Redundantly Amplified Information Suppresses Quantum Correlations in Many-Body Systems

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We establish bounds on quantum correlations in many-body systems. They reveal what sort of information about a quantum system can be simultaneously recorded in different parts of its environment. Specifically, independent agents who monitor environment fragments can eavesdrop only on amplified and redundantly disseminated—hence, effectively classical—information about the decoherence-resistant pointer observable. We also show that the emergence of classical objectivity is signaled by a distinctive scaling of the conditional mutual information, bypassing hard numerical optimizations. Our results validate the core idea of quantum Darwinism: objective classical reality does not need to be postulated and is not accidental, but rather a compelling emergent feature of quantum theory that otherwise—in the absence of decoherence and amplification—leads to "quantum weirdness." In particular, a lack of consensus between agents that access environment fragments is bounded by the information deficit, a measure of the incompleteness of the information about the system.

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Introduction.—Is classical reality, reflected in the consensus between independent agents about the properties of physical systems [1], a consequence of quantum laws? Quantum weirdness makes it difficult to reconcile human perception with our most successful scientific theory. In particular, quantum systems display stronger correlations than those admitted by classical physics [2–4]. They enable the advantages of quantum information processing [5–8]. Despite their importance in quantum science, our understanding of genuinely quantum correlations is limited: their identification and quantification in large scale quantum systems—the focus of quantum-classical transition—is an often intractable problem [9–15].

Here, we prove universal, quantitative bounds on quantum correlations in many-body systems: they are bounded by the shared classical information among their parts. As an important consequence, objectivity of measurement results arises only when quantum correlations between an information source and a network of recipients are selectively suppressed. That is, consensus responsible for objective classical reality is an emergent attribute of quantum mechanics.

First, we consider a quantum universe consisting of a system $\mathcal{S}$ and an environment $\mathcal{E}$. We prove an upper bound on quantum discord, which quantifies genuinely quantum correlations [16]. The simultaneous creation of quantum discord between $\mathcal{S}$ and different environment fragments $\mathcal{F}$ and $\mathcal{E}/\mathcal{F}$ is restricted. The upper limit is determined by how much classical information about $\mathcal{S}$ is concurrently available to observers monitoring the two distinct fragments.

Then, we extend our study to the multipartite case. Quantum correlations are generally *not* monogamous, and almost ubiquitous in Hilbert space [17–21]. Nevertheless, we prove an upper bound on the *average* bipartite quantum discord, and, remarkably, also on the entanglement of formation that can exist between $\mathcal{S}$ and any of N subsystems ε_i of the environment. Simultaneous classical correlations between $\mathcal{S}$ and each ε_i imply that quantum discord (almost) vanishes throughout the universe. Hence, quantum information about $\mathcal{S}$ is inaccessible to independent observers that monitor different ε_i.

This result supports quantum Darwinism, pinpointing the origin of classical reality within quantum theory [22]. Its core insight is that independent observers (such as humans) find out about $\mathcal{S}$ by eavesdropping on ε_is—e.g., scattered or emitted photons in our everyday $\mathcal{E}$ [23–29]. Only information that has been replicated throughout the environment [30,31], resulting in multiple records, is widely accessible—only pointer states that survive decoherence intact and can be shared by many observers become the subject of consensus, acquiring a classically objective nature [1,32].

The newfound bounds on quantum correlations confirm that agreement among independent observers suppresses quantumness. Only large fragments (i.e., $\mathcal{F} \geq \mathcal{E}/\mathcal{F}$) retain quantum information about $\mathcal{S}$. Moreover, we show that when disjoint environment fragments establish sufficient correlations with $\mathcal{S}$, they store predominantly information about a *unique* observable. We compute bounds on such classical correlations between environment fragments, obtaining an information-theoretic characterization of objective classical reality. These bounds generalize previous findings [33–40], highlighting that redundancy of information available to independent observers implies uniqueness of objective reality.

