

Universality Class of Ising Critical States with Long-Range Losses

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We show that spatial resolved dissipation can act on d -dimensional spin systems in the Ising universality class by qualitatively modifying the nature of their critical points. We consider power-law decaying spin losses with a Lindbladian spectrum closing at small momenta as $\propto q^\alpha$, with α a positive tunable exponent directly related to the power-law decay of the spatial profile of losses at long distances, $1/r^{(\alpha+d)}$. This yields a class of soft modes asymptotically decoupled from dissipation at small momenta, which are responsible for the emergence of a critical scaling regime ascribable to the nonunitary counterpart of the universality class of long-range interacting Ising models. For $\alpha < 1$ we find a nonequilibrium critical point ruled by a dynamical field theory described by a Langevin model with coexisting inertial ($\sim \partial_t^2$) and frictional ($\sim \partial_t$) kinetic coefficients, and driven by a gapless Markovian noise with variance $\propto q^\alpha$ at small momenta. This effective field theory is beyond the Halperin-Hohenberg description of dynamical criticality, and its critical exponents differ from their unitary long-range counterparts. Our Letter lays out perspectives for a revision of universality in driven open systems by employing dark states tailored by programmable dissipation.

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Introduction.—The search for nonequilibrium criticality in driven open quantum systems has become an exciting research frontier, both for its fundamental relevance in statistical mechanics and for the variety of atomic molecular and optical physics platforms where it can be concretely explored [1–19]. The aim is the discovery of universality classes which cannot be encompassed by established classifications of dynamical criticality [20–23] nor can be related to out-of-equilibrium scaling in isolated systems [24–36], where, in sharp contrast, both energy and total number of particles are conserved. A common obstruction against the realization of this program is the occurrence of an effective thermal behavior for the soft modes relevant at the critical points of driven dissipative systems [37–42]: although the full momentum distribution of the nonequilibrium steady state manifestly breaks detailed balance, low momenta can thermalize at an effective temperature set by the interplay of drive, noise, and losses. This forces several instances of driven open criticality to fall into known equilibrium universality classes [20,43], with few exceptions represented by the appearance of novel independent anomalous exponents associated with decoherence [44–46], or by exotic features as nonequilibrium multicritical points [47].

The culprit for effective thermalization is a noise variance (dictated by dissipation) with a nonvanishing gap at small momenta and/or frequencies, which sets the temperature of infrared modes in several circumstances of interest [37–41,43,44,48]. Softening such gap and allowing the noise to scale down to zero at small momenta is the route for instances of driven open criticality without

thermal counterpart. In quadratic fermionic models [49,50] or interacting quantum wires [51–53], dissipation with nonlocal support in real space acting on neighboring sites in a correlated fashion [54] has been employed to achieve nonequilibrium quantum criticality. In these cases the noise variance vanishes at infrared momenta, and it exposes a set of modes asymptotically decoupled for $q \rightarrow 0$ from the decohering and thermalizing effect of the environment. These forms of nonlocal dissipation can steer a system into a many-particle dark state with nontrivial quantum correlations—a state preparation protocol with interesting perspectives for applications in quantum information and technology [55–66].

In this Letter, we consider spin losses with a controllable spatial profile decaying algebraically at long distances [67–70]. Their Lindbladian spectrum scales with momentum softly as $\propto q^\alpha$ in the infrared; the tunability of α allows us to explore a dissipative analog of the universality class of long-range interacting quantum magnets. Our results are based on renormalization group (RG) and therefore pertinent to a whole family of spin models distinct by RG irrelevant perturbations at the Ising critical point. Modern cavity QED quantum simulators [68,71–73] in the regime of strong cavity loss, have the potential to expose unconventional forms of dynamical criticality, since they can imprint on atomic ensemble decay channels with tunable spatial profiles [70]. This is in sharp contrast with previous contributions on driven open criticality where the structure of dissipation supporting dark modes is not flexible and given by the specific implementation at hand [52,55,57,59]. In particular, we discuss here the instance of critical spin

ensembles subject to long-range spatial emission, whose universal properties are ruled by a Langevin theory [20,46] where inertial ($\propto \partial_t^2$) and frictional ($\propto \partial_t$) kinetic coefficients coexist and with a gapless driving noise scaling proportionally to q^α in the infrared. Upon tuning α , one can control the degree of RG relevance of the operators necessary for a consistent description of these novel critical states, and interpolate among different universality classes.

