N})^2}{r^2}. \tag{16}$$

The expansion of ϕ suggests that $(\gamma_{\rm PPN}-1)MG_{\rm N}$ is the "dilaton charge." Indeed, multiplying $\sqrt{-g}$ to Eq. (6) and using $\sqrt{-g}\Box(e^{-2\phi}) = \partial_\mu(\sqrt{-g}\partial^\mu e^{-2\phi})$, we can compute the dilaton charge from the stress-energy tensor of the star,

$$(\gamma_{\rm PPN}-1)MG_{\rm N} = \frac{1}{4\pi}\oint_\infty dS_i\partial^i e^{-2\phi}$$
$$= \frac{1}{4\pi}\int d^3x\, e^{-2d}\left(K_\mu{}^\mu - T_{(0)} + \frac{1}{6}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right), \tag{17}$$

where the H flux has been fixed by Eqs. (12) and (13). Similarly, from Eq. (16) we identify $\sqrt{\beta_{\rm PPN}-1}MG_{\rm N}$ as the "electric H flux charge" that is through Eqs. (12) and (13) also related to the stress-energy tensor,

$$\sqrt{\beta_{\rm PPN}-1}MG_{\rm N} = \left|\oint_\infty \frac{dS_i}{16\pi}\epsilon^{ijk}H_{tjk}\right|$$
$$= \left|\frac{K_{(tr)}g^{rr}(e^{-2d}/\sin\vartheta)}{2\int_r^{r_\star}dr'(e^{-2d}K^{[\vartheta\varphi]})}\right|. \tag{18}$$

For consistency, the fractional quantity at the end of Eq. (18) must be independent of r, while the ϑ dependency is trivially canceled since $K_{[\vartheta\varphi]} \propto \sin\vartheta$, $K^{[\vartheta\varphi]} \propto 1/\sin\vartheta$, and

$$e^{-2d} = e^{-2\phi(r)}\sqrt{-g_{tt}(r)g_{rr}^3(r)}r^2\sin\vartheta. \tag{19}$$

Lastly, from Eqs. (11), (15), and (17), the Newtonian mass [Eq. (14)] has its own integral expression [17],

$$M = \frac{1}{4\pi G_{\rm N}}\int d^3x\, e^{-2d}\left(-K_t{}^t - \frac{1}{2}H_{t\mu\nu}H^{t\mu\nu}\right). \tag{20}$$

Substituting the expression of $-K_t{}^t > 0$ for particles [Eq. (5)] into Eq. (20), we identify m_* in terms of the rest mass,

$$m_* = 8\pi G_{\rm N}m, \tag{21}$$

and further confirm that, like in GR, the Newtonian mass M is given by the sum of the energy $m\dot{t} = m/\sqrt{1-v^2}$ rather than the rest mass. However, in contrast to GR, it is *a priori* $-4K_t{}^t$ rather than the conventional energy, or

$\propto -2K_t{}^t + T_{(0)}$, that enters the mass formula. This difference is significant for gauge bosons, in particular gluons: as seen from Eq. (4) only the electric field F_{ti} contributes to the mass, but the magnetic field F_{ij} does not.

Since e^{-2d} is the rightful integral measure in DFT, all the integral expressions above are "proper," i.e., invariant under static (radial) diffeomorphisms, which is rather notoriously not the case for the Schwarzschild mass in GR.

$\beta_{\rm PPN} = 1$ *from a weak energy condition.*—The exact expressions of the Newtonian mass [Eq. (20)] and the magnetic H flux [Eq. (12)] are in good agreement with the linearized general solutions [Eq. (9)]. The former [Eq. (20)] is obvious and the latter [Eq. (12)] is due to a shell theorem. However, the electric H flux given through Eqs. (15) and (18) appears highly nonperturbative. In fact, for the linearized expression [Eq. (9)], when $K_{[\mu\nu]}$ therein is static and spherical, we should have $\partial_t K_{[\mu\nu]} = 0$ and $K_{[ti]} = x_i f(r)$ for some radial function $f(r)$. Consequently, $\partial_{[t}K_{ij]}$ vanishes and the linear version of the electric H flux must be trivial. We resolve this discrepancy by arguing that, the electric H flux in the exact solution should be trivial, too. With Eq. (19), its contribution to the Newtonian mass [Eq. (20)] is an $1/r^2$ integral, $\int_0^\infty dr\, e^{-2\phi}h^2/(G_{\rm N}\sqrt{-g_{tt}g_{rr}}r^2)$, which—provided the geometry at the center of the star is nonsingular—would diverge unless $h = 0$. Since the mass M is finite and a weak energy condition $-K_t{}^t > 0$ should hold, the electric H flux must be inevitably trivial. Besides, the last expression of Eq. (18) ought to be independent of r. In its small r limit, from Eq. (19), the numerator vanishes. Hence, we arrive at the same conclusion $h = 0$. Consequently, from Eq. (13) $K_{(tr)} = 0$ and from Eq. (15) $\beta_{\rm PPN} = 1$.