Finally, we introduce an analytical witness of objectivity. Testing quantum Darwinism in complex systems is hard, because quantifying correlations requires daunting numerical optimizations [11,41]. We overcome this limitation and show that redundancy of classical correlations (in its strongest form) is signaled by a characteristic scaling of the conditional mutual information, an analytical function of quantum states [42].

Trade-off relations for quantum correlations.—We consider a quantum system $\mathcal{S}$ of dimension $d_\mathcal{S}$ and an N-partite environment $\mathcal{E} := \cup_{i=1}^N \varepsilon_i$ of dimension $d_\mathcal{E} = \Pi_{i=1}^N d_{\varepsilon_i}$. We call $\mathcal{F}_k := \cup_{\#i=k} \varepsilon_i$ a fragment of $k < N$ elements, and $\mathcal{E}_{/k} := \mathcal{E}/\mathcal{F}_k$ its complement. The information shared by $\mathcal{S}$ and $\mathcal{F}_k$ is quantified by the mutual information $I(\mathcal{S}:\mathcal{F}_k) := H(\mathcal{S}) + H(\mathcal{F}_k) - H(\mathcal{S}\mathcal{F}_k)$, where $H(\mathcal{X}) := -\mathrm{tr}\{\rho_\mathcal{X} \log_2 \rho_\mathcal{X}\} \le \log_2 d_\mathcal{X}$ is the von Neumann entropy of the state $\rho_\mathcal{X}$ of $\mathcal{X}$. The mutual information consists of classical and quantum components [16,43]. The classical part is the (maximal) mutual information that is left after a local measurement $\mathbf{M}_k := \{\mathbf{M}_\alpha, \sum_\alpha \mathbf{M}_\alpha^\dagger \mathbf{M}_\alpha = \mathbb{I}_{d_{\mathcal{F}_k}}\}$ on $\mathcal{F}_k$. Given the postmeasurement state

$$\rho_{\mathcal{S}\mathcal{F}_{k,\mathbf{M}_k}} = \sum_\alpha (\mathbb{I}_{d_\mathcal{S}} \otimes \mathbf{M}_\alpha)\rho_{\mathcal{S}\mathcal{F}_k}(\mathbb{I}_{d_\mathcal{S}} \otimes \mathbf{M}_\alpha^\dagger), \tag{1}$$

classical correlations are quantified as the maximal information about $\mathcal{S}$ an observer can extract by measurements on $\mathcal{F}_k$: $J(\mathcal{S}:\check{\mathcal{F}}_k) := \max_{\mathbf{M}_k} I(\mathcal{S}:\mathcal{F}_{k,\mathbf{M}_k})$ [43,44]. This quantity is upper bounded by $H(\mathcal{S})$. *Quantum discord*, the most general kind of quantum correlation, is then defined as the difference between premeasurement and postmeasurement mutual information,

$$D(\mathcal{S}:\check{\mathcal{F}}_k) := I(\mathcal{S}:\mathcal{F}_k) - J(\mathcal{S}:\check{\mathcal{F}}_k). \tag{2}$$

Note that classical and quantum correlations are generally not invariant under subsystem swapping: $J(\mathcal{S}:\check{\mathcal{F}}_k) \neq J(\check{\mathcal{S}}:\mathcal{F}_k)$, and $D(\mathcal{S}:\check{\mathcal{F}}_k) \neq D(\check{\mathcal{S}}:\mathcal{F}_k)$.

Quantum discord $D(\mathcal{S}:\check{\mathcal{F}}_k)$ is the minimum *quantum information* about $\mathcal{S}$ that $\mathcal{F}_k$ loses when a local measurement $\mathbf{M}_k$ is performed [3,45,46]. Quantum discord can exist even in nonentangled states [16,21], as it can be created by local operations and classical communication (LOCCs) [47]. Specifically, $D(\mathcal{S}:\check{\mathcal{F}}_k) = 0$ if and only if there exists a measurement $\tilde{\mathbf{M}}_k$ such that $\rho_{\mathcal{S}\mathcal{F}_k} = \rho_{\mathcal{S}\mathcal{F}_{k,\tilde{\mathbf{M}}_k}}$.