Ising criticality with nonlocal losses.—We consider a quantum Ising chain in d dimensions

$$H = -\sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x + h \sum_i \sigma_i^z, \quad (1)$$

subject to a spin loss Lindblad channel which is nonlocal and shaped by a spatial structure factor $\gamma_{i,j}$:

$$\dot{\rho} = i[\rho, H] + \sum_{i,j} \gamma_{i,j} \left(\sigma_i^- \rho \sigma_j^+ - \frac{1}{2} \{ \sigma_j^+ \sigma_i^-, \rho \} \right). \quad (2)$$

The open quantum system in (1) and (2) is not exactly solvable and, even with state-of-art numerics, dynamics could be extracted only for small system sizes and intermediate times. Instead, here we rely on nonequilibrium RG to inspect the long-distance or long-time scaling properties of the system at criticality. In this regard, any RG irrelevant perturbation at the Ising critical point in (1) (e.g., short-range spin-spin interactions along the \hat{z} direction) will not affect our results, which are therefore pertinent to the whole set of spin models belonging to the Ising universality class. Correlated spin losses as in Eq. (2) can be realized in cavity QED [70] or photonic crystal waveguides [67,68], where tunable interactions and losses between pairs of spins at arbitrary distances can be controlled through a combination of spatial-dependent energy level shifts and external pump fields [70]. The former typically result from a background magnetic field gradient coupling to levels in a hyperfine manifold, and they give spatial resolution to dissipation, while a Raman drive with several sidebands enables control on the functional form of $\gamma_{i,j}$, which can be made translational invariant, $\gamma(r)$ with $r = |i - j|$.

In the following we analyze the impact of a class of $\gamma(r)$, which scales asymptotically as an inverse power law of r , on Ising critical systems [i.e., $\gamma(r) \propto 1/r^{\alpha+d}$ for large r , cf. with Fig. 1]. The details of the implementation of $\gamma(r)$ are contained in Refs. [68,70] (see also [74]). Diagonalizing Eq. (2) in Fourier space with this shape of $\gamma(r)$, we obtain for the decay rate of the infrared modes the expression $\Gamma_q \sim \Gamma_1 q^\alpha$. These infrared modes are dark, i.e., asymptotically decoupled from dissipative effects (Γ_q vanishes as $q \rightarrow 0$). Following the analogy with criticality in long-range interacting spin systems where instead the Hamiltonian spectral gap scales as $m \sim q^\alpha$ [75–77], we can interpret the instance of criticality

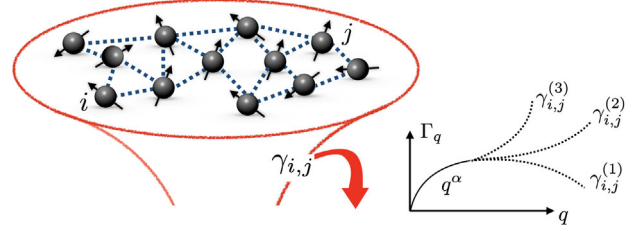


FIG. 1. Schematic portrait of a spin lattice subject to nonlocal losses, $\gamma_{i,j}$, acting on pairs of spins at positions i and j .

inspected in this Letter as a nonunitary counterpart of critical long-range interacting Ising systems. This analogy is further supported by the fact that the anticommutator in the Lindblad equation (2) can be regarded as a non-Hermitian long-range interaction term [70]. We notice that the shape of $\gamma(r)$ at short or intermediate distances is inconsequential for our results which are relevant for soft, infrared modes at criticality. Therefore, our analysis applies to a broad class of spatial profiles $\gamma(r)$, provided they entail the infrared scaling of the Lindbladian spectrum mentioned above, $\Gamma_q \sim \Gamma_1 q^\alpha$ (cf. with the inset in Fig. 1). In this respect, our setup also extends previous instances on preparation and criticality of open quantum systems with dark states in cold atoms or quantum optics platforms, where, in contrast, the shape of nonlocal losses is not tunable [49–52,57]. For the case discussed in this Letter, the exponent α can be flexibly varied by a proper choice of the amplitudes of the Raman sidebands (see note [73], or for more details Ref. [70]). Notice that the $\alpha = 0$ case will not display any interesting instance of dissipative criticality since it does not support dark states (Γ_q constant for $q \rightarrow 0$).

