Neutrino Propagation When Mass Eigenstates and Decay Eigenstates Mismatch

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We point out that the Hermitian and anti-Hermitian components of the effective Hamiltonian for decaying neutrinos cannot be simultaneously diagonalized by unitary transformations for all matter densities. We develop a formalism for the two-flavor neutrino propagation through matter of uniform density, for neutrino decay to invisible states. Employing a resummation of the Zassenhaus expansion, we obtain compact analytic expressions for neutrino survival and conversion probabilities, to first and second order in the "mismatch parameter" $\bar{\gamma}$.

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Introduction.—Neutrino oscillation experiments have unequivocally established that neutrinos have masses, and their flavors mix. However, data still allow the possibility of new physics effects at a subleading level. Neutrino decay to lighter invisible states [1–3] is one such possibility. Solutions to neutrino anomalies via a combination of oscillation and decay have been studied [4–16]. Most of these papers have analyzed neutrino oscillation probabilities in vacuum, taking the mass eigenstates to be identical with the decay eigenstates for their analytic treatment. Matter effects, if relevant, have been implemented numerically.

The effective Hamiltonian for neutrino decay is non-Hermitian, with its Hermitian component corresponding to the energy and the anti-Hermitian component corresponding to decay. The assumption of identifying the mass (energy) eigenstates to decay eigenstates is not valid in general. Indeed, even in vacuum, these two components need not commute, and hence need not be diagonalizable simultaneously by unitary transformations. Even for the special circumstances or models where the mass eigenstates and decay eigenstates coincide in vacuum, matter effects make this mismatch inevitable.

The non-Hermitian Hamiltonian itself may be diagonalized by a similarity transformation employing a nonunitary matrix. Using this principle, the oscillation probabilities in the two-flavor scenario in vacuum were approximately calculated in [17]. A similar exercise has also been performed in [18], albeit in the context of visible neutrino decays in matter, but no compact analytic expressions for probabilities have been presented.

In this Letter, we present a novel prescription for computing the neutrino survival or conversion probabilities for the scenario with simultaneous oscillation and invisible decay of neutrinos propagating in matter of uniform density. We represent the effective Hamiltonian matrix by $\mathcal{H}_m$, where

$$\mathcal{H}_m = H_m - i\Gamma_m/2. \tag{1}$$

Here H_m and Γ_m are Hermitian matrices. We choose to work in the basis where the Hermitian part of the Hamiltonian is diagonalized. This is the same as the basis of neutrino mass eigenstates in matter in the absence of decay. In this basis, H_m is a diagonal matrix whose elements depend on neutrino mass squared differences, neutrino energy, and Earth matter potential. The flavor evolution of neutrinos takes the form

$$\nu(t) = e^{-i\mathcal{H}_m t}\nu(0). \tag{2}$$

Note that since $[H_m, \Gamma_m] \neq 0$ in general, $\mathcal{H}_m$ is not a normal matrix, and $e^{-i\mathcal{H}_m t} \neq e^{-iH_m t}e^{-\Gamma_m t/2}$. Thus, one has to express $e^{-i\mathcal{H}_m t}$ in terms of a chain of commutators using the inverse Baker-Campbell-Hausdorff (BCH) formula, also known as the Zassenhaus formula [19,20]. The standard form of this formula cannot be truncated to a finite number of terms in the current scenario, therefore we employ a resummation technique using its series expansion [21]. The procedure facilitates a perturbative expansion of the neutrino survival and conversion probabilities, in terms of a small parameter $\bar{\gamma}$ that characterizes the mismatch between the eigenstates of H_m and Γ_m.

Our prescription leads to explicit analytic forms for two-flavor neutrino probabilities in matter. The probabilities in vacuum, as well as those calculated by using the assumption of coincident mass and decay eigenstates, emerge as special cases. This formulation is completely new, and provides a clear framework for analyzing neutrino decay in vacuum and matter on the same footing. Moreover, the techniques can be applied to any situation where quantum mechanical evolution in terms of non-Hermitian Hamiltonian is to be calculated.

