$\Delta I = 2$ Bifurcation as a Characteristic Feature of Scissors Rotational Bands

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We report microscopic many-body calculations indicating that rotational bands based on nuclear scissors vibrations exhibit systematic splitting between neighboring spin states ($\Delta I = 2$ bifurcation) in which the magnitude of the moment of inertia oscillates between states having even and odd spins. We show that this unexpected result is caused by self-organization of the deformed proton and neutron bodies in the scissors motion, which is further amplified by the $K^\pi = 1^+$ two-quasiparticle configurations near the scissors states. We propose that the puzzling excited state found above the 1^+ scissors state in ^{156}Gd [Phys. Rev. Lett. **118**, 212502 (2017)] is the first evidence of this effect, and predict that bifurcation may generally appear in all other scissors rotational bands of deformed nuclei, and possibly in other systems exhibiting collective scissors vibrations.

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In quantum many-body systems collective motion often appears as the spontaneous emergence of ordered motion for a large number of self-organized identical particles [1]. The scissors mode in deformed atomic nuclei depicts the collective vibration of the proton and neutron systems with respect to each other. This vibration, first predicted in the 1970s [2], may be classified, together with the well-known β and γ vibrations in nuclei [3], as one of the lowest collective excitations of the ground state [4]. Since the pioneering experiment for ^{156}Gd in 1984 [5], the $I^\pi = 1^+$ scissors-mode state has been investigated experimentally [6]. This research has also triggered considerable theoretical interest in nuclear physics [7–18]. Extending the idea that the superfluidity of neutron and proton "clouds" oscillate in deformed nuclei, similar collective excitations were first predicted [19] and then observed [20] in Bose-Einstein condensation. The scissors-mode state was also predicted in other many-body systems including metallic clusters [21], quantum dots [22], superfluid Fermi gases [23], and anisotropic crystals [24].

Measurement of the magnetic dipole ($M1$) response for nuclei with even numbers of neutrons and protons allowed some general features of the scissors-mode vibration to be obtained from light to very heavy mass regions. For the well-studied rare earth region, for example, the excitation energies of the scissors-mode states in the $N = 82$–126 major-shell nuclei are found to be remarkably constant (~3 MeV) [25]. Moreover, the total $B(M1)$ strength of the scissors mode is found to be proportional to the square of the deformation parameter [26], which is an important indicator of collective motion in nuclei.

Surprisingly, almost nothing is known about *rotational* behavior of the scissors-mode states (except the puzzling result reported by the Darmstadt group [27] that we shall discuss below). In a deformed nucleus executing scissors-like motion, the neutron and proton distributions in the deformed nucleus are thought to conduct small-angle vibrations with respect to each other *while they are rotating*. Collective rotation coupled with the scissors vibration is conceptually complicated. It is expected that such a system would break all known symmetries with regard to rotation, even though the potentials for neutrons and protons are considered to be separately axial symmetric.

In a microscopic description of the scissors mode, two of us (Y. S. and M. G.) and collaborators, made a prediction for rotational bands built on multiphonon scissors modes in deformed nuclei [10]. The theoretical method for that study was an extension of the projected shell model (PSM) [28]. In order to describe the relative motion between deformed neutron and proton fields in a microscopic way, instead of a single BCS vacuum the angular momentum projection was performed for separate neutron and proton deformed BCS vacua. The (angular-momentum) projected neutron and proton states were then coupled through the diagonalization of a pairing plus quadrupole interaction in this basis. It was shown that the procedure gives the usual ground-state rotational band corresponding to a strongly coupled BCS condensate of neutrons and protons, but also leads to a set of excited states arising from a more complex vacuum that incorporates motions in the relative orientation of the neutron and proton fields (For the theoretical framework of Ref. [10] as well as its further improvement, see Ref. [29]).

Specifically, a *rotational band* on top of the $I^\pi = 1^+$ state at about 3 MeV was predicted [10] and further discussed [12], but had not been observed until the recent experiment by the Darmstadt group. In Ref. [27], Beck *et al.* reported their discovery of the 2^+ state above the 1^+ scissors band head in ^{156}Gd. While this is the first candidate observation of an excited state of the scissors mode state, its excitation energy is a puzzle. The measured energy of the new 2^+ state lies only 19 keV above the known 1^+ scissors state. If indeed the 2^+ state seen at 3.089 MeV excitation is the first rotational excitation of the 1^+ scissors state at 3.070 MeV in ^{156}Gd, two possibilities have to be considered. The scissors mode either has a rotational moment of inertia that is considerably larger than the rigid-body value anticipated for ^{156}Gd, or the scissors rotational band must exhibit a significant energy splitting, assuming the average moment of inertia of the scissors-mode band is similar to the one of the ground band. An attempt to reproduce the experimental data by using large decoupling-parameter values under various assumptions failed [35].