Canonical scaling with long-range losses.—We now map the lattice model in Eq. (2) into a long wavelength nonequilibrium field theory [43,78]. In particular, we will discuss how the effective field theory governing nonequilibrium critical behavior for $\alpha < 1$ is ascribable to a Langevin model [20,46] with coexisting inertial and frictional terms, driven by a gapless noise $\propto q^\alpha$ at small momenta. Following the usual prescription [79–81] we map the spin operators in terms of bosons $\sigma_i^- \rightarrow a_i$ and $\sigma_i^z \rightarrow 2a_i^\dagger a_i$, and we implement the hard-core constraint with a large on-site nonlinearity $a_i^\dagger a_i^\dagger a_i a_i$. By taking the continuum limit and coarse graining over short wavelengths [40,79–81], the Ising interaction in Eq. (1) yields a second derivative in space within a leading order derivative expansion (Kq^2 in momentum space), while the nonlinearity yields the usual φ^4 potential. As detailed in Refs. [40,43,82], the quantum master equation for an Ising model with losses (2) can be mapped into a Keldysh path integral in terms of the classical and quantum components of the real field, $\varphi_{c/q}(Q)$, which in Fourier space, $Q = (\mathbf{q}, \omega)$, reads

$$\mathcal{S}_G = \int_Q (\varphi_c(-Q), \varphi_q(-Q)) \begin{pmatrix} 0 & P^A \\ P^R & P^K \end{pmatrix} \begin{pmatrix} \varphi_c(Q) \\ \varphi_q(Q) \end{pmatrix} \quad (3)$$

with

$$\begin{aligned} P^{R/A} &= -\omega^2 - 2Kq^2 \mp 2i\Gamma_q\omega + m - \Gamma_q^2/2, \\ P^K &= i\Gamma_q \equiv i(\Gamma_0 + \Gamma_1q^\alpha), \end{aligned} \quad (4)$$

the retarded or advanced and Keldysh inverse Green's functions [78]. The former contain spectral information on the excitations: in our case, the distance from the critical point, m , and the decay rate of the infrared modes, Γ_q . The Keldysh component of the quadratic action P^K is instead directly related to the momentum Fourier transform of the noise variance [43].

In Γ_q we have included a constant term Γ_0 which takes into account the spontaneous emission of the spins into free space and which is often the main adversary in schemes implementing dissipative engineering with dark states [83,84]. Its detrimental role is to locally measure the atoms and therefore suppress the entangling effect of nonlocal dissipation at long times (or small wave vectors). We will inspect the critical properties of our model in the regime where $\Gamma_1q^\alpha \gg \Gamma_0$ and therefore P^K scales effectively as q^α , and, at the same time, also the retarded or advanced sectors become gapless, $P^{R/A} \sim q^{2\alpha}$, since we tune the spectral mass to zero as well. The former can be implemented in the RG scheme via the scaling ansatz $\Gamma_0 \sim \Gamma_1q^\alpha$ (see also Ref. [85]). As pointed out in [86–88] weak dissipation can expose novel critical behavior for a long temporal window before thermalizing effects set in. In our setup, this is mirrored by the fact that for $q \ll (\Gamma_0/\Gamma_1)^{1/\alpha}$ incoherent emission takes over, and the Gaussian action in (3) reduces to a Langevin action with $\Gamma_q \simeq \Gamma_0$ and no ω^2 term which notoriously thermalizes [20,89]. This crossover is analogous to the suppression of equilibrium quantum criticality at distances larger than the de Broglie thermal length [21–23].

Approaching criticality ($m \rightarrow 0$) and for $\Gamma_0 \rightarrow 0$, we can adopt the following canonical scaling ansatz [46,90] for the dynamical critical exponent z controlling the relative scaling of frequency and momentum [89]: $\omega \sim q^z$, with $z = \alpha$. This results in the terms $\propto \omega\Gamma_q$ and $\propto \omega^2$ both equally scaling like $\sim q^{2\alpha}$ in the infrared. This is in contrast to relaxational Langevin models, where the inertial term proportional to the second derivative in time ($\sim \omega^2$) is subleading compared to the frictional first order time derivative ($\sim \omega$). Therefore, we recover a scalar dynamical field theory with coexisting inertial and frictional kinetic coefficients, driven by a gapless Markovian noise, which is a model beyond the Halperin-Hohenberg classification [20]. An effective field theory resembling some of these features has recently appeared in [86,88]. As a nontrivial extension here we encompass a family of RG fixed points

upon tuning the exponent α of the soft Langevin noise. This results in corrections not only to dynamical critical exponents as in [86,88] but also to static ones, as we discuss in the following.