Formalism.—The effective Hamiltonian may be written in the basis of neutrino mass eigenstates in matter as

$$\mathcal{H}_m = \begin{pmatrix} a_1 - ib_1 & -\frac{1}{2}i\gamma e^{i\chi} \\ -\frac{1}{2}i\gamma e^{-i\chi} & a_2 - ib_2 \end{pmatrix}, \tag{3}$$

where a_i, b_i, γ, χ are real. Since Γ_m needs to be positive semidefinite, $b_i \geq 0$ and $\gamma^2 \leq 4b_1b_2$. The sign of γ is taken to be positive; this defines the value of χ uniquely. The Hermitian part of this Hamiltonian is diagonal, which is ensured by the choice of basis. The anti-Hermitian part is composed of the diagonal components involving b_i, and the off-diagonal components involving γ. Note that $b_i = [\Gamma_m]_{ii}/2$, and $\gamma e^{i\chi} = [\Gamma_m]_{12} = [\Gamma_m]^*_{21}$.

For future convenience, we define the complex parameter $d_i \equiv a_i - ib_i$, the differences $\Delta_a \equiv a_2 - a_1$, $\Delta_b \equiv b_2 - b_1$, $\Delta_d \equiv d_2 - d_1$, and the dimensionless ratios

$$\bar{\gamma} \equiv \frac{\gamma}{|\Delta_d|}, \qquad \bar{\Delta}_a \equiv \frac{\Delta_a}{|\Delta_d|}, \qquad \bar{\Delta}_b \equiv \frac{\Delta_b}{|\Delta_d|}. \tag{4}$$

Then, in terms of the identity matrix $\mathbb{I}$ and

$$\mathbb{X} \equiv -\frac{i\Delta_d t}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbb{Y} \equiv -\frac{\gamma t}{2}\begin{pmatrix} 0 & e^{i\chi} \\ e^{-i\chi} & 0 \end{pmatrix}, \tag{5}$$

one may write

$$-i\mathcal{H}_m t = -\frac{it}{2}(d_1 + d_2)\mathbb{I} + \mathbb{X} + \mathbb{Y}. \tag{6}$$

The commutator of $\mathbb{X}$ and $\mathbb{Y}$ is

$$\mathcal{L}_{\mathbb{X}}\mathbb{Y} \equiv [\mathbb{X}, \mathbb{Y}] = i\frac{\gamma\Delta_d t^2}{2}\begin{pmatrix} 0 & -e^{i\chi} \\ e^{-i\chi} & 0 \end{pmatrix}, \tag{7}$$

which will play a key role in our analysis.

Zassenhaus expansion.—In order to calculate the evolution matrix $e^{-i\mathcal{H}_m t}$, keeping aside the term proportional to the identity matrix, we need to calculate the quantity $e^{\mathbb{X}+\mathbb{Y}}$. This may be written in terms of the Zassenhaus expansion [19,20] as

$$e^{\mathbb{X}+\mathbb{Y}} = e^{\mathbb{X}} e^{\mathbb{Y}} e^{-\frac{1}{2}[\mathbb{X},\mathbb{Y}]} e^{\frac{1}{6}(2[\mathbb{Y},[\mathbb{X},\mathbb{Y}]]+[\mathbb{X},[\mathbb{X},\mathbb{Y}]])} \cdots. \tag{8}$$

Note that $|\mathbb{Y}| \sim \bar{\gamma}|\mathbb{X}|$ and $\mathcal{L}_{\mathbb{X}}\mathbb{Y} \sim \bar{\gamma}|\mathbb{X}|^2$, where the absolute sign $(|\cdots|)$ represents a typical nonzero element in the corresponding matrix. This implies that, in general, for higher-order commutators, $\mathcal{L}^{k-1}_{\mathbb{X}}\mathbb{Y} \sim \bar{\gamma}|\mathbb{X}|^k$. Therefore, it is not possible to truncate the expansion in Eq. (8) at any fixed order of commutators. One needs to collect $O(\gamma^k)$ terms from commutators of all orders by performing a resummation procedure. We therefore employ the expression for the Zassenhaus expansion in terms of a series [21]:

$$e^{\mathbb{X}+\mathbb{Y}} = \left(1 + \sum_{p=1}^{\infty} \sum_{n_1,\ldots,n_p=1}^{\infty} \frac{n_p\ldots n_1}{n_p(n_p+n_{p-1})\ldots(n_p+\ldots+n_1)} \mathcal{Y}_{n_p}\ldots\mathcal{Y}_{n_1}\right) e^{\mathbb{X}}, \tag{9}$$

where $\mathcal{Y}_n = (1/n!)\mathcal{L}^{n-1}_{\mathbb{X}}\mathbb{Y}$.

