

Experimental Demonstration of Genuine Tripartite Nonlocality under Strict Locality Conditions


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 (Received 7 March 2022; revised 10 May 2022; accepted 11 July 2022; published 2 August 2022)

Nonlocality captures one of the counterintuitive features of nature that defies classical intuition. Recent investigations reveal that our physical world's nonlocality is at least tripartite; i.e., genuinely tripartite nonlocal correlations in nature cannot be reproduced by any causal theory involving bipartite nonclassical resources and unlimited shared randomness. Here, by allowing the fair sampling assumption and postselection, we experimentally demonstrate such genuine tripartite nonlocality in a network under strict locality constraints that are ensured by spacelike separating all relevant events and employing fast quantum random number generators and high-speed polarization measurements. In particular, for a photonic quantum triangular network we observe a locality-loophole-free violation of the Bell-type inequality by 7.57 standard deviations for a postselected tripartite Greenberger-Horne-Zeilinger state of fidelity $(93.13 \pm 0.24)\%$, which convincingly disproves the possibility of simulating genuine tripartite nonlocality by bipartite nonlocal resources with globally shared randomness.

DOI: [10.1103/PhysRevLett.129.060401](https://doi.org/10.1103/PhysRevLett.129.060401)

Quantum theory allows correlations between spatially separated systems that are incompatible with local realism [1]. The most well-known manifestation is the correlation in bipartite systems—Bell nonlocality [1,2] that originally lies in the nature of quantum entanglement. As confirmed via loophole-free violations of Bell inequalities [3–7], Bell nonlocality has found novel applications in many quantum information tasks such as device-independent quantum cryptography [8,9] and randomness certification [10,11].

In contrast to bipartite systems, multipartite systems display much richer and complex correlation structures [2,12]. Historically, multipartite entanglement conventionally understood as the property of nonseparability [13] was used to violate Bell-like inequalities (e.g., Mermin's inequality [14]) for multipartite nonlocality [15,16]. However, one could reproduce such Bell-like violations by using entanglement of partial separability. This fact was first pointed out by Svetlichny in 1987 [17], who derived an inequality such that it is obeyed by three-particle biseparable states but its violation shows the states are truly three-particle nonseparable. This has motivated great interest in the study of the strongest form of multipartite nonlocality—genuine multipartite nonlocality (GMN).

In an effort to contribute to this line of research, Svetlichny's genuine tripartite nonlocality has been experimentally verified [18,19] and generalized to scenarios

featuring an arbitrary number of particles [20,21] as well as arbitrary dimensions [22,23]. However, Svetlichny's GMN is relative to local operations and classical communication (LOCC) [24,25]. This is inconsistent with the situation involving spacelike separated parties that enforces a nonsignaling condition [25,26], which has already been shown in a tabletop experiment [27]. Notably, restricted by nonsignaling conditions, Svetlichny's GMN can also be observed in any network built by sharing only bipartite nonlocal resources, e.g., bipartite entanglement [28]. Moreover, some correlations that display the forms of genuine tripartite nonlocality [17,26] can be replicated by bipartite systems [29]. Realistically, all parties can have access to a common source of shared randomness. Also, instead of quantum theory, one could exploit alternative physical theories such as boxworld [30] for nonclassical resources (e.g., Popescu-Rohrlich boxes [31]). Interestingly, it has been shown that boxworld theory cannot reproduce all quantum correlations even if we allow globally shared classical randomness [32,33]. Furthermore, bipartite resources are not enough to reproduce tripartite phenomena in a theory-independent analysis; however, shared randomness is not involved in the analysis [34]. Thus, it is interesting to study GMN relative to local operations and shared randomness (LOSR) [35] from a theory-agnostic perspective, i.e., whether there are correlations in nature