$\gamma_{\rm PPN} \simeq 1$ *from subhadronic pressure.*—With the vanishing electric H flux ($h = 0$), the volume integrals of the total mass [Eq. (20)] and $\gamma_{\rm PPN}$ [Eq. (17)] are now all restricted to the star's interior,

$$\gamma_{\rm PPN} = 1 - \frac{\int_{\rm star} d^3x\, e^{-2d}(K_\mu{}^\mu - T_{(0)} + \frac{1}{6}H_{ijk}H^{ijk})}{\int_{\rm star} d^3x\, e^{-2d}K_t{}^t}, \qquad (22)$$

where $\frac{1}{6}H_{ijk}H^{ijk} = H_{r\vartheta\varphi}H^{r\vartheta\varphi}$ is set by $K^{[\vartheta\varphi]}$ from Eq. (12). Clearly, $\gamma_{\rm PPN}$ depends on the stress-energy tensor for which we introduce volume-averaged, equation-of-state parameters (c.f. [45]),

$$w_K = \frac{\int_{\rm star} d^3x\, e^{-2d}\frac{1}{3}K_i{}^i}{\int_{\rm star} d^3x\, e^{-2d}(-K_t{}^t)}, \qquad w_T = \frac{\int_{\rm star} d^3x\, e^{-2d}T_{(0)}}{\int_{\rm star} d^3x\, e^{-2d}(-K_t{}^t)}, \qquad (23)$$

and further for the magnetic H flux part,

$$\delta_{H-\rm flux} = \frac{16\pi\int_0^{r_\star} dr\, r^2 e^{2\phi}\sqrt{g_{rr}^3/(-g_{tt})}(\int_r^{r_\star} dr'\, e^{-2d}K^{[\vartheta\varphi]})^2}{\int_{\rm star} d^3x\, e^{-2d}(-K_t{}^t)}. \qquad (24)$$

Note from the identification of the usual energy-momentum tensor $\mathcal{T}_{\mu\nu} = e^{-2\phi}(2K_{(\mu\nu)} - g_{\mu\nu}T_{(0)})$, c.f. Eq. (27), the conventional equation-of-state parameter reads

$$w = \frac{\mathcal{T}_i{}^i}{-3\mathcal{T}_t{}^t} = \frac{p}{\rho} = \frac{w_K - \frac{1}{2}w_T}{1 + \frac{1}{2}w_T}. \qquad (25)$$

The post-Newtonian parameter [Eq. (22)] amounts to

$$\gamma_{\rm PPN} = 3w_K - w_T + \delta_{H-\rm flux}. \qquad (26)$$

For the ideal gas of particles, we have $w_T = \delta_{H-\rm flux} = 0$ and hence simply $\gamma_{\rm PPN} = 3w$. In terms of the averaged speed v, this is equivalent to $\gamma_{\rm PPN} \simeq v^2$ since from Eq. (5) we have $K_\mu{}^\mu/K_t{}^t = -1/(g_{tt}\dot{t}^2) = -g^{tt}(1 - v^2) \simeq 1 - v^2$.

If a star were composed of nonrelativistic pressureless dust, we would get $\gamma_{\rm PPN} = 0$, which certainly fails to explain the observations. This is actually the case with the usual analysis of the Brans-Dicke theory having $\omega_{\rm BD} = -1$ [Eq. (8)] and nonrelativistic sources. More realistic stars would exert pressure along the radial direction to balance the gravitational force. In the Newtonian gravity, the pressure is fixed to meet $(dp(r)/dr) = -M(r)G_{\rm N}\rho/r^2$ and leads to $w = (\langle p\rangle/\rho) = (MG_{\rm N}/2r_\star)$ (volume averaged). The Sun would have then merely $\gamma_{\rm PPN} = 3w \sim 3\times 10^{-6}$, which is far less than unity. In GR, the outer Schwarzschild geometry of a star depends on the total mass only, being inert to the pressure. In DFT, the outer geometry [Eq. (10)] depends on the equation-of-state parameters [Eq. (23)], yet the Einstein equations thereof [Eq. (1)] still do not impose any restriction on them, as seen, e.g., from Eqs. (17) and (20). In flat spacetime, the conservation of the (ordinary) stress-energy tensor implies that the volume integral of the pressure vanishes, which is known as the von Laue condition [46]. To correctly include the stringy gravitational effect, we call the on-shell conservation $\nabla_A T^A{}_B = 0$ of Eq. (1) [17] (c.f. [47,48]), which becomes more concretely $\nabla_\mu(e^{-2\phi}K^{[\mu\nu]}) = 0$ and