To obtain the expansion up to $O(\bar{\gamma})$ and $O(\bar{\gamma}^2)$, we need to perform the summation for $p = 1$ and $p = 1, 2$, respectively, since every $\mathbb{Y}$ comes with a factor of $\bar{\gamma}$. Thus for an accuracy of $O(\bar{\gamma}^2)$, we can truncate

$$e^{\mathbb{X}+\mathbb{Y}} \approx \left(1 + \sum_{n_1=1}^{\infty} \mathcal{Y}_{n_1} + \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} \frac{n_1}{(n_1+n_2)} \mathcal{Y}_{n_2}\mathcal{Y}_{n_1}\right) e^{\mathbb{X}}, \tag{10}$$

with the double summation term not needed for $O(\bar{\gamma})$ accuracy. One may use

$$\mathcal{Y}_n = \frac{1}{n!}(i\Delta_d t)^{n-1}\sigma_3^{n-1}\mathbb{Y} \tag{11}$$

in order to get closed functional forms for the infinite sums. Here σ_3 is the Pauli matrix.

Neutrino flavor conversions up to $O(\bar{\gamma})$.—The truncation of the right-hand side of Eq. (10) to the first summation gives

$$e^{\mathbb{X}+\mathbb{Y}} = \left(1 + \frac{\sin(\Delta_d t)}{\Delta_d t}\mathbb{Y} - \frac{\cos(\Delta_d t) - 1}{\Delta_d t} i\sigma_3\mathbb{Y}\right) e^{\mathbb{X}}. \tag{12}$$

The amplitude matrix in the mass basis in matter is then

$$\mathcal{A}_m \equiv e^{-i\mathcal{H}_m t} = \begin{pmatrix} e^{-id_1 t} & -i\frac{\gamma e^{i\chi} g_-(t)}{\Delta_d} \\ -i\frac{\gamma e^{-i\chi} g_-(t)}{\Delta_d} & e^{-id_2 t} \end{pmatrix}, \tag{13}$$

where the functions $g_{\pm}(t)$ are defined as

$$g_{\pm}(t) = \frac{1}{2}(e^{-id_2 t} \pm e^{-id_1 t}). \tag{14}$$

The neutrino flavor conversion probability $P_{\beta\alpha}$ for $\nu_\beta \to \nu_\alpha$ conversion may be obtained by calculating the flavor conversion amplitude

$$[\mathcal{A}_f]_{\alpha\beta} = [U_m e^{-i\mathcal{H}_m t} U_m^\dagger]_{\alpha\beta}, \tag{15}$$

and further, $P_{\beta\alpha} = |\mathcal{A}_{\alpha\beta}|^2$. In the two-flavor system,

$$U_m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \tag{16}$$

is the unitary rotation matrix. One can write

$$\mathcal{A}_f = \begin{pmatrix} g_-(t)A(\chi) + g_+(t) & g_-(t)B(\chi) \\ g_-(t)B(-\chi) & -g_-(t)A(\chi) + g_+(t) \end{pmatrix}, \tag{17}$$

where $A(\chi)$ and $B(\chi)$ are given in Table I. The χ dependence of A and B is implicit wherever not stated explicitly.

The survival probability of a neutrino of flavor α is

$$\begin{aligned} P_{\alpha\alpha} = \frac{e^{-(b_1+b_2)t}}{2}[&(1+|A|^2)\cosh(\Delta_b t). \\ &+ (1-|A|^2)\cos(\Delta_a t) - 2\mathrm{Re}(A)\sinh(\Delta_b t) \\ &+ 2\mathrm{Im}(A)\sin(\Delta_a t)]. \end{aligned} \tag{18}$$

The survival probability $P_{\beta\beta}$ for the other flavor may be obtained from $P_{\alpha\alpha}$ with the replacement $A \to -A$. The probability for $\nu_\beta \to \nu_\alpha$ conversion is

$$P_{\beta\alpha} = \frac{e^{-(b_1+b_2)t}}{2}|B(\chi)|^2[\cosh(\Delta_b t) - \cos(\Delta_a t)]. \tag{19}$$

The conversion probability $P_{\alpha\beta}$ is obtained by the replacement $\chi \to -\chi$. The explicit expressions for the terms in Eqs. (18) and (19) are given in Table II.