We show in this Letter that the unusually low excitation energy of the 2^+ state in ^{156}Gd [27] is a consequence of energy-level staggering in the scissors rotational band. We further point out that this may be a new type of staggering that is collective in nature, appearing specifically for systems executing scissorslike motion. Our discussion leads to a speculation that *all* scissors rotational bands in deformed nuclei should exhibit a $\Delta I = 2$ bifurcation in band energies and electromagnetic transition strengths. Thus we propose that bifurcation is a characteristic feature of a scissors-mode rotational band.

Our theoretical discussion is based on an extended version of the projected shell model [28] (for details of the model extension, see [29]), which employs a well-established Hamiltonian for nuclear structure studies in the form of a pairing plus quadrupole force [36,37] with inclusion of a quadrupole-pairing force [38]. It is written in the isospin formalism with three parts: $\hat{H} = \hat{H}_\nu + \hat{H}_\pi + \hat{H}_{\nu\pi}$, where H_τ ($\tau = \nu, \pi$) is

$$\hat{H}_\tau = \hat{H}_\tau^0 - \frac{\chi_{\tau\tau}}{2}\sum_\mu \hat{Q}_\tau^{\dagger\mu}\hat{Q}_\tau^\mu - G_M^\tau \hat{P}_\tau^\dagger \hat{P}_\tau - G_Q^\tau \sum_\mu \hat{P}_\tau^{\dagger\mu}\hat{P}_\tau^\mu, \quad (1)$$

denoting the neutron and proton Hamiltonian. The neutron-proton interaction $H_{\nu\pi}$ is of quadrupole-quadrupole form

$$\hat{H}_{\nu\pi} = -\chi_{\nu\pi}\sum_\mu \hat{Q}_\nu^{\dagger\mu}\hat{Q}_\pi^\mu. \quad (2)$$

It is important to note that we place no constraint on the relative orientation of the neutron and proton bodies and their rotational direction. The effective moment of inertia of the whole system is not fixed in our model, but is determined by the self-organization implicit in solving the eigenvalue equation.

The quasiparticle vacua are defined as $|0\rangle = |0_\nu\rangle|0_\pi\rangle$, which are obtained by the BCS calculations for deformed Nilsson single-particle states separately for neutrons and protons with an appropriate deformation. The basis for the diagonalization is obtained by angular momentum projection [28] onto the vacuum:

$$|I\rangle = \mathcal{N}^I \hat{P}^I|0\rangle \equiv \mathcal{N}^I[\hat{P}^{I_\nu}|0_\nu\rangle \otimes \hat{P}^{I_\pi}|0_\pi\rangle]^I \equiv \mathcal{N}^I[I_\nu \otimes I_\pi]^I, \quad (3)$$

where $\hat{P}^I$ is the angular momentum projection operator [39] and $\mathcal{N}^I$ the normalization constant. For details of the angular-momentum coupling calculation, see [29].

Diagonalization of the Hamiltonian in the basis (3) leads naturally to a strikingly regular pattern for rotational bands built on top of multiphonon scissors vibrations with weak anharmonicity (see Fig. 1 of Ref. [10]). The pattern can be understood as the manifestation of a nearly perfect SU(3) symmetry: all bands can be well reproduced by an SU(3) fermion dynamical symmetry model [40] if the projected neutron and proton BCS vacuum states are considered to be two SU(3) representations coupled through the $Q_n - Q_p$ interaction. The emergence of the low-energy $K = 1$ band from the calculation has an obvious correspondence with the scissors rotational-vibrational mode of a coupled-rotor model [2].

^{156}Gd is the first nucleus where the 1^+ scissors mode was confirmed experimentally [5], and also is the first nucleus where the 2^+ member of the 1^+ scissors band [27] has been reported. In Fig. 1, we show two rotational bands from the calculation: the ground-state band (labeled as the 0^+_{gs} band), and the one at ~3 MeV above it, the 1^+ scissors rotational