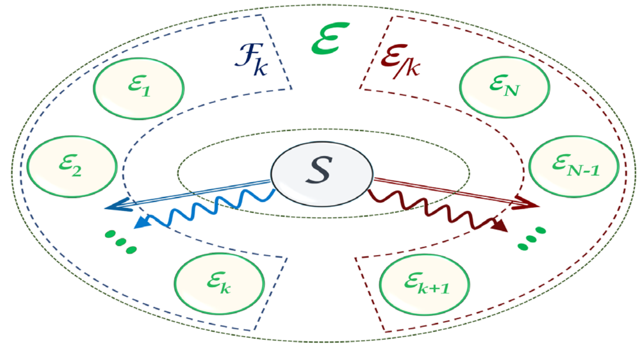

FIG. 1. We demonstrate quantitative bounds on quantum correlations between a system $\mathcal{S}$ and fragments $\mathcal{F}_k, \mathcal{E}_{/k}$ of an N-partite environment $\mathcal{E}$. Equation (5) is an upper limit to quantum discord (wavy lines) in terms of the consensus about classical information (double lines) that is broadcast from $\mathcal{S}$ to $\mathcal{F}_k$ and $\mathcal{E}_{/k}$.

Quantum discord signals the presence of quantum coherence [48]. It can be converted into entanglement [49,50], and it is a resource for quantum metrology [51]. For pure states, it is equal to the entanglement entropy, $D(\mathcal{S}:\check{\mathcal{F}}_k) = D(\check{\mathcal{S}}:\mathcal{F}_k) = H(\mathcal{S})$, while in general its maximal value is $H(\mathcal{F}_k)$ [9].

In the following, we derive constraints on quantum correlations between $\mathcal{S}$ and any fragment $\mathcal{F}$. First, we evaluate upper bounds to $D(\mathcal{S}:\check{\mathcal{F}}_k)$, that is, how much quantum information about $\mathcal{S}$ is accessible to an observer who knows the state of $\mathcal{F}_k$ (Fig. 1). Koashi and Winter discovered a trade-off between the entanglement of formation $E_f(\mathcal{S}:\mathcal{F}_k)$ in $\mathcal{S}\mathcal{F}_k$ [52], and classical correlations in $\mathcal{S}\mathcal{E}_{/k}$ [17,53],

$$E_f(\mathcal{S}:\mathcal{F}_k) \le H(\mathcal{S}) - J(\mathcal{S}:\check{\mathcal{E}}_{/k}). \tag{3}$$

Without loss of generality, we assume now that $\mathcal{S}\mathcal{E}$ is in a pure state $|\psi\rangle_{\mathcal{S}\mathcal{E}}$. Then, the inequality in Eq. (3) is saturated. This surprising relation between classical and quantum features does not hold if we replace entanglement with quantum discord [17,54].

Yet, we can establish an exact bound on quantum discord between $\mathcal{S}$ and environment fragments. We quantify the (lack of) agreement between the classical information about $\mathcal{S}$ that is accessible via $\mathcal{F}_k$ and $\mathcal{E}_{/k}$, i.e., *classical objectivity* [37,55], by introducing the information deficit

$$\delta := \frac{J(\mathcal{S}:\check{\mathcal{E}}) - \min\{J(\mathcal{S}:\check{\mathcal{F}}_k), J(\mathcal{S}:\check{\mathcal{E}}_{/k})\}}{H(\mathcal{S})} \in [0,1]. \tag{4}$$

The information deficit disappears if and only if classical information about $\mathcal{S}$ is simultaneously stored into $\mathcal{F}_k$ and $\mathcal{E}_{/k}$, and it is maximal if and only if there is maximal discrepancy [56]. The information deficit δ was employed in previous Quantum Darwinism literature as a free parameter. The definition in Eq. (4) is key in the proof that classical objectivity restricts the proliferation of quantum correlations.