$$\nabla^\rho[e^{-2\phi}K_{(\rho\mu)}] + \frac{1}{2}e^{-2\phi}[H_{\mu\rho\sigma}K^{[\rho\sigma]} - \partial_\mu T_{(0)}] = 0. \qquad (27)$$

For the spherically symmetric static case of our interest where $K_{tr} = 0 = H_{t\vartheta\varphi}$ and Eq. (12) holds, writing the covariant derivative in Eq. (27) explicitly as

$$\begin{aligned} r\sqrt{-g}\nabla^\rho[e^{-2\phi}K_{(\rho r)}] = {}& \frac{d}{dr}\left[re^{-2d}\left(K_r{}^r - \frac{1}{3}K_i{}^i\right)\right] \\ &+ \frac{1}{3}r^3\frac{d}{dr}(r^{-2}e^{-2d}K_i{}^i) \\ &- \frac{1}{2}re^{-2d}\left(\frac{\dot{g}_{tt}}{g_{tt}}K_t{}^t + \frac{\dot{g}_{rr}}{g_{rr}}K_i{}^i\right), \end{aligned}$$

we relax the von Laue condition rather inevitably:

$$\int d^3x\, e^{-2d}\left(K_i{}^i - \frac{3}{2}T_{(0)} + \frac{1}{8}H_{ijk}H^{ijk}\right)$$
$$= \int d^3x\, e^{-2d}\left[\frac{r\dot{g}_{tt}}{2g_{tt}}\left(-K_t{}^t + \frac{1}{2}T_{(0)} + \frac{1}{24}H_{ijk}H^{ijk}\right) - \frac{r\dot{g}_{rr}}{2g_{rr}}\left(K_i{}^i - \frac{3}{2}T_{(0)} + \frac{1}{8}H_{ijk}H^{ijk}\right) - r\dot{\phi}\left(T_{(0)} + \frac{1}{12}H_{ijk}H^{ijk}\right)\right]. \tag{28}$$

While $(\dot{g}_{tt}/2g_{tt})(-K_t{}^t)$ corresponds to the gravitational force, the presence of the self-reflective term of $-(r\dot{g}_{rr}/2g_{rr})(K_i{}^i - \frac{3}{2}T_{(0)} + \frac{1}{8}H_{ijk}H^{ijk})$ invalidates the aforementioned Newtonian underestimation of the total pressure. In fact, Eqs. (27) and (28) are identities for any matter and geometric data $\{K_{\mu\nu}, T_{(0)}, g_{\mu\nu}, B_{\mu\nu}, \phi\}$ that satisfy the Einstein equations [Eq. (2)], since $\nabla_A G^A{}_B = 0$ is a "Bianchi" identity in DFT [48,49].

For $\gamma_{\rm PPN} \simeq v^2$ to be close to unity within the stringent observational bound of 2.1×10^{-5} [31,36], the constituting particles—if not strings—should be ultrarelativistic. In terms of temperature $3k_BT = m[(1/\sqrt{1-v^2}) - 1]$, we get $\gamma_{\rm PPN} \simeq 1 - [m/(m + 3k_BT)]^2$. Assuming $T = 1.57 \times 10^7$ K at the center of the Sun [50], from $[m/(m + 3k_BT)]^2 < 2.1 \times 10^{-5}$, the particle mass m should not exceed 20 eV. Certainly no such a light atom exists. While so, a recent experiment has revealed a pressure inside protons as high as 10^{35} Pa ~ 0.005 GeV^4 [51] comparable to their mass density 10^{18} kg/m^3 ~ 0.004 GeV^4 $\sim (\Lambda_{\rm QCD})^4$ [52]. Henceforth, we look into the energy and pressure inside baryons, or the QCD matter of Eq. (3). The deviation $\gamma_{\rm PPN} - 1$ [Eq. (22)] is then given by the sum of $\delta_{H-{\rm flux}}$ [Eq. (24)] and an integral,

$$\frac{1}{4\pi M G_{\rm N}}\int_{\rm star} d^3x\, \frac{1}{4}e^{-2d}{\rm Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{2}\bar{\psi}\left(m + i\frac{1}{12}\gamma^{ijk}H_{ijk}\right)\psi. \tag{29}$$