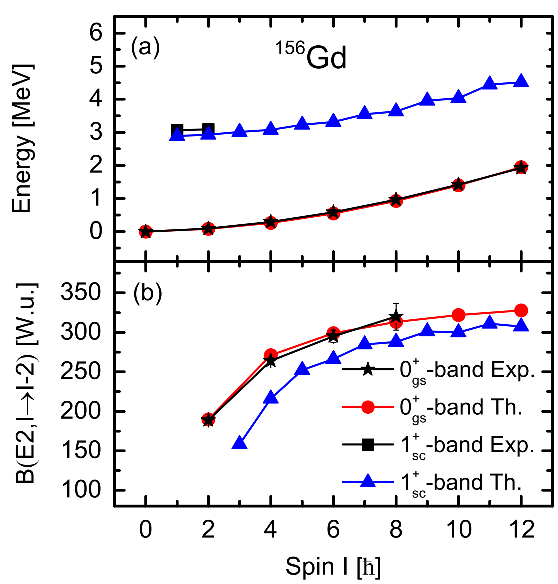

FIG. 1. Calculated (a) energies and (b) $B(E2, I \to I-2)$ values for the ground-state band (0^+_{gs} band) and scissors-mode rotational band (1^+_{sc} band) in ^{156}Gd. Available experimental data [41] are shown for comparison. In the $B(E2)$ calculations here and later in the paper, standard effective charges $e_\pi = 1.5$ and $e_\nu = 0.5$ are used.

band (labeled as the 1^+_{sc} band). It can be seen that for the 0^+_{gs} band, both the calculated energies and $B(E2)$ values compare very well with the known experimental data [41]. In Fig. 1(a), the 1^+_{sc} band exhibits an overall curvature similar to the 0^+_{gs} band, suggesting that it has a comparable moment of inertia as the 0^+_{gs} band. In Fig. 1(b), the calculated $B(E2)$ values for the 1^+_{sc} band have comparable values as for the 0^+_{gs} band, indicating that the scissors rotational band has a similar but slightly weaker $E2$ collectivity.

In Fig. 1(a), our calculation compares well with the experimental energies of the first two states with $I = 1$ and 2 in the 1^+_{sc} band [27]. The remaining states in the band are our predictions. It is striking that the calculated 1^+_{sc} band is not a smooth rotational band, but exhibits zigzags between the odd-spin and even-spin members. The calculated $B(E2, I \to I-2)$ values for the 1^+_{sc} band also show staggerings [see Fig. 1(b)]. To amplify the zigzag behavior, we replot in Fig. 2(a) the band energies in the form of energy differences $\Delta E(I) = E(I) - E(I-1)$. The zigzag phase, with the only-known first data point $\Delta E(2)$ being reproduced qualitatively [27], is such that all $\Delta E(I)$s for $I =$ even are lower in energy. The zigzag curve in Fig. 2(a) follows the same staggering phase as the $B(E2, I \to I-2)$ curve in Fig. 1(b). The calculated intraband $B(M1, I \to I-1)$ values shown in Fig. 2(b) are very small in magnitude, but also exhibit clear zigzags having the staggering phase opposite to that of $\Delta E(I)$ in Fig. 2(a).

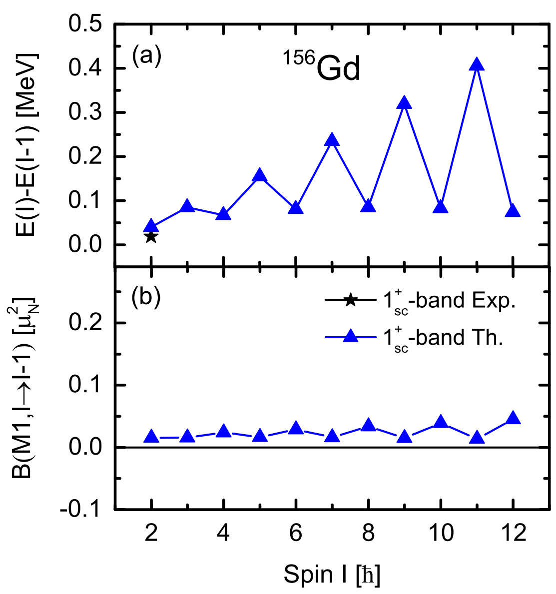

FIG. 2. Staggering features predicted for the scissors-mode rotational band (1^+_{sc}-band) in ^{156}Gd. (a) Energy differences $\Delta E = E(I) - E(I-1)$ and (b) intraband $B(M1, I \to I-1)$ values. In the $B(M1)$ calculations here and in Fig. 4, we use standard free-nucleon values for g_l and g_s, with g_s damped by a usual 0.75 factor.