Result 1. For any state of the universe $|\psi\rangle_{\mathcal{SE}}$,

$$D(\mathcal{S}:\check{\mathcal{F}}_k) + D(\mathcal{S}:\check{\mathcal{E}}_{/k}) \leq 2\delta H(\mathcal{S}). \tag{5}$$

Proof.—Since $J(\mathcal{S}:\check{\mathcal{E}}) = H(\mathcal{S})$ for pure states $|\psi\rangle_{\mathcal{SE}}$, one has

$$\begin{aligned} & I(\mathcal{S}:\mathcal{F}_k)+I(\mathcal{S}:\mathcal{E}_{/k})=2H(\mathcal{S}) \\ \Rightarrow & D(\mathcal{S}:\check{\mathcal{F}}_k)+D(\mathcal{S}:\check{\mathcal{E}}_{/k})=2H(\mathcal{S})-J(\mathcal{S}:\check{\mathcal{F}}_k)-J(\mathcal{S}:\check{\mathcal{E}}_{/k}) \\ & \leq 2H(\mathcal{S})+2\delta H(\mathcal{S})-2J(\mathcal{S}:\check{\mathcal{E}}) \\ & \leq 2\delta H(\mathcal{S}). \end{aligned}$$

Hence, consensus between two observers accessing $\mathcal{F}_k, \mathcal{E}_{/k}$, respectively, about classical information on $\mathcal{S}$ prevents proliferation of quantum correlations. Note that for $\delta \to 0$, neither fragment can share quantum discord with $\mathcal{S}$.

We extend the result, by proving a bound on this concurrent sharing of quantum information about $\mathcal{S}$ with N environment constituents ε_i, i.e., to $N > 2$ observers (Fig. 2). As a special case of Eq. (4), we quantify the (lack of) consensus of two observers accessing ε_i and $\mathcal{E}_{/i}$ by

$$\delta_i := \frac{J(\mathcal{S}:\check{\mathcal{E}}) - \min\{J(\mathcal{S}:\check{\varepsilon}_i), J(\mathcal{S}:\check{\mathcal{E}}_{/i})\}}{H(\mathcal{S})} \in [0,1]. \tag{6}$$

It is also useful to define the average information deficit $\delta := \sum_{i=1}^N \delta_i/N$. For $N = 2$, it is the quantity in Eq. (4) [57]. Then, there exists a universal bound on quantum discord in many-body systems [56].

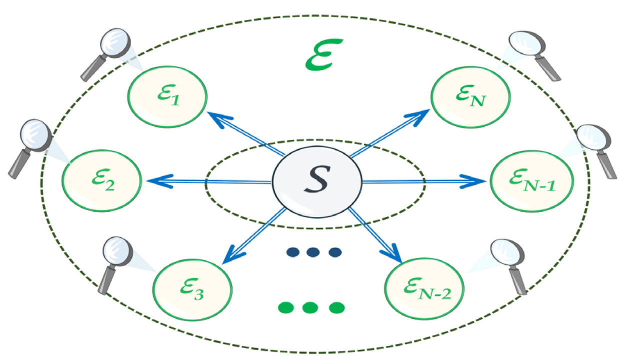

FIG. 2. Quantum Darwinism recognizes that information about a system $\mathcal{S}$ is obtained from disjoint environment fragments consisting of distinct subsystems ε_i's of $\mathcal{E}$ by independent observers. Unconstrained proliferation of classical correlations means that only the information about a pointer observable is accessible. Equation (7) implies that, whenever classical objectivity manifests, bipartite quantum correlations are suppressed.

Result 2. For any state of the universe $|\psi\rangle_{\mathcal{SE}}$,

$$\begin{aligned} \bar{D}(\mathcal{S}:\check{\varepsilon}_i) &:= \frac{1}{N}\sum_{i=1}^N D(\mathcal{S}:\check{\varepsilon}_i), \\ \bar{D}(\mathcal{S}:\check{\varepsilon}_i) &\leq \delta H(\mathcal{S}). \end{aligned} \tag{7}$$

The tightness of this bound only depends on the δ, i.e., (lack of) objectivity. Also, since $E_f(\mathcal{S}:\varepsilon_i) = H(\mathcal{S}) - J(\mathcal{S}:\check{\mathcal{E}}_{/i})$, and $\delta_i = 1 - \min\{J(\mathcal{S}:\check{\varepsilon}_i), J(\mathcal{S}:\check{\mathcal{E}}_{/i})\}/H(\mathcal{S})$, the entanglement of formation is upper bounded: $E_f(\mathcal{S}:\varepsilon_i) \leq \delta_i H(\mathcal{S})$. Averaging over all subsystems ε_i, we get the following.

