Geometric Induction in Chiral Superfluids

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We explore the properties of chiral superfluid thin films coating a curved surface. Because of the vector nature of the order parameter, a geometric gauge field emerges and leads to a number of observable effects such as anomalous vortex-geometric interaction and curvature-induced mass and spin supercurrents. We apply our theory to several well-known phases of chiral superfluid ^{3}He and derive experimentally observable signatures. We further discuss the cases of flexible geometries where a soft surface can adapt itself to compensate for the strain from the chiral superfluid. The proposed interplay between geometry and chiral superfluid order provides a fascinating avenue to control and manipulate quantum states with strain.

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Geometric phases, rooted in the concept of parallel transport and related to topology, figure prominently in a startling variety of physical contexts, ranging from optics and hydrodynamics to quantum field theory and condensed matter physics [1]. In classical systems, for example, the geometric phase shift of the Foucault pendulum is equal to the enclosed solid angle subtended at Earth's center [2]. Other classical examples of geometric phases include the motion of deformable bodies [3] and tangent-plane order on a curved substrate [4,5]. In quantum mechanics, the geometric phases arise from slowly transporting an eigenstate round a circuit C by varying parameters $\mathbf{R}$ in its Hamiltonian $\hat{\mathbf{H}}(\mathbf{R})$ [6]. For example, the geometric phase of a single-electron Bloch wave function in the Brillouin zone is essential for topological states of matter such as the quantum Hall effect and topological insulators [7].

Beyond the single-electron picture, the concept of geometric phase has become a defining property of topological superconductors, where Cooper pairs can directly inherit their geometric phases from the two paired electrons [8]. Chiral superconductors, a particularly interesting class of topological superconductors [9], have received great attention due to their promise of hosting Majorana zero modes in vortex cores and at edges, which are central to several proposals for topological quantum computation [10,11].

In a chiral p-wave superconductor, the Cooper pairs carry orbital angular momentum (OAM) of $\hbar$, and the order parameter is a complex vector defined in the tangent plane of a two-dimensional (2D) surface $|\Psi\rangle = \psi(\hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2)/\sqrt{2}$ with $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ the local orthogonal basis and ψ the complex amplitude [11–13]. Here the $\pm$ sign denotes the chirality and the direction of the OAM. When such an order parameter with positive chirality evolves in a circuit on a curved 2D surface (Fig. 1 as an illustration), a geometric phase arises according to the formula $(1/\langle\Psi|\Psi\rangle)\oint_C \langle\Psi|i\partial_\mu|\Psi\rangle dl^\mu = \oint_C \omega_\mu dl^\mu$. Here $\omega_\mu = \hat{\mathbf{e}}_1 \cdot \partial_\mu \hat{\mathbf{e}}_2$ is the geometric connection whose curl is Gaussian curvature (see Sec. S-I in the Supplemental Material, where we present the mathematical foundation of geometric connection [14]). Generalization to a chiral ℓ-wave order parameter, describing a condensate of Cooper pairs with orbital angular momentum $\ell\hbar$, yields a geometric phase $\ell \oint_C \omega_\mu dl^\mu$ [14]. The geometric connection ω_μ may lead to a number of intriguing effects, such as the geo-Meissner effect [18] and the geometric Josephson effect [19], which serve as definitive signatures of chiral superconductivity.

In this Letter, we study the interplay between chiral superfluidity and geometry. We are motivated by the following observations: (i) Chiral superfluids are charge-neutral condensates. Therefore, the corresponding electromagnetic signature must be qualitatively different from that of superconductors. (ii) Unlike chiral superconductors, chiral superfluids are observed in nature (^{3}He-A phase) [20] and provide a test bed for our proposed geometric induction theory. (iii) The study of interactions between chiral-superfluid vortices and geometry, while

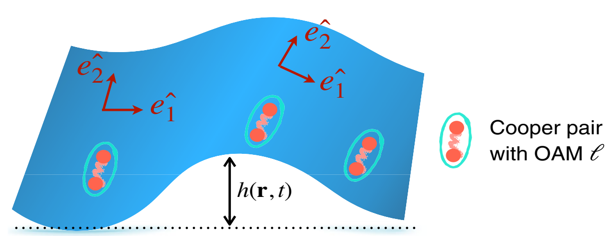

FIG. 1. Schematic illustration of transporting a vectorial order on a curved surface. The height $h(\mathbf{r}, t)$ measures the deviation of a curved surface from a plane.

experimentally feasible, is still lacking in the literature. (iv) Geometry may provide a practical knob to manipulate novel quantum states, such as the Majorana zero mode in a vortex. Thus it may offer a unique route to quantum manipulation including braiding—central to topological quantum computation [21,22].