For an axially symmetric nucleus such as ^{156}Gd, the usual rotational picture is that the neutron and proton ellipsoids take a *common* symmetry axis and the system rotates around the axis perpendicular to the symmetry axis (called principal-axis rotation in the literature). There exists a D_2 symmetry with respect to a 180° rotation around the principal axis [3]. It is difficult to imagine such a D_2 symmetry in cases of rotations with the scissors motion. One may then think of a more general picture of tilted rotation (nonprincipal axis rotation) discussed in deformed nuclei, which implies unavoidably an explicit breaking of the D_2 symmetry [42]. However, the nonprincipal axis rotation usually leads to enhanced intraband M1 transitions [43], in contrast to our results in Fig. 2(b) with much suppressed intraband M1 strength. Palumbo [44], using a two-rotor model, obtained as a general feature that the intraband magnetic transition amplitudes in the scissors-mode rational band vanish, qualitatively consistent with our $B(M1)$ results in Fig. 2(b).

Moments of inertia (MOI) in a bifurcated scissors rotational band can be discussed with the usual definition for MOIs

$$\mathcal{J}(I) = \frac{2I-1}{E(I) - E(I-2)}. \tag{4}$$

We display the calculated $\mathcal{J}(I)$ for the 1^+_{sc} band of ^{156}Gd in Fig. 3, separately for two $\Delta I = 2$ branches with $I =$ even or odd. One sees that on average, $\mathcal{J}(I)_{\mathrm{even}}$ is about 10% larger than $\mathcal{J}(I)_{\mathrm{odd}}$, suggesting that in a scissors rotational motion the $\mathcal{J}(I)$ alternates in magnitude between odd- and even-spin states. For a rotational body with conserved angular momentum, the condition $I = \mathcal{J}\omega$ requires that either ω changes discretely with the changing $\mathcal{J}$, meaning that the rotation is slower (faster) for even (odd) spin states as compared to smooth rotations when a fixed rotational axis is assumed, or, the rotational axis must periodically *change its orientation* for different spins in response to angular momentum conservation. The latter possibility suggests a very unusual rotational picture for isolated many-body systems in scissor motion [45].

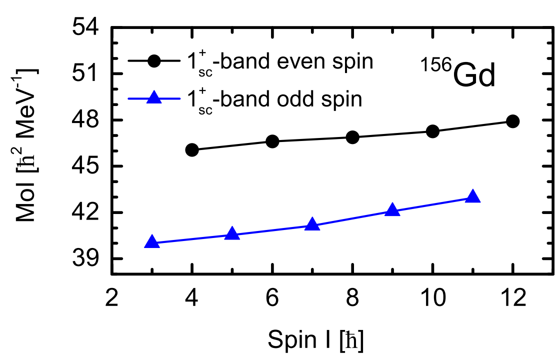

FIG. 3. Comparison of moments of inertia for $I =$ even and odd branches in the 1^+_{sc} band in ^{156}Gd.

The above-discussed scissors states are of collective orbital motion in nature. In the vicinity of the scissors-state energy (∼3 MeV), there exist many excited quasiparticle (qp) states characterized by individual orbitals. The situation is quite different from γ bands in ^{156}Gd [46] and in many other nuclei [47–49], which lie at much lower energy but may also show energy staggering. How do these qp states affect the bifurcation feature found in the scissors band? Will they enhance, destroy, or have little effect on the collective scissors motion? To answer this question, we enlarge the model space by adding 2-qp states in the calculation (see [29]). The quasiparticles are associated separately with the deformed neutron or proton potential, and therefore we can label them by using the quantum numbers K_ν or K_π (K is the projection of the single-particle angular momentum on the symmetry axis). Among the 2-qp $K = 1$ states, those of the intruder orbits with the configurations $(\nu i_{13/2})^2$ and $(\pi h_{11/2})^2$ are included.

Comparing to Fig. 2(a), the coupling of 2-qp states in Fig. 4(a) amplifies the staggering and pushes the $2^+_{\rm sc}$ state down to the exact experimental value [27]. In order to keep the same description for the gs-band properties, here we increase the proton pairing strength in (1) by a 1.2 factor to compensate the weakened pairing by inclusion of quasiparticles. We find that among the 2-qp states, the $K = 1$ proton 2-qp state coupled from $K = 5/2$ and $7/2$ of $\pi h_{11/2}$ contributes clearly to the enhanced scissors staggering. The reason is that the 2-qp states with two quasiparticles from