We now focus on kinetic coefficients proportional to spatial derivatives. At the level of canonical power counting, there is a threshold value $\alpha < 1$ at which the second derivative in space ($\propto Kq^2$ term) is subleading in the infrared compared to the $\Gamma_1^2q^{2\alpha}$ fractional derivative resulting from “long-range” losses in the spectral sector ($P^{R/A}$). This should be contrasted with critical long-range interacting Ising models where such threshold is set at $\alpha = 2$ [75–77,91] besides small corrections resulting from anomalous dimensions [92–98]. These different thresholds occur because Hermitian long-range interactions compete with Kq^2 through a q^α term in the R/A sector (see Refs. [75–77,91]), while non-Hermitian ones through $q^{2\alpha}$ terms resulting from $\omega\Gamma_q$ and Γ_q^2 [cf. with Eq. (4)]. We notice that the RG procedure generates only analytical terms and thus cannot renormalize the terms scaling with the exponent α (see also Refs. [75–77]). The only term which can acquire an anomalous dimension is the kinetic coefficient of the inertial term ($\sim \omega^2$), as we will further discuss below. This makes unviable a fine compensation of the anomalous dimensions of the retarded and Keldysh sectors, which would signal, whenever occurring, effective infrared thermalization [44,85]. Therefore, the RG fixed point discussed in the following explicitly breaks fluctuation-dissipation relations and cannot have an equilibrium counterpart, distinctly from other instances of nonequilibrium open criticality [37–42]. We now study the critical regime $m \rightarrow 0$ for $\alpha < 1$.

RG fixed point and criticality for $\alpha < 1$.—We will now complement the Gaussian action in Eq. (3) with nonlinear terms following the canonical power counting just discussed. For $\alpha < 1$, we have $P^{R/A} \sim q^{2\alpha}$ and $P^K \sim q^\alpha$, with canonical scaling dimensions for the classical and quantum fields $\varphi_c \sim q^{d/2-\alpha}$, $\varphi_q \sim q^{d/2}$, and accordingly a lower critical dimension of $d_l = 2\alpha$. Below the upper critical dimension $d_u = 4\alpha$ the classical nonlinear term $(u_c/4!)\varphi_c^3\varphi_q$ is relevant. The next RG leading nonlinearity appears at $d < 3\alpha$ where the additive noise term $i(\kappa/2)\varphi_c^2\varphi_q^2$ and the sextic term $(\lambda/5!)\varphi_c^5\varphi_q$ are both RG relevant. Quantum vertices with higher powers of quantum fields are always irrelevant hinting at the semiclassical nature of the fixed point, and marking a difference with previous studies on quantum criticality induced by dark states [49–52]. Notice that similarly to the long-range interacting model A of the Halperin-Hohenberg classification [99] we have a dynamical critical exponent $z = \alpha$, but different lower and upper critical dimensions due to the gapless nature of the noise, suggesting that the scaling regime studied here belongs to a different universality class. Similarly there are neat differences with the canonical

power counting of the zero-temperature critical long-range Ising model, where $z = \alpha/2$.

In order to find the interacting fixed point, we run a one-loop resummed RG on the effective potential including relevant nonlinearities [43,80,100]. Technical details are reported in the Supplemental Material [101]. We employ a sharp cutoff in momentum space $k < q < \Lambda$ where k is the running RG scale and Λ an uv regulator. In the following we parametrize the flow of the couplings in terms of the RG time $t = \ln k$. We first consider a leading order ϵ expansion, right below the upper critical dimension $\epsilon \equiv d_u - d \ll 1$ (where $d_u = 4\alpha$). We follow the canonical rescaling discussed above, $\tilde{m} \sim m/k^{2\alpha}$, $\tilde{\Gamma}_0 \sim \Gamma_0/(\Gamma_1 k^\alpha)$, and $\tilde{u}_c \sim u_c/k^{4\alpha-d}$, and we find from the following rescaled beta functions:

$$\begin{aligned} \partial_t \tilde{m} &= -2\alpha \tilde{m} + \frac{\tilde{u}_c[-2\tilde{m} + (1 + \tilde{\Gamma}_0)^2]}{4(1 + \tilde{\Gamma}_0)^6}, \\ \partial_t \tilde{u}_c &= -\epsilon \tilde{u}_c - \frac{3\tilde{u}_c^2}{2(1 + \tilde{\Gamma}_0)^6}, \quad \partial_t \tilde{\Gamma}_0 = -\alpha \tilde{\Gamma}_0, \end{aligned} \quad (5)$$

a Wilson-Fisher (WF) fixed point at $(\tilde{m}^*, \tilde{\Gamma}_0^*, \tilde{u}_c^*) = [-\epsilon/(12\alpha), 0, -2\epsilon/3]$, with a correlation length critical exponent $\xi \sim m^{-\nu}$, $\nu = 1/(2\alpha) + \epsilon/(12\alpha^2)$. This fixed point has an additional unstable RG direction corresponding to perturbations around the fixed point value $\tilde{\Gamma}_0^* = 0$, in agreement with the requirement to fine-tune the Lindbladian gap ($\Gamma_0 \rightarrow 0$) in addition to the closing of the spectral one ($m \rightarrow 0$). This is in full analogy with the RG relevance of temperature at equilibrium quantum critical points, which is as well responsible for the onset of an additional RG unstable direction [21–23]. At $\mathcal{O}(\epsilon^2)$ we find $z \simeq \alpha + \epsilon^2/[24(1 + 4\alpha^2)]$ following similar calculations performed for critical Langevin models [46,77].

In Eqs. (5) the flow of $\tilde{\Gamma}_0$ is solely governed by its canonical dimension. To find a nontrivial WF fixed point for Γ_0 , we need the multiplicative noise $i(\kappa/2)\varphi_c^2\varphi_c^2$ to be RG relevant. As discussed above, this occurs for $\alpha > d/3$, giving to the Gaussian noise sector, P^K , a one-loop dressing proportional to $\sim \kappa \int_Q G^K(Q)$. For consistency with RG relevance we have also to include the sextic vertex $\propto \lambda$ (see Refs. [101] for details). By evaluating the one-loop resummed RG flow at $d = 2$ and $\alpha = 0.7$, we find a

WF fixed point $(\tilde{m}^*, \tilde{\Gamma}_0^*, \tilde{u}_c^*, \tilde{\kappa}^*, \tilde{\lambda}^*) = (0.04, 0.23, 2.53, -1.98, -0.93)$ with still two unstable directions; the one associated to the spectral mass yields $\nu \simeq 0.71$. Loop corrections to Γ_0 in the vicinity of this fixed point renormalize the condition $\Gamma_0(k) \simeq \Gamma_1 k^\alpha$ for suppression of the dark mode from incoherent spontaneous emission. Following a calculation contained in Ref. [85] (summarized also in [101]), we find that at distances larger than the inverse of $k^* \simeq 10^{-6}\Lambda_G$, the novel scaling is superseded by a conventional noncritical thermal Ising theory (as also mentioned above). Here Λ_G is the so-called Ginzburg scale [80]: at distances larger than Λ_G^{-1} , correlation functions scale universally with the critical exponents of the WF fixed point. For distances smaller than Λ_G^{-1} correlation functions are instead dominated by nonuniversal corrections (lattice effects, RG irrelevant spin interactions, etc.). From the side of dynamics, upon initializing the spin model (1)–(2) sufficiently away from the eventual steady state, it will enter, after a transient ($t \lesssim \Lambda_G^{-1}$), into a self-similar scaling regime where spatial- and time-resolved spin correlations are governed by the critical exponents of the WF fixed point. Such dynamical scaling regime persists until spontaneous emission will “heat” the dark modes at times larger than the inverse of k^* ; at these times, the critical long-wavelength theory will crossover into a conventional Langevin theory.