The Letter is organized as follows: We first develop the necessary formalism for 2D chiral superfluids covering a curved surface. We then study the interaction between vortices and geometry, aiming at controlling quantum states with geometry. Next, we derive mass current and spin current induced by Gaussian curvature in several well-known phases of chiral superfluid ^{3}He, and we obtain the associated electromagnetic signatures. Finally, we study the quantum backaction of a chiral superfluid on a flexible surface.

Emergent geometric gauge fields.—The order parameter of a chiral ℓ-wave superfluid can be generically written as a rank-ℓ tensor, i.e.,

$$\Psi = \psi \underbrace{\epsilon_\pm \otimes \epsilon_\pm \ldots \otimes \epsilon_\pm}_{\ell \text{times}}, \tag{1}$$

where $\epsilon_\pm = (1/\sqrt{2})(\hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2)$ denote chiral basis, and $\psi = \sqrt{\rho}e^{i\theta}$ is the complex amplitude in terms of the superfluid density ρ and phase θ. $\ell = 1$ ($\ell = 2\ldots$) corresponds to the order parameter of chiral p-wave (d-wave…) superfluids. In this Letter, we consider the positive chirality. The negative chirality cases can be obtained by reversing the sign of ℓ in our formulas.

On a curved surface (substrate), the minimal Lagrangian of a chiral ℓ-wave superfluid reads

$$\mathcal{L}_{\rm sf} = i\hbar\psi^* D_t\psi - \frac{\hbar^2 g^{ij}}{2m}(D_i\psi)^*(D_j\psi) - V(|\psi|), \tag{2}$$

where g^{ij} is inverse the metric tensor g_{ij}, m is the mass of a Cooper pair, and $V(|\psi|)$ is a symmetry-breaking potential. Since the order parameter $\psi(t, \mathbf{r})$ depends on the choice of orthonormal basis $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, one needs to use the covariant derivatives D_μ ($\mu = 0, 1, 2$) defined by

$$D_\mu = \partial_\mu + i\ell\omega_\mu, \tag{3}$$

where $\omega_\mu = \hat{\mathbf{e}}_1 \cdot \partial_\mu \hat{\mathbf{e}}_2$ is the geometric connection originating from parallel transport of a vector on a curved surface [14]. The geometric connection ω_μ is a geometric gauge field akin to the electromagnetic vector potential, with the Gaussian curvature playing the role of a magnetic field. It was shown that a similar Lagrangian can induce Hall viscosity [23] and thermal Hall effect [24]. From the covariant derivatives, we can obtain the total field strength tensor $T_{\mu\nu} = i[D_\mu, D_\nu] = -\ell G_{\mu\nu}$, where $G_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ is the geometric field tensor, and correspondingly, we define the electric- and magneticlike field strength,

$$\mathcal{E}^i = \frac{1}{2}\frac{\epsilon^{i\mu\nu}}{\sqrt{g}}G_{\mu\nu}, \qquad \mathcal{B} = \frac{1}{2}\frac{\epsilon^{0ij}}{\sqrt{g}}G_{ij}, \tag{4}$$

with i, j taking values 1 or 2. We will discuss a number of effects that originate from the geometric gauge field.

Anomalous vortex-geometry interaction.—To discuss vortex physics, we rewrite Eq. (2) in terms of superfluid density ρ and phase θ, i.e., set $\Psi = \sqrt{\rho}e^{i\theta}$ to get

$$\mathcal{L}_{\rm sf} = i\hbar\rho(\partial_0\theta + \ell\omega_0) - \frac{\hbar^2\rho g^{ij}}{2m}(\partial_i\theta + \ell\omega_i)(\partial_j\theta + \ell\omega_j) - V(\rho), \tag{5}$$

where the potential $V(\rho) = A(\rho - \bar{\rho})^2$ guarantees that the superfluid acquires a finite average density $\bar{\rho}$. Upon integrating out the fluctuations of density, one obtains