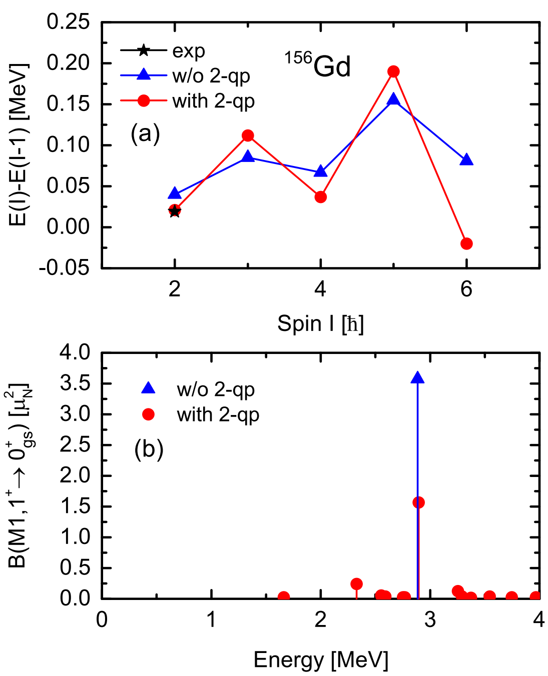

FIG. 4. Effect of quasiparticle configurations. (a) Comparison of calculated staggering in the scissors rotational band with and without two-quasiparticle states, and (b) Comparison of calculated B(M1, $1^+ \rightarrow 0^+_{\rm gs}$) distribution with and without two-quasiparticle states.

the same j-orbit must follow the general staggering rule [28] so that they stagger *in phase* with the scissors band (see explanation in Sec. II of [29]). Figure 4(b) shows B(M1, $1^+ \rightarrow 0^+_{\rm gs}$) values. The unperturbed B(M1) is found to be $\sim 3.5\ \mu_N^2$, in contrast to the much suppressed ones from the QRPA calculation [50]. Mixture of the 2-qp states significantly *reduces* it to $\sim 1.5\ \mu_N^2$, causing at the same time a *fragmented* distribution around the scissors M1. Our calculated B(E2, $1^+_{\rm sc} \rightarrow 2^+_{\rm gs}$) is however larger by more than 2 orders of magnitude than the experimentally-suggested small value [0.037(26) W.u. in [27]]. A theoretical value (a few W.u.) for the inter-B(E2) is comparable to that of a γ-vibrational state in deformed nuclei, which is usually expected for a transition from an excited collective state to the ground state. A table of comparison for measured [27] and calculated transition strengths of the $1^+_{\rm sc}$ scissors state to the ground band in ^{156}Gd can be found in Ref. [29].

In conclusion, we have discovered a characteristic feature of the scissors mode that has not been noticed before: A scissors-mode rotational band is never smooth, but staggers between odd and even spin states in even–even nuclei, resulting in a $\Delta I = 2$ bifurcation within the band. We have discussed that during the scissors-mode rotation the states showing such staggerings change their moments of inertia back and forth between odd and even spins, in response to angular momentum conservation. We consider the previously unexplained small separation for the excited 2^+ state above the 1^+ scissors-mode state in ^{156}Gd [27] as the first experimental evidence of such staggerings.

We have calculated systematically the even–even isotopes of ^{156}Gd with $N = 90$–102 for fixed $Z = 64$ and isotones with $Z = 60$–70 for fixed $N = 92$, and found that for all cases the staggering pattern is qualitatively similar to that shown in Fig. 2(a) (See Fig. 2 of Ref. [29]). In particular, they all have a lower-than-usual 2^+ above the $1^+_{\rm sc}$ state. Note that these nuclei represent the mass region in which the Fermi levels lie across from the low-K to high-K $i_{13/2}$ ($h_{11/2}$) orbitals for neutrons (protons), indicating that the $\Delta I = 2$ bifurcation in the scissors rotational band may be a *general feature*. Addition of 2-qp configurations tends to *enhance* the staggering, making the bifurcation even more pronounced. Furthermore, mixture of qp configurations with the pure scissors state *weakens* the B(M1, $1^+_{\rm sc} \rightarrow 0^+_{\rm gs}$) strength, causing at the same time fragmentation of M1 distribution. We are aware that coupling of qp states to collective motion is a complicated dynamical problem that can depend sensitively on details. In the present ^{156}Gd example qp states do not destroy, but enhance the scissors-band bifurcation.

We emphasize that the discussion in the present work does not require specific effective interactions to produce the bifurcation. The bifurcation seems to appear by self-arrangement of the system as long as an appropriate restoring force between the two "blades" of the scissors is present. If so, the feature discussed here may exist in similar scissors systems of other fields as well. Finally, we note that in order to confirm the bifurcation feature, it is crucial to identify experimentally the excited states with $I^{\pi} \geq 3^{+}$ in the scissors rotational bands.

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