Remark. For any $|\psi\rangle_{\mathcal{SE}}$,

$$\bar{E}_f(\mathcal{S}:\varepsilon_i) := \frac{1}{N}\sum_{i=1}^N E_f(\mathcal{S}:\varepsilon_i) \leq \delta H(\mathcal{S}). \tag{8}$$

That is, quantum correlations are tightly constrained whenever multiple observers reach agreement concerning classical information about $\mathcal{S}$. Note that the inequality $\bar{J}(\mathcal{S}:\check{\varepsilon}_i) := (1/N)\sum_{i=1}^N J(\mathcal{S}:\check{\varepsilon}_i) \leq H(\mathcal{S})$ can be saturated: independent observers can achieve arbitrarily small δ, making quantum correlations (almost) vanish. We support this statement with an example. (See the Supplemental Material [56] for details.) A qubit $\mathcal{S}$ and an N-qubit environment $\mathcal{E}$ are in the initial state $|+\rangle_{\mathcal{S}}|0\rangle_{\mathcal{E}}^{\otimes N}$, with $|+\rangle \equiv \sqrt{1/2}(|0\rangle + |1\rangle)$. We quantify classical and quantum correlations that are created by a unitary $\mathbf{U}_{\mathcal{SE}}(a) \equiv \Pi_{i=1}^N \mathbf{U}_{\mathcal{S}\varepsilon_i}(a)$, where each $\mathbf{U}_{\mathcal{S}\varepsilon_i}(a)$ implements the controlled gate $\mathbb{I}_2 \oplus \begin{pmatrix} a & \sqrt{1-a^2} \\ \sqrt{1-a^2} & -a \end{pmatrix}, a \in [0,1]$, on $\mathcal{S}\varepsilon_i$. Their average values, in this case, are the values calculated for any $\mathcal{S}\varepsilon_i$ bipartition. This dynamical "c-maybe" model [58] is significant: it can represent the correlation pattern of a system $\mathcal{S}$ interacting with a photonic environment. The universe is therefore in a *singly branching* state [23,59]. The plots in Fig. 3 highlight how the newfound bound to quantum discord is much tighter than the entropic limit $H(\varepsilon_i)$ in the most interesting regime, when system and environment are highly correlated [$a \to 0$; the universe is in a generalized Greenberger–Horne–Zeilinger (GHZ) state].

While limits to quantum information sharing are manifest in GHZ states, the generality of Eqs. (7) and (8) is surprising. Quantum discord and the entanglement of formation are generally nonmonogamous: $D(\mathcal{S}:\check{\mathcal{E}}) \ngeq \sum_i D(\mathcal{S}:\check{\varepsilon}_i)$ [17–20,54,61,62]. Also, there are infinitely many kinds of entanglement structures, i.e., classes of states that cannot be transformed into each other by (stochastic) LOCCs [63]. Our bounds therefore capture a universal feature of many-body quantum systems which cannot be inferred from the structure of the GHZ class, nor by monogamy relations.