$$\mathcal{L}_{\rm sf} = \frac{\gamma_0}{2}(\partial_0\theta + \ell\omega_0)^2 - \frac{\gamma_s}{2}(\nabla\theta + \ell\boldsymbol{\omega})^2, \tag{6}$$

where $\gamma_0 = \hbar^2/2A$ indicates fluctuation strength and $\gamma_s = \hbar^2\bar{\rho}/m$ denotes the superfluid stiffness. Upon rescaling temporal and spatial coordinates, we arrive at an effective Lagrangian density of the Lorentz-invariant form,

$$\mathcal{L}_{\rm eff} = \frac{\gamma}{2}(\partial_\mu\theta + \ell\omega_\mu)^2. \tag{7}$$

To discuss vortex interactions and dynamics, we introduce the alternative form

$$\mathcal{L}_{\rm eff} = -\frac{1}{2\gamma}\xi_\mu^2 + \xi^\mu(\partial_\mu\theta + \ell\omega_\mu), \tag{8}$$

which gives Eq. (7) after integrating out the auxiliary field ξ^μ. Without loss of generality, one can take the phase θ as a smoothly fluctuating field, except at vortices where it winds around 2π [25]. Therefore, one can write $\partial_\mu\theta = \partial_\mu\theta_{\rm smooth} + \partial_\mu\theta_{\rm vortex}$ and plug it into the Eq. (8), yielding

$$\mathcal{L}_{\rm eff} = -\frac{1}{2\gamma}\xi_\mu^2 + \xi^\mu(\partial_\mu\theta_{\rm smooth} + \partial_\mu\theta_{\rm vortex} + \ell\omega_\mu). \tag{9}$$

Integrating out $\theta_{\rm smooth}$, we get the constraint for $\partial_\mu\xi^\mu = 0$, which can be automatically satisfied by the substitution $\xi^\mu \equiv \varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$. Notice that, on a curved surface, $\varepsilon^{\mu\nu\lambda} \equiv \epsilon^{\mu\nu\lambda}/\sqrt{g}$, and a_μ can be understood as a gauge field, since $a_\mu \to a_\mu + \partial_\mu\Gamma$ does not change ξ^μ. With this substitution, we can write the action in terms of a_μ,

$$\mathcal{S}_{\rm eff} = \int dt d^2r\sqrt{g}\left[-\frac{f_{\mu\nu}^2}{4\gamma} + a_\lambda\varepsilon^{\lambda\nu\mu}\partial_\nu(\partial_\mu\theta_{\rm vortex} + \ell\omega_\mu)\right],$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the strength tensor of the a_μ field. We reveal the physical meaning of the second term of the above equation. Integrating the zero component $\varepsilon^{0\mu\nu}\partial_\nu\partial_\mu\theta_{\rm vortex}$ over a region containing a vortex yields

$\int d^2r\sqrt{g}\epsilon^{0\mu\nu}\partial_\mu\partial_\nu\theta_{\text{vortex}} = \oint d\mathbf{r}\cdot\nabla\theta_{\text{vortex}} = 2\pi$. We thus recognize $\epsilon^{0\mu\nu}\partial_\mu\partial_\nu\theta_{\text{vortex}}$ as the density of vortices, i.e., the time component of vortex current density

$$j^\lambda_{\text{vor}} = \epsilon^{\lambda\mu\nu}\partial_\mu\partial_\nu\theta_{\text{vortex}}. \qquad (10)$$

One the other hand, we realize that $\epsilon^{0\mu\nu}\partial_\mu\omega_\nu = \mathcal{B}$ and $\epsilon^{i\mu\nu}\partial_\mu\omega_\nu = \mathcal{E}^i$ are the geometric field strength defined in Eq. (4). Therefore, we identify a geometric current

$$j^\lambda_{\text{geo}} = \epsilon^{\lambda\mu\nu}\partial_\mu\omega_\nu = (\mathcal{B}, \mathcal{E}^1, \mathcal{E}^2). \qquad (11)$$

Substituting vortex current and geometric current into the effective action, we obtain the effective Lagrangian density for vortices and geometry

$$\mathcal{L}_{\text{vor-geo}} = -\frac{1}{4\gamma}f^2_{\mu\nu} + a_\lambda(j^\lambda_{\text{vor}} + j^\lambda_{\text{geo}}). \qquad (12)$$