It should be noted that in the two-flavor approximation in the absence of neutrino decay, i.e., $b_1 = b_2 = \gamma = 0$, we have $P_{\alpha\alpha} = P_{\beta\beta}$ and $P_{\beta\alpha} = P_{\alpha\beta}$. These equalities no longer hold in the presence of decay.

TABLE I. The terms in the amplitude matrix $\mathcal{A}_f$ in the flavor basis used in Eq. (17), calculated up to $O(\bar{\gamma})$.

Term	Expression
$A(\chi) \equiv A^{(0)} + \gamma A^{(1)}$	$-\cos 2\theta_m - i(\gamma/\Delta_d)\sin 2\theta_m \cos\chi$
$B(\chi) \equiv B^{(0)} + \gamma B^{(1)}$	$\sin 2\theta_m - i(\gamma/\Delta_d)(\cos 2\theta_m \cos\chi + i\sin\chi)$

Neutrino flavor conversions up to $O(\bar{\gamma}^2)$.—For probabilities accurate up to order $\bar{\gamma}^2$, we need to calculate the term in Eq. (10) that involves a double summation. This sum may be rewritten as

$$\frac{1}{2}\sum_{n_1=1}^{\infty}\left(\sum_{n_2=1}^{\infty}\mathcal{Y}_{n_2}\mathcal{Y}_{n_1} + \sum_{n_2=n_1}^{\infty}\frac{n_1-n_2}{n_1+n_2}[\mathcal{Y}_{n_2},\mathcal{Y}_{n_1}]\right), \tag{20}$$

whose closed form may be obtained using the observation

$$[\mathcal{Y}_{n_2},\mathcal{Y}_{n_1}] = \frac{(-1)^{n_2}-(-1)^{n_1}}{4n_1!n_2!}(i\Delta_d t)^{n_2+n_1-2}(\gamma t)^2\sigma_3. \tag{21}$$

The eigenvalues of $\mathcal{H}_m$ get corrections at $O(\bar{\gamma}^2)$, and it is convenient to write the probabilities at this (and higher) order in terms of the difference of the exact eigenvalues

$$\Delta_D = \sqrt{\Delta_d^2 - \gamma^2}. \tag{22}$$

The probabilities at $O(\bar{\gamma}^2)$ can be written in the same form as Eqs. (18) and (19) with the replacements

$$\Delta_a \to \mathrm{Re}(\Delta_D), \qquad \Delta_b \to -\mathrm{Im}(\Delta_D), \tag{23}$$

and the entries in Table I replaced by

$$A(\chi) \to A^{(0)} + \gamma A^{(1)} - \gamma^2\cos 2\theta_m/(2\Delta_d^2), \tag{24}$$

$$B(\chi) \to A^{(0)} + \gamma A^{(1)} + \gamma^2\sin 2\theta_m/(2\Delta_d^2), \tag{25}$$

The entries corresponding to Table II can be calculated using Eqs. (24) and (25).

Exact results.—For the two-flavor system, it is also possible to obtain the exact expressions for neutrino survival and conversion probabilities by expressing $-i\mathcal{H}_m t$ as a linear combination of Pauli matrices [22]. For any 2×2 matrix $\mathbb{K}$, one can write

$$e^{\mathbb{K}} = e^{k_0}\left[\mathbb{I}\cosh k + \frac{\vec{k}\cdot\vec{\sigma}}{k}\sinh k\right], \tag{26}$$

where $k_\mu \equiv \mathrm{Tr}(\mathbb{K}\cdot\sigma_\mu)/2$, and $k \equiv \sqrt{k_1^2+k_2^2+k_3^2}$. For the matrix $\mathbb{K} = -i\mathcal{H}_m t$ as in Eq. (3), this corresponds to

TABLE II. The terms to be used in the probabilities shown in Eqs. (18) and (19), calculated up to $O(\bar{\gamma})$.

Term	Expression
$\mathrm{Re}(A)$	$-\cos 2\theta_m + \bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos\chi$
$\mathrm{Im}(A)$	$-\bar{\gamma}\bar{\Delta}_a \sin 2\theta_m \cos\chi$
$\lvert A\rvert^2$	$\cos^2 2\theta_m - 2\bar{\gamma}\bar{\Delta}_b \sin 2\theta_m \cos 2\theta_m \cos\chi$
$\lvert B\rvert^2$	$\sin^2 2\theta_m + 2\bar{\gamma}\sin 2\theta_m(\bar{\Delta}_a \sin\chi + \bar{\Delta}_b \cos 2\theta_m \cos\chi)$

$$k_0 = -\frac{it}{2}(d_1 + d_2), \qquad k = \frac{it\Delta_D}{2}. \tag{27}$$

This leads to the exact probabilities that can be written in the same form as Eqs. (18) and (19), with the replacements given in Eq. (23), and

$$A(\chi) \to \frac{\Delta_d}{\Delta_D} A(\chi), \qquad B(\chi) \to \frac{\Delta_d}{\Delta_D} B(\chi), \tag{28}$$

in Table I. The entries corresponding to Table II can be calculated using Eq. (28).