Fixed point for $\alpha > 1$.—By inspection of Eqs. (4) we notice that for $\alpha > 1$ the kinetic coefficient $\sim Kq^2$ in the advanced or retarded sector dominates over the $\propto \Gamma_1^2 q^{2\alpha}$ term resulting from nonlocal dissipation. This leads to a dynamical critical exponent $z = 1$, with the term $\propto \omega\Gamma_q$ now negligible in the infrared; in other words, we have an Ising model with short-range interactions and a $\propto \Gamma_1 q^\alpha$ Markovian noise. This changes the critical properties of the theory as summarized in Table I. At this WF fixed point quantum terms such as the quartic $u_q \varphi_q^3 \varphi_c$ are irrelevant, unless $\alpha > 2$. However, as α increases the spatial support of losses quickly shrinks [70], retrieving uninteresting local dissipation effects similar to Γ_0 .

Competing long-range interactions and losses.—Finally, we consider the scenario where long-range Ising interactions, $\sum_{\langle i,j \rangle} (J/|i-j|^{1+\alpha}) \sigma_i^x \sigma_j^x$, compete with long-range losses. Such term adds a Jq^α contribution to $P^{R/A}$ [75–77,91]. By inspection of Eqs. (4), we realize that

TABLE I. Nonequilibrium criticality with long-range (LR) losses. The third column displays the lower (d_l) and upper (d_u) critical dimensions, while the fourth one summarizes the effective dynamical field theory valid at criticality. Below the lower critical dimension separate scaling and RG analyses are required, and an Ising-type field theory description does not apply. The thresholds d_l and d_u implicitly bound the values of α compatible with the universality class discussed in this Letter.

	Critical exponent	$d_l < d < d_u$	Effective field theory
LR losses with $\alpha < 1$	$\nu = 1/(2\alpha) + \epsilon/(12\alpha^2)$	$2\alpha < d < 4\alpha$	Coexisting inertial/frictional derivatives + soft noise
LR losses with $\alpha > 1$	$\nu = 1/2 + \epsilon/12$	$3 - \alpha < d < 5 - \alpha$	Short-range Ising model + soft noise
LR losses and interactions ($\alpha < 2$)	$\nu = 1/\alpha + \epsilon/(3\alpha^2)$	$\alpha/2 < d < 3\alpha/2$	Long-range Ising model + soft noise

for $\alpha > 2$ we recover the same scaling discussed above for long-range losses with $\alpha > 1$. For $\alpha < 2$, instead, we find a leading scaling $P^{R/A} \sim Jq^\alpha$, since long-range interactions suppress at small momenta the contribution of nonlocal losses in the spectral sector ($z = \alpha/2$). This is equivalent to the critical scaling of a long-range interacting Ising model driven by a $\propto \Gamma_1 q^\alpha$ Markovian noise, and it is a limit where classical and quantum vertices scale alike, $u_{c,q} \sim q^{3\alpha/2-d}$. Such quantum scaling regime is formally equivalent to a critical zero-temperature long-range interacting Ising model [75,76]. The associated critical dimensions and exponents are summarized in Table I (they do not hold for $d = 3$).

Perspectives.—A recent cavity QED experiment [73] demonstrates the tunability of nonlocal spin couplings, suggesting that the exploration of programmable nonunitary interactions [70] in critical spin systems may belong to near-term implementations. An interesting follow-up research direction would consist in focusing on richer driven open platforms, where incoherent losses or pumps and dephasing channels with nonlocal spatial character can compete. For instance, revising driven open condensates with an $O(2)$ order parameter [44,45,52,102,103] appears a natural perspective. In the same spirit, the effective field theory derived in this Letter can be considered as a natural starting point for an extension to models with different symmetries or equipped with global conservation laws, in the pursuit of an Halperin-Hoening [20] classification of critical theories with tunable dark states. It also appears important to access quantitatively the value of the critical exponents (and the radius of convergence of the ϵ expansion) using methods, like functional RG, which are technically suited to perform loop resummations in models with long-range interactions [95]. However, the ϵ expansion of our Letter is expected to describe critical properties at least qualitatively, as it also occurs in Hermitian long-range Ising models [76,77,95], or as it would be for the large- N version [104] of the field theory (3). In all these respects, our findings can be regarded as a seed for technically richer explorations.

Finally, we believe it would be extremely interesting to study the effect of long-range losses on the paramagnet and ferromagnet separated by the critical point. This appears, however, as a technically challenging task since it requires to solve the nondiagonal Liouvillian in (2) beyond semi-classical limits where its many-body dynamics have been efficiently simulated so far [105]. Quantum kinetic equations based on Majorana fermions representation of spins [106] could represent a possible avenue to find correlations in this case.

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