This central equation governs the dynamics and interactions of vortices and geometry in a chiral superfluid covering a curved surface. There are three types of interactions mediated by the gauge field a_μ, namely "vortex-vortex" interaction, "geometry-geometry" interaction, and "vortex-geometry" interaction. The vortex-geometry interaction resembles the quasiparticle-geometry coupling (the Wen-Zee term [26]) in quantum Hall effect (QHE). In the Supplemental Material [14], we derive an alternative form of Eq. (12), revealing the similarity and differences between chiral superfluidity and QHE. While the analogy has been realized in the literature [27,28], the field theory of chiral superfluidity has two key differences with QHE: the gauge field action is Maxwell-like instead of Chern-Simons–like and Aharonov-Casher gauge potential is absent in QHE. These differences lead to qualitatively different electromagnetic responses.

In the static limit, Eq. (12) can be understood by analogy to the Coulomb gas model: the Gaussian curvature $\mathcal{B}(\mathbf{r})$ plays the role of a nonuniform background charge distribution and the vortices appear as pointlike sources with electrostatic charges equal to their winding number. As a result, the vortices tend to position themselves so that the Gaussian curvature is screened: the negative (positive) vortices on positive (negative) curvature.

Let us quantify the strength of vortex-geometric interaction by considering a vortex in a rotational symmetric 2D surface specified by a three-dimensional vector $\mathbf{R}(r,\varphi) = (r\cos\varphi, r\sin\varphi, h_0\exp(-r^2/2r_0^2))$, where r and φ are plane polar coordinates. Clearly, $\mathbf{R}(r,\varphi)$ describes a static Gaussian bump with a maximum height h_0 and spatial extent $\sim r_0$. It is useful to characterize the deviation of the bump from a plane in terms of a dimensionless aspect ratio $\alpha \equiv h_0/r_0$. We can define local orthonormalized basis vectors $\hat{\mathbf{e}}_\mathbf{r}$ and $\hat{\mathbf{e}}_\varphi$ by normalizing two orthogonal tangent vectors $\mathbf{t}_r = \partial\mathbf{R}/\partial r$ and $\mathbf{t}_\varphi = \partial\mathbf{R}/\partial\varphi$. The components of

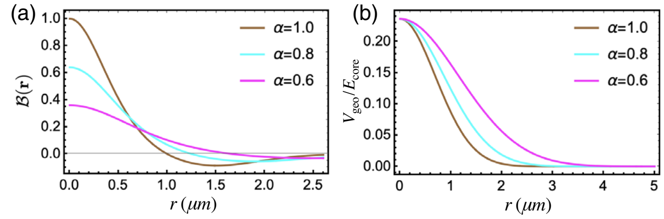

FIG. 2. (a) The spatial-dependent Gaussian curvature of a Gaussian bump of aspect ratios $\alpha = 1$, 0.8, and 0.6. (b) The corresponding spatial-dependent geometric potential, and $E_{\text{core}} \approx \hbar^2\rho_s/m$ is a typical 2D vortex core energy [29].

the geometric gauge field introduced in Eq. (3) are given by $\omega_i = \hat{\mathbf{e}}_\mathbf{r}\cdot\partial_i\hat{\mathbf{e}}_\varphi$, i.e., $\omega_r = 0$ and $\omega_\varphi = -1/\sqrt{c(r)}$ with $c(r) \equiv 1 + (\alpha^2r^2/r_0^2)\exp(-(r^2/r_0^2))$. Consequently, the Gaussian curvature of the bump can be obtained $\mathcal{B}(r) = (\alpha^2/r_0^2c(r)^2)(1-(r^2/r_0^2))\exp(-r^2/r_0^2)$, which generates a geometric potential

$$V_{\text{geo}}(\mathbf{r}) = \int d^2r'\sqrt{g(\mathbf{r}')}\mathcal{B}(\mathbf{r}')\Gamma(\mathbf{r},\mathbf{r}') \qquad (13)$$

via the propagator $\Gamma(\mathbf{r}',\mathbf{r})$ of the gauge field a_μ. Here $g(\mathbf{r}') = c(r')$ is the determinant of the metric. The geometric potential embodies the vorticity and Gaussian curvature attachment that was obtained previously in the literature [14,23,28]. One can employ a conformal transformation to obtain the propagator $\Gamma(\mathbf{r}',\mathbf{r})$ and then the geometric potential [14]

$$V_{\text{geo}}(r) = \frac{\hbar^2\rho_s}{m}\int_r^\infty dr'\frac{\sqrt{c(r')}-1}{r'}. \qquad (14)$$

Figure 2 shows the Gaussian curvature and geometric potential of a Gaussian bump. Note that when the self-energy of a vortex is considered, there exists an additional geometric interaction (see details in the Supplemental Material [14]).