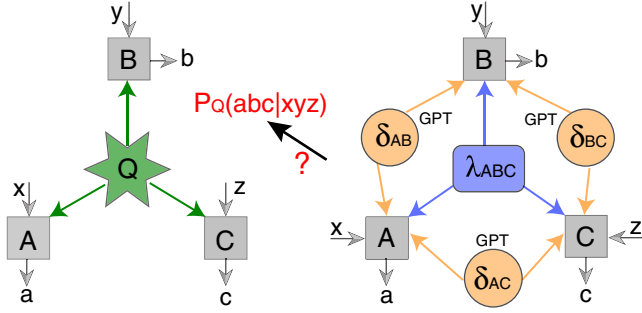


FIG. 1. A triangular network features three observers (gray squares) A , B , and C for Alice, Bob, and Charlie, respectively, with (x, a) , (y, b) , and (z, c) being their inputs and outputs. The question of interest is whether or not the correlations $P_Q(abc|xyz)$ observed on the network on the left-hand side, in which each observer receives a particle from the tripartite-entangled quantum source (green starburst) and performs local measurements, can be simulated by the correlations obtained on the network on the right-hand side, in which the observers are connected by nonclassical bipartite resources (δ_{ij} with $i, j = A, B, C$) and shared randomness λ_{ABC} .

irreproducible by sharing only fewer-partite nonlocal resources with LOSR (Fig. 1).

Recently, Coiteux-Roy *et al.* answered positively by taking into account all causal theories compatible with device replication [i.e., refer to generalized probabilistic theories (GPTs)], including classical theory, quantum theory, nonsignaling boxes, and any hypothetical causal theory [36,37]. In the framework of LOSR, they refined Svetlichny's GMN to genuine LOSR multipartite nonlocality or GMN in network. With the inflation technique widely used in analyzing theory-independent correlations [38,39], they derived a device-independent Bell-type inequality that is satisfied by all multipartite correlations arising from sharing fewer-partite nonlocal resources and global randomness. From the violations to the Bell-type inequality by N -partite Greenberger-Horne-Zeilinger (GHZ) states for all finite N , they thus proved that nature's nonlocality must be boundlessly multipartite in any causal GPTs.

In this Letter, we aim to show genuine LOSR tripartite nonlocality in a state-of-the-art photonic quantum network under strict locality constraints; i.e., all the parties involved are spacelike separated. This requirement is crucial in analyzing Bell-type inequality violation as potential locality loopholes might be exploited by adversaries and also enforces the nonsignaling conditions with classical communication between the parties being forbidden. In detail, we adopt postselection and prepare a triggered three-photon GHZ state from two independent entangled pair sources and distribute the state to three spacelike separated observers, Alice, Bob, and Charlie. The locality loophole is closed by spacelike separating relevant events and using fast quantum random number generators (QRNGs) and

high-speed polarization analyzers. We require the fair sampling assumption and postselection in the experiment, and show that the produced tripartite correlations cannot be simulated by any bipartite nonlocal resources with LOSR; i.e., they are genuinely LOSR tripartite nonlocal. We expect our work will stimulate further experimental investigation of genuinely multipartite nonlocality to better understand nature.

The genuine LOSR tripartite nonlocality proposed by Coiteux-Roy *et al.* is guaranteed by violations to the device-independent inequality arising from combining two intertwined games [36,37], respectively detecting (1) some nonclassical resources albeit possibly bipartite and (2) some tripartite resource albeit possibly classical. For (1), the Bell game, they exploit the standard Clauser-Horne-Shimony-Holt (CHSH) Bell test between Alice and Bob, conditioned on Charlie's output result $C_1 = 1$, which reads

$$I_{\text{Bell}}^{C_1=1} := \langle A_0 B_0 \rangle_{C_1=1} + \langle A_0 B_1 \rangle_{C_1=1} + \langle A_1 B_0 \rangle_{C_1=1} - \langle A_1 B_1 \rangle_{C_1=1}, \quad (1)$$

where subscripts represent the observer's setting choices and all observables take either ± 1 . In a standard Bell game, $I_{\text{Bell}}^{C_1=1}$ can reach $2\sqrt{2}$, which necessitates nonclassical resources. For (2), all observers are required to give the same outputs, which can take either of the two values ± 1 . In this tripartite consistency game (i.e., same game), the correlation is defined as [36]

$$I_{\text{same}} := \langle A_0 B_2 \rangle + \langle B_2 C_0 \rangle, \quad (2)$$

and the perfect score is $I_{\text{same}} = 2$.