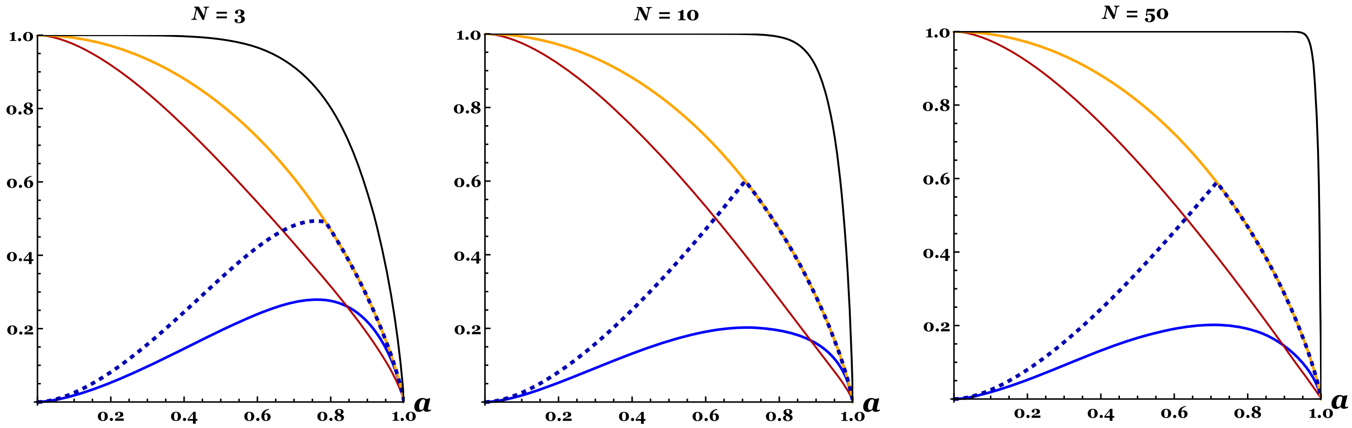

FIG. 3. By employing known methods [58,60], for different values of N, we compute the following quantities in the state $\mathbf{U}_{\mathcal{SE}}(a)|+\rangle_{\mathcal{S}}|0\rangle_{\mathcal{E}}^{\otimes N}$ ([56]): quantum discord, $\bar{D}(\mathcal{S}:\check{\varepsilon}_i)$ (blue line); known upper bound $H(\varepsilon_i)$, (orange line); minimum among the upper bound from Eq. (7) and $H(\varepsilon_i)$ (dashed blue line); classical correlations $\bar{J}(\mathcal{S}:\check{\varepsilon}_i)$ (red line); $H(\mathcal{S})$ (black line). For $N=2$, the bound in Eq. (7) is even saturated, $\bar{D}(\mathcal{S}:\check{\varepsilon}_i)=\delta H(\mathcal{S})$. Overall, it is much more informative than the entropic limit for $a\to 0$, while classical correlations attain $H(\mathcal{S})$.

We stress that *redundancy* of amplified—hence, classical—information is sufficient to suppress quantum correlations. For pure states $|\psi\rangle_{\mathcal{SE}}$,

$$\bar{J}(\mathcal{S}:\check{\varepsilon}_i)\geq(1-\delta)H(\mathcal{S})\Rightarrow\bar{D}(\mathcal{S}:\check{\varepsilon}_i)\leq\delta H(\mathcal{S}). \quad (9)$$

Moreover, if at least $(1-\delta)H(\mathcal{S})$ bits of classical correlations are shared between $\mathcal{S}$ and R_δ subsystems ε_i, then [56]

$$\bar{D}(\mathcal{S}:\check{\mathcal{F}}_k)\leq[1-R_\delta(1-\delta)/N]H(\mathcal{S}), k\leq N/2, \quad (10)$$

where the average is computed over all $\mathcal{F}_k$'s. When classical information is redundantly broadcast ($R_\delta\approx N,\delta\approx 0$), only large fragments ($k>N/2$) display any traces of quantum correlations with $\mathcal{S}$.

Recent works discovered bounds on entanglement sharing for generic $\mathcal{SE}$ dynamics [33–35]. Specifically, a state $\rho_{\mathcal{S}\varepsilon_i}=\mathrm{tr}_{\mathcal{E}/i}\{\mathbf{U}_{\mathcal{SE}}|\psi\rangle_{\mathcal{SE}}\}$ is always close to a separable state, displaying zero discord in the ideal limit $N\to\infty$. Here, we have obtained exact, physically meaningful bounds on quantum correlations assuming a realistic, finite environment. In particular, when observers accessing different ε_i's agree with each other (small δ), quantum information is inaccessible, while classical information can spread into the environment. This is how consensus that defines classical reality emerges from a quantum