The vortex-geometry interaction provides a unique route to control the position of a vortex. And since a localized Majorana mode is associated with a vortex in a chiral superfluid, one can adiabatically braid Majorana modes by

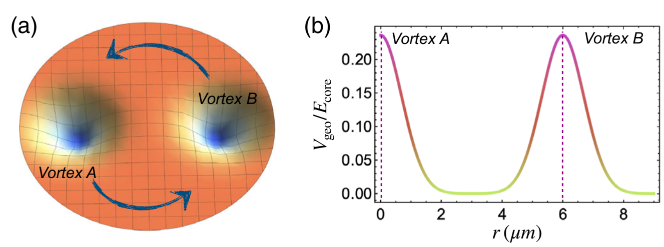

FIG. 3. (a) Schematic demonstration of quantum braiding by engineering geometric curvature. (b) The geometric potential versus distance with aspect ratio $\alpha = 0.8$ for each valley.

mechanically engineering geometric curvature, as is illustrated in Fig. 3(a). We plot the geometric potential (for vortices) generated by two valleys in Fig. 3(b). It shows that the geometric potential is comparable to the self-energy of a vortex. Therefore, the vortex-geometry interaction offers a promising route to perform topological quantum computing in the future.

Anomalous mass and spin supercurrent in ^{3}He *superfluid thin film.*—We apply geometric induction theory in chiral-superfluid ^{3}He film. While both ^{3}He and ^{4}He are superfluids at sufficiently low temperature, the superfluidity in ^{3}He more closely resembles superconductivity than the superfluid ^{4}He. Because, unlike ^{4}He, ^{3}He atoms are fermions that have to be paired to become superfluid. In ^{3}He the strong repulsive force exerted by the atomic cores prevents s-wave pairing: instead, the pairs form an orbital p-wave state, with L and S both equal to $\hbar$. We will consider the ^{3}He-A phase where Cooper pairs possess finite angular momentum in the z direction L_z. Near a surface, surface scattering favors the orbital angular momentum L_z perpendicular to the surface [30]. As a result, our geometric induction theory applies.

In ^{3}He-A (A_1, A_2) phase the spin-up and spin-down components have the same chirality, and the corresponding order parameter reads [20]

$$\Psi_A = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_\mathbf{x} + i\hat{\mathbf{e}}_\mathbf{y})(\sqrt{\rho_\uparrow}e^{i\theta_\uparrow}|\uparrow\rangle + \sqrt{\rho_\downarrow}e^{i\theta_\downarrow}|\downarrow\rangle), \qquad (15)$$

where $\rho_{\uparrow/\downarrow}$ and $\theta_{\uparrow/\downarrow}$ are the superfluid density and phase of the spin-up and spin-down component, respectively. Depending on the relative magnitude of $\rho_\uparrow$ and $\rho_\downarrow$, this order parameter can describe ^{3}He-A phase ($\rho_\uparrow = \rho_\downarrow$), A_1 phase (either $\rho_\uparrow$ or $\rho_\downarrow$ vanishes), or A_2 phase ($\rho_\uparrow \neq \rho_\downarrow$). Assuming constant superfluid density, we can obtain the Ginzburg-Landau (GL) Lagrangian density for ^{3}He superfluid thin film embedded on a curved surface

$$\mathcal{L}_A = \frac{\gamma_\uparrow}{2}(\partial_\mu\theta_\uparrow + \omega_\mu + \mathcal{A}^{ac}_\mu)^2 + \frac{\gamma_\downarrow}{2}(\partial_\mu\theta_\downarrow + \omega_\mu - \mathcal{A}^{ac}_\mu)^2 + \text{interacting terms} + \text{potential}\ldots, \qquad (16)$$

where $\gamma_{\uparrow/\downarrow} = (\rho_{\uparrow/\downarrow}/m)$ denotes the stiffness for the spin-up, spin-down component; $(\mathcal{A}^{ac}_0, \mathcal{A}^{ac}_k) = (\mu_i B_i, \varepsilon_{ijk}E^i\mu^j)$ is the Aharonov-Casher (AC) gauge field arising due to a magnetic moment $\boldsymbol{\mu}$ moving in an electromagnetic field $(\mathbf{E}, \mathbf{B})$ [31,32].