We note that $A_0 := A_{x=0}$ appears in both games; thus, Alice cannot distinguish which of the two games she is participating in. This prevents her from playing the two games separately and she has to optimize Eqs. (1) and (2) simultaneously with her input $x = 0$. Actually, it is impossible for Alice to decouple the two games, which indicates that performing well at both games (1) and (2) would require dependence on a genuinely LOSR tripartite nonlocal resource [36].

With the inflation techniques [38,39], Coiteux-Roy *et al.* then combine the two aforementioned games in one scenario. If each two parties from three spacelike separated observers Alice, Bob, and Charlie share a bipartite nonlocal resource and each party performs some local measurements, e.g., A_x , B_y , and C_z with random inputs $x \in \{0, 1\}$, $y \in \{0, 1, 2\}$, and $z \in \{0, 1\}$ [Fig. 2(a)], with outcomes $a, b, c = \pm 1$, then the resulting joint outcome probabilities $p(abc|xyz)$ satisfy the following device-independent Bell-type inequality (in slightly different but equivalent form):

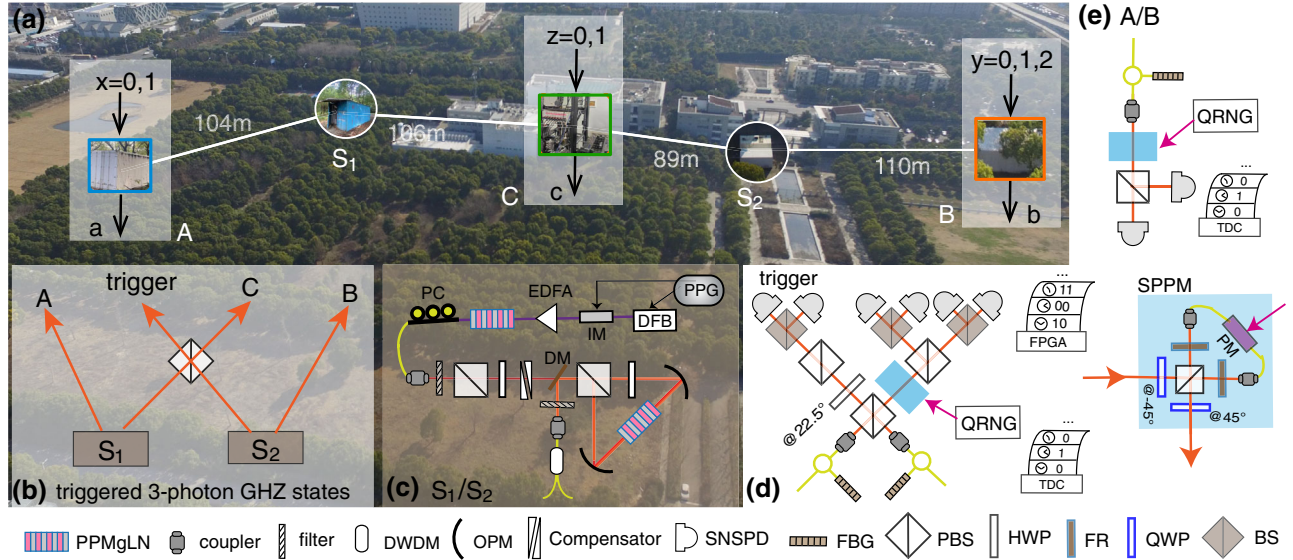


FIG. 2. Scheme of the experiment. (a) Three spacelike separated observers (Alice, Bob, and Charlie) perform their local measurements. Two EPR sources S_1 and S_2 emit entangled photon pairs such that they are combined at a PBS for preparing a $|\text{GHZ}_3\rangle$ state. The beeline distances are 104, 106, 89, and 110 m between Alice- S_1 , S_1 -Charlie, Charlie- S_2 , and S_2 -Bob, respectively. Their optical fiber links are 112.63, 124.9, 109.6, and 125.48 m respectively. (b) The $|\text{GHZ}_3\rangle$ state is created by projecting one photon over the diagonal basis $|+\rangle$ (trigger) from a four-photon GHZ state after postselection. (c) In each source, a pair of polarization-entangled photons in state $|\Phi^+\rangle$ is prepared by pumping a PPMgLN crystal in a Sagnac loop (for details, see text and Ref. [42]). (d),(e) Each observer performs local measurements on their received photon. The measurement choices are decided by their quantum random number generators (QRNGs). In each node, a high-speed single-photon polarization modulation (SPPM) is implemented to vary the direction of local polarization analysis. PPMgLMN, periodically poled MgO doped lithium niobate; PC, polarization controller; DWDM, dense wavelength-division multiplexer; DM, dichroic mirror; OPM, off-axis parabolic mirror; FBG, fiber Bragg grating; BS, beam splitter; SNSPD, superconducting nanowire single-photon detector.