In our normalization [Eq. (2)], from Eq. (21), $K_{\mu\nu}$, $T_{(0)}$ are order of $8\pi G_{\rm N}$. Hence the magnetic H flux [Eq. (12)] gives a subleading contribution and may be ignored. Then, Eq. (29) boils down faithfully to the integral of the gluon and quark condensates (vacuum expectation value) [53,54]. According to [55–63], the condensates have local support restricted to the hadrons' interior. Put differently, the energy, pressure, and condensates specified by the components of $K_{\mu\nu}$, $T_{(0)}$ are all confined. Although the condensates may have the natural order of magnitudes $(\Lambda_{\rm QCD})^4$ and $m \times (\Lambda_{\rm QCD})^3$, respectively, while the latter can be chirally rotated and averaged away, the empirical measurements of the former [64–68] show significant scatter and even differences in sign [58], allowing zero as 0.006 ± 0.012 GeV^4 [64]. Perhaps, at (four-dimensional) thermal equilibrium, the equal number of electric and magnetic gluon field-strength-squared cancel each other to suppress the gluon condensate normalized by the baryon mass density (c.f. [60]). This ratio essentially estimates Eq. (29) and the observation $\gamma_{\rm PPN} \simeq 1$ is consistent with the suppression.

Discussion.—To conclude, DFT sets $\beta_{\rm PPN} = 1$ and lets $\gamma_{\rm PPN}$ depend on the equation-of-state parameters [Eq. (26)]. Rather than ruling out the theory, applied to baryons' interior where the energy and pressure are both confined, the apparently universal observations $\gamma_{\rm PPN} \simeq 1$ including the Sun and the Earth [31,36,38] signify the ultrarelativistic equation of state inside baryons, $w_K \simeq \frac{1}{3}$, $w_T \simeq 0$, through the suppression of the (massless) gluon condensate [Eq. (29)]. We call for both theoretical and experimental verifications of this rather drastic conclusion of ordinary matter being ultrarelativistic at a subhadronic level.

Some comments are in order. In terms of the adiabatic index c_P/c_V, one gets $w = c_P/c_V - 1$, which becomes $1/3$ for ultrarelativistic ideal fluid having $c_P = 4k_B$ and $c_V = 3k_B$, e.g., [69].

Any two-derivative potential-free effective scalar theory, if $\boldsymbol{O}(D,D)$-symmetric, should be of the form

$$S_{\rm eff} = -\int d^4x\, e^{-2d}g^{\mu\nu}\partial_\mu\Phi^I\partial_\nu\Phi^J\mathcal{G}_{IJ}(\Phi), \tag{30}$$

which gives $K_\mu{}^\mu = g^{\mu\nu}\partial_\mu\Phi^I\partial_\nu\Phi^J\mathcal{G}_{IJ} = T_{(0)}$, $K^{[\vartheta\varphi]} = 0$, and thus rather precisely $\gamma_{\rm PPN} = 1$.

The B field is dual to a scalar axion in four dimensions. Recently, a static Kähler axion has been suggested to make $\gamma_{\rm PPN}$ close to unity (for $w = 0$ and $\omega_{\rm BD} = \frac{3}{2}$) [70]. In our case, the axion dual to the magnetic H flux would be time, rather than radial, dependent.

Since $\delta_{H-{\rm flux}} \geq 0$, if any star were ever found to feature $\gamma_{\rm PPN} > 1$, it should be stringy. From the superextensive multi-integrals appearing in $\delta_{H-{\rm flux}}$ (24), bigger stringy stars or galaxies would have larger value of $\gamma_{\rm PPN}$.

The equations of state $w_K \simeq \frac{1}{3}$, $w_T \simeq 0$ for $\gamma_{\rm PPN} \simeq 1$ prevent also ϕ from the cosmological time evolution [45,71]. This may satisfy an extremely tight bound on the time variation of the fine structure constant [72], since e^{-2d} is an overall factor hence affecting the coupling constant of the gauge boson Lagrangian [Eq. (3)].

We wish to thank Robert Brandenberger, Minkyoo Kim, Bum-Hoon Lee, Chang-Hwan Lee, Hocheol Lee, Taekoon Lee, Wonwoo Lee, Shinji Mukohyama, Sang-Jin Sin, and Clifford Will for useful comments and correspondences. This work is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) Grants No. NRF-2016R1D1A1B01015196, NRF-2018R1A2B2007163, and NRF-2020R1A6A1A03047877 (Center for Quantum Space Time).

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