One can obtain the current density of the spin-up and spin-down components from the Lagrangian density $\mathcal{L}_A$ $j^{\uparrow/\downarrow}_\mu = \gamma_{\uparrow/\downarrow}[\partial_\mu\theta_{\uparrow/\downarrow} + \omega_\mu \pm \mathcal{A}^{ac}_\mu]$. Defining a total mass current $j^m_\mu = j^\uparrow_\mu + j^\downarrow_\mu$ and a total spin current $j^s_\mu = j^\uparrow_\mu - j^\downarrow_\mu$ yields the matrix formula

$$\begin{pmatrix} j^m_\mu \\ j^s_\mu \end{pmatrix} = \begin{pmatrix} \gamma^m & \gamma^s \\ \gamma^s & \gamma^m \end{pmatrix} \cdot \begin{pmatrix} \omega_\mu \\ \mathcal{A}^{ac}_\mu \end{pmatrix}, \qquad (17)$$

where $\gamma^{m/s} \equiv \gamma_\uparrow \pm \gamma_\downarrow$, and the phase gradient term is absorbed into the ω_μ and $\mathcal{A}^{ac}_\mu$ by a gauge transformation. One can immediately make several useful predictions from Eq. (17). In ^{3}He-A phase $\gamma^s = 0$ indicates that Gaussian curvature drives a mass current, whereas the AC gauge field drives a spin current. In ^{3}He-A_1 or A_2 phase, however, γ^s is finite so that either Gaussian curvature or an AC gauge field can drive both mass current and spin current, simultaneously. Symmetry may allow a spin-spin interaction term [33] such as $j^\uparrow_\mu j^{\mu\downarrow}$, which effectively shifts the strength of mass or spin stiffness in Eq. (17) [14].

Electromagnetic signature.—We obtain the electromagnetic signature of chiral superfluids induced by geometric gauge fields, and for definiteness we take ^{3}He-A phase as an example. Minimization of GL action with respect to the four-vector potential $A_\mu = (\phi, \mathbf{A})$ leads to the effective electric charge and electric current density [14]

$$\sigma_c = -\gamma^s\mu\mathcal{B}(\mathbf{r}), \qquad \mathbf{J}_c = \gamma^s\boldsymbol{\mu}\times\boldsymbol{\mathcal{E}}(\mathbf{r}), \qquad (18)$$

where $\mathcal{B}(\mathbf{r})$ and $\boldsymbol{\mathcal{E}}(\mathbf{r})$ are the magnetic- and electriclike geometric field strength in Eq. (4); $\boldsymbol{\mu} = \mu\hat{\mathbf{e}}_\mathbf{3}$ is the magnetic moment perpendicular to the surface. The definition of the geometric field strength leads to the Maxwell-like equation $\boldsymbol{\nabla}\times\boldsymbol{\mathcal{E}} = \partial_t\mathcal{B}$, which further guarantees the current conservation $\partial_t\sigma_c + \boldsymbol{\nabla}\cdot\mathbf{J}_c = 0$. Similar reasoning enables us to obtain the effective electric charge density and current density for several other chiral phases of ^{3}He [14]. We assume a superfluid density $\rho \approx 10^{22}/\text{m}^2$ and a Gaussian curvature $\mathcal{B} \approx 1/(100\ \mu\text{m})^2$. The effective charge density can induce an electric field $E \approx 10^{-3}$ V/m.