$$F := I_{\text{Bell}}^{C_1=1} + \frac{4I_{\text{same}} - 8}{1 + \langle C_1 \rangle} \leq 2, \quad (3)$$

where F is the three-party correlation function; see calculations from $p(abc|xyz)$ in Supplemental Material [40]. A violation to the above inequality indicates the genuine LOSR tripartite nonlocality.

There are quantum correlations that violate the Bell-type inequality above. For example, we distribute the tripartite GHZ state $|\text{GHZ}_3\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$ in a triangular network and set Alice's, Bob's, and Charlie's measurements as $A_x \in \{Z, X\}$, $B_y \in \{[(Z+X)/\sqrt{2}], [(Z-X)/\sqrt{2}], Z\}$, and $C_z \in \{Z, X\}$, respectively. Here Z and X are standard Pauli operators. In this case, the tripartite quantum strategy yields a maximum violation of $F = 2\sqrt{2}$. For a mixture of the $|\text{GHZ}_3\rangle$ state with white noise, violation of Eq. (3) requires a fidelity of $\geq 93\%$. Note that Eq. (3) can be directly generalized to the N -party GHZ state; however, the required state fidelity greatly increases with the system size N [37] (for details, see Ref. [40]).

Our setup is shown in Fig. 2. To violate Eq. (3), we first prepare the $|\text{GHZ}_3\rangle$ state that can be efficiently created by combining two Einstein-Podolsky-Rosen (EPR) sources (S_1 and S_2) at a polarization beam splitter (PBS), as shown in Fig. 2(b). We use a pulse pattern generator (PPG) to send

out 250 MHz trigger signals, and the PPG in source S_2 acts as the master clock to synchronize all operations. In each source, a distributed feedback (DFB) laser is triggered to emit a 2 ns, 1558 nm laser pulse, which is carved into 80 ps with an intensity modulator (IM). The laser pulses are frequency doubled in a PPMgLN crystal after passing through an erbium-doped fiber amplifier (EDFA). We then use the produced 779 nm pump laser to drive a type-0 spontaneous parametric down-conversion (SPDC) process in the second PPMgLN crystal in a polarization-based Sagnac loop. Each source produces pairs of photons in the Bell state $|\Phi^+\rangle = (|HH\rangle + |VV\rangle)/\sqrt{2}$, where H and V denote horizontal and vertical polarization, respectively [see Fig. 2(c) and Ref. [42] for details). By interfering two photons at a PBS at Charlie's node, we get a four-photon GHZ state through the postselection of fourfold coincidences, which is used for creating the $|\text{GHZ}_3\rangle$ state when we measure the trigger photons in diagonal basis $|+\rangle$.