Numerical comparison.—We now demonstrate the convergence of our analytic results towards the exact neutrino oscillation probabilities, when higher and higher order terms in $\bar{\gamma}$ are included. For the sake of illustration, we choose the survival probability of ν_μ with energy $E \sim$ GeV, for a baseline of 295 km. This would correspond to the probability relevant for the T2K and T2HK experiments. Since matter effects in the $\nu_\mu - \nu_\tau$ sector are negligible at small baselines, we choose $\theta_m = \theta_{\rm atm} = 45°$. We take $\Delta_a = 2.56 \times 10^{-3}$ eV$^2/(2E)$, with the decay parameters $(b_1, b_2, \gamma) = (3, 6, 8) \times 10^{-5}$ eV$^2/(2E)$, $\chi = \pi/4$, where the $1/E$ dependence accounts for time dilation. Note that our parameter choices satisfy the desired conditions $b_1, b_2 \ll |\Delta_a|$ (the effects of decay should be subdominant to those of oscillations), and $\gamma^2 < 4b_1 b_2$ (positive definiteness of the decay matrix).

The left panel in Fig. 1 shows the probability $P_{\mu\mu}(E)$ without decay, and successive approximations at $O(\bar{\gamma})$ and $O(\bar{\gamma}^2)$ in the presence of decay. The incorrect approximation that neglects the commutator $[\mathbb{X}, \mathbb{Y}]$ is also indicated. The convergence towards the exact solution is more clearly demonstrated in the right panel of Fig. 1, where we show the values of the error

$$\Delta P_{\mu\mu} \equiv P_{\mu\mu}(\text{analytic}) - P_{\mu\mu}(\text{exact}) \tag{29}$$

on a logarithmic scale. Clearly, the inclusion of $O(\bar{\gamma})$ and $O(\bar{\gamma}^2)$ terms reduces the error by orders of magnitude.

Comparison with earlier results.—In Ref. [17], neutrino decay in vacuum was analyzed using diagonalization of the non-Hermitian Hamiltonian with $\mathcal{H}_{\rm diag} = N^{-1}\mathcal{H}N$, using a nonunitary matrix N. We find that the most general form of the nonunitary matrix that would diagonalize a non-Hermitian $\mathcal{H}$ is

$$N = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} 1 & -i\frac{\gamma e^{i\chi}}{\Delta_D + \Delta_d} \\ i\frac{\gamma e^{-i\chi}}{\Delta_D + \Delta_d} & 1 \end{pmatrix}. \tag{30}$$

Note that since $(\Delta_D + \Delta_d)$ is complex, the off-diagonal elements of the second matrix in Eq. (30) are not complex conjugates of one another, an assumption implicitly made in Ref. [17]. This introduces corrections in the neutrino conversion probabilities of $\sim O(\bar{\gamma}\bar{\Delta}_b)$. These may be neglected if one assumes $\bar{\Delta}_b \sim O(\bar{\gamma})$; however, these will then contribute to $O(\bar{\gamma}^2)$ corrections.

For the special case where only the mass eigenstate ν_2 in vacuum decays (with lifetime τ_2), the probabilities in matter may be obtained by the following identifications:

$$a_{1,2} = \frac{\tilde{m}_{1,2}^2}{2E}, \qquad b_{1,2} = \frac{\alpha_2}{4E}[1 \mp \cos[2(\theta - \theta_m)], \tag{31}$$

$$\chi = 0, \qquad \gamma = \frac{\alpha_2}{2E}\sin[2(\theta - \theta_m)]. \tag{32}$$

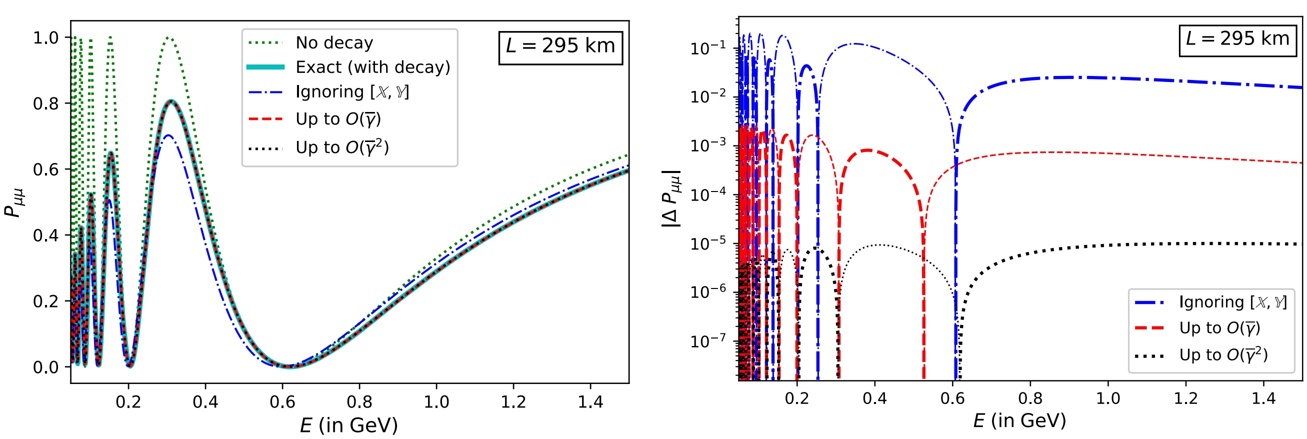

FIG. 1. The left panel shows the survival probability $P_{\mu\mu}$ calculated exactly, and by using the analytic expressions in the text, in the presence and absence of neutrino decay. The right panel shows the differences between the analytic expressions and the exact results for decay. The thick (thin) lines correspond to $\Delta P_{\mu\mu} > 0$ ($\Delta P_{\mu\mu} < 0$). The sharp dips in the right panel correspond to those energies where the analytic expressions give the same values as the exact ones.

Here, $\tilde{m}_i(m_i)$ and $\theta_m(\theta)$ are the mass eigenvalues and mixing angle in matter (vacuum), and $\alpha_2 = m_2/\tau_2$. Note that all the elements of the Γ_m matrix are now nonzero ($b_1, b_2, \gamma \neq 0$), and hence both the neutrino mass eigenstates show decaying behavior. This prescription gives the correct analytic probability expressions for decaying neutrinos in matter, which are hitherto not explicitly given in the literature. In the vacuum limit ($\theta_m \to \theta$, $\tilde{m}_i \to m_i$, $b_1 \to 0$ and $\gamma \to 0$) the standard probabilities in vacuum [23] are obtained.

Concluding remarks.—Neutrino decay is characterized by a non-Hermitian Hamiltonian, which cannot be diagonalized by a unitary transformation. Further, there is no guarantee that decay eigenstates are the same as the mass eigenstates, although it is usually assumed. We point out that even if these two sets are the same in vacuum, matter effects necessarily change this simple picture, warranting a more careful treatment.

In this Letter, we develop a novel formalism which can address the above two issues, and allows one to obtain compact analytic forms for two-flavor probabilities even in matter. The crucial step is to perform the analysis in the basis of mass eigenstates in matter in the absence of decay, so that the Hermitian component of the Hamiltonian is diagonal. The anti-Hermitian decay matrix is not diagonal in this basis, and does not commute with the Hermitian part. We introduce a resummation of commutators in the Zassenhaus expansion for the time evolution matrix. Using this, we compute the neutrino probabilities perturbatively in the small parameter $\bar{\gamma}$ which characterizes the mismatch between mass and decay eigenstates. This is the first time such a formulation has been used to treat propagation of unstable neutrinos in matter.

In this work, we have presented the exact expressions as well as perturbative expansions for two-flavor neutrino probabilities. The framework of the latter may be easily extended to three flavors [24]. Moreover, the approximate analytic expressions are useful to bring out the underlying physics. The scope of application of this method goes beyond just the neutrino decay hypothesis; the formalism may be applied to various other phenomena such as the combined treatment of oscillations and absorption for high energy neutrinos, axion-photon oscillations in an optically semi-opaque medium, or even the neutral-meson mixing systems.

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