Geometric induction in a flexible superfluid thin film.—We consider the geometric induction theory of a chiral superfluid embedded on a flexible surface. The flexibility of the surface provides additional degrees of freedom to minimize the total GL action,

$$S_{\text{tot}} = \int dtd^2r\sqrt{g}\left\{\frac{\gamma}{2}(\partial_\mu\theta + \ell\omega_\mu)^2 + \left[\frac{\kappa_0}{2}(\partial_t h)^2 - \frac{\kappa_r}{2}(\nabla^2 h)^2\right]\right\}, \qquad (19)$$

where the first and second term represent the Lagrangian of chiral superfluid and geometry, respectively. To describe a flexible surface, we use height $h(x, y, t)$—the deviation of a curved surface from a plane—to parametrize a 2D surface. The geometric stiffnesses κ_0 and κ_r measure the softness of the surface [34]. The geometric connection $\omega_\mu = \frac{1}{2}\varepsilon^{0\beta\gamma}\partial_\gamma(\partial_\beta h\partial_\mu h)$ embodies the essential interaction between a chiral superfluid and geometry. Minimizing the

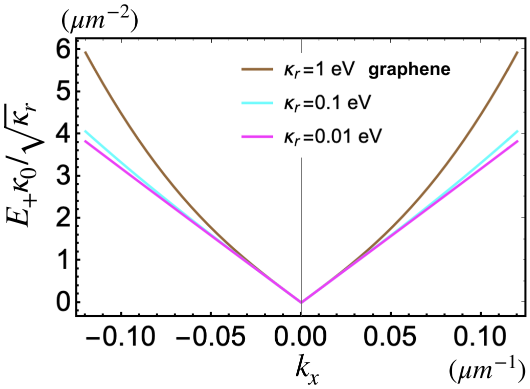

FIG. 4. The energy-momentum dispersion of h is shown for three stiffnesses κ_r. For comparison, a suspended graphene has a stiffness $k_r = 1$ eV. Numerically, we have assumed a reasonable superfluid density $\rho \approx 10^{22}/\mathrm{m}^2$ and superfluid current gradient $\Gamma = 10^{-7}$ J/m^2. We set $\ell = 1$ and $k_y = 0$ in the plot.

GL action with respect to the h, one obtains the equation of motion for geometry to linear order in height and supercurrent density j_μ,

$$\kappa_0 \partial_t^2 h - \kappa_r \nabla^4 h = \ell(\partial_\mu \partial_\beta h)\epsilon^{0\beta\gamma}\partial_\gamma j^\mu. \qquad (20)$$

The dynamic of geometry is qualitatively modified by chiral superfluid. To quantify the influence of chiral superfluidity on geometry, we study the energy-momentum dispersion of h (called flexural modes) by assuming a supercurrent in the x direction with a gradient $\Gamma \equiv \partial_y j^x$ in the y direction. We obtain the modified dispersion relation due to the backaction of the chiral superfluid,

$$E_\pm = \pm\sqrt{\frac{\kappa_r}{\kappa_0}(k_x^2 + k_y^2)^2 + \frac{\ell\Gamma}{\kappa_0}k_x^2}. \qquad (21)$$

In Fig. 4 we see that the energy-momentum dispersion in the x direction becomes Dirac type at small momentum, i.e., $k_x \ll k_c \equiv \sqrt{|\ell\Gamma/\kappa_r|}$, with the critical speed $v_c = \sqrt{\ell\Gamma/\kappa_0}$. Given $\ell = 1$, $\Gamma = 10^{-7}$ J/m^2, and the geometric stiffness $\kappa_0 \approx 7.6 \times 10^{-8}$ g/cm^2 (values taken from graphene [35]), we can estimate the emergent critical speed $v_c \approx 0.36$ m/s.

Summary.—We have studied the intriguing interplay between chiral superfluidity and geometry. A geometric gauge field emerges and induces anomalous dynamics and interactions. Based on chiral vortex-geometry interaction, we proposed a mechanical approach to control the positions of vortices, creating a new route for quantum braiding. We further show that both mass and spin supercurrent can be driven by a Gaussian curvature. And we also obtained the geometry-induced electromagnetic signatures. Finally, we study the backaction of chiral superfluidity on geometry. We find that the dispersion of geometry shifts from quadratic to linear due to the presence of chiral superfluidity.

Proposed effects illustrate the opportunities of controlling quantum states with strain, e.g., pseudo-electromagnetic fields in topological semimetals [36], uniaxial-pressure control of competing orders [37], and strain-induced superconductivity [38]. While it is known that strain can affect the superconductivity, this Letter highlights the opportunities to induce and modify spin and mass currents in superfluids.

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