The observers then perform local measurements on their photon from the produced $|\text{GHZ}_3\rangle$ state. Alice and Charlie perform one of two measurements A_x and C_z , respectively, while Bob measures one of three bases B_y . Their setting choices x , z , and y are decided in real time by a fast quantum random number generator situated there. The QRNG at each station randomly outputs a sequence of bits

with equal probabilities. Note that the QRNG at Bob's station outputs four distinct sequences of two bits; however, we can discard one of the four outputs in order to have only three setting choices with equal probabilities for Bob under fair-sampling assumptions. All random bits from QRNG sources pass the NIST randomness tests [43] (see Ref. [44] for more information about the QRNGs). To realize the fast measurement setting choice, we implemented a high-speed high-fidelity single-photon polarization modulation (SPPM), which consists of two fixed quarter-wave plates (QWPs), two Faraday rotators (FRs), and an electro-optical phase modulator (PM), shown in Fig. 2 (see Supplemental Material in Ref. [44] for details). The SPPM varied the photons' polarization at a rate of 250 MHz with a fidelity of $\sim 99\%$ with random inputs. For Charlie, his setting choices decided by his QRNG were recorded with a time-to-digital converter (TDC). All his photon detections were analyzed in time and recorded by a field-programmable gate array (FPGA). Alice's and Bob's photon detection and setting results from QRNGs were recorded by their TDC, respectively. All the data were locally collected at the remote ports and sent to a separate computer to evaluate the three-party correlation function F .

The timing and layout of the experiment are critical to close locality loopholes, such that, for example, any observer's measurement results are causally independent from the other's setting choices. Now considering Charlie and Alice, we spacelike separate the events of Charlie completing the QRNG for setting choices (QRNG_C) from the events of finishing single-photon detection by Alice (M_A), and vice versa. In each trial, the time elapses of a QRNG to generate a random bit that determines the setting choices for the received photon are both 53 ± 2 ns for QRNG_C and QRNG_A. The time elapse of measurement events is defined as the interval between a photon enters the loop interferometer in the SPPM (Fig. 2) and the time of superconducting nanowire single-photon detector (SNSPD) outputs a signal for M_C and M_A are 44.9 ± 0.5 and 44.6 ± 0.5 ns, respectively. Their analyses are described in the left-hand panel of Fig. 3(a), where M_A is 156.3 ± 4 ns outside the light cone of QRNG_C and M_C is 73.5 ± 4 ns outside the light cone of QRNG_A, satisfying the locality condition here. We summarize all relevant results for the other two slices in Fig. 3(a) (middle and right-hand panels), with the labels defined using the same conversion. All the time-space relations are drawn to scale. The analysis is summarized and detailed in the Supplemental Material [40].

To estimate the fidelity of our prepared state after postselection with respect to the ideal state $|\text{GHZ}_3\rangle$, we perform a fidelity witness that can be evaluated with only a few measurements. The average triggered three-photon rate is 0.3 Hz and the fidelity is calculated to be $93.13 \pm 0.24\%$. We also perform a quantum state tomography to additionally characterize our prepared state (see Ref. [40]). We then evaluate the experimental violation of the inequality given

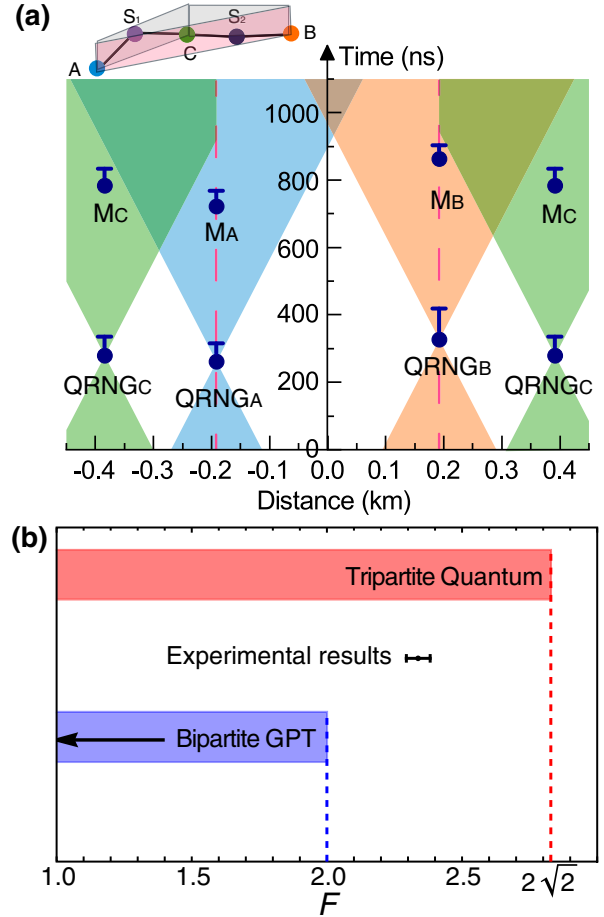


FIG. 3. Space-time analysis and experimental results. (a) The left-hand panel shows the spacelike separations between the setting choice events by Charlie (QRNG_C) and Alice (QRNG_A), and between QRNG_C (QRNG_A) and measurement events by Alice (M_A) (Charlie, M_C), respectively. Similarly, the middle and right-hand region (split by red dashed lines) are for events at Alice's and Bob's nodes, and events at Bob's and Charlie's nodes, respectively. Blue vertical bars denote the time elapsing for events, with start and end marked by circles and horizontal line, respectively. (b) The experimental result is $2.338(44)$, displayed as a black dot. Error bars indicate one standard deviations in the experiment. Compared with the bounds of bipartite GPT (blue) and tripartite quantum (red), our result surpasses the bipartite GPT bound of 2 by 7.57 standard deviations, indicating genuinely LOSR tripartite nonlocality.

by Eq. (3) and record 33 770 fourfold coincidence detection events over 171 725 s. As shown in Fig. 3(b), we obtain the correlation of $F = 2.338 \pm 0.044$, which is beyond the bipartite GPT bound by 7.57 standard deviations. That means the observed correlations via the three-photon GHZ state cannot be reproduced by any two-way GPT resources with local operations and unlimited shared randomness; i.e., it is genuinely LOSR tripartite nonlocal [36,37].

Based on an optical quantum network under strict locality constraints, we have experimentally demonstrated that nature's tripartite nonlocality cannot be simulated from any bipartite nonlocal causal theories. In our experiment,

the locality loophole between the three parties is addressed by spacelike separating relevant events and employing fast QRNGs and high-speed polarization analyzers. In this way, no information is exchanged among the three parties in each trail, leading to the LOSR paradigm [36,37]. Our demonstration requires fair-sampling assumptions and admits the postselection loophole as well as the detection loopholes. To analyze Eq. (3), we exclusively consider postselection of the cases where the detectors click results in an unbiased sample under fair-sampling assumptions, which usually relates to detection loopholes [45] that may be closed in the future with high-efficiency photon sources [46] and detectors.

Another important issue is the postselection for the entanglement generation process [47], as our $|\text{GHZ}_3\rangle$ state depends on postselecting a four-photon GHZ state, wherein one photon as a trigger colocated with Charlie's photon. With fair-sampling assumptions in each trial, we only consider performing postselection on each port such that every party receives exactly one photon, although there are multiphoton events present that could decrease the prepared state fidelity. In the presence of postselection, one could have selection bias that arises due to conditioning or restricting the data generated in the experiment [47], which might lead to correlations breaking Bell-like inequality without necessarily claiming the genuine LOSR tripartite nonlocality. For example, if we use three independent bipartite-entangled states shared by Alice, Bob, and Charlie, and allow them measuring local parity operators, only postselection on the interested events in the outcomes will lead to the statistics that show genuinely tripartite nonlocal features [48,49]. However, one could potentially close the postselection loophole at the sources by preparing states in a heralded event-ready manner such as using cascaded SPDC sources [19,50] or using on-demand single-photon sources with fusion gates [51] in the future. Beyond the tripartite scenarios, a future interesting direction is to explore genuinely LOSR multipartite nonlocality in more complex networks, albeit it is experimentally challenging.

We thank Chang Liu and Quantum Ctek for providing the components used in the quantum random number generators. This work was supported by the National Natural Science Foundation of China, the Chinese Academy of Sciences, the National Fundamental Research Program, and the Anhui Initiative in Quantum Information Technologies.

Note added.—Recently, we became aware of two similar optical tabletop experimental works without closing locality loopholes [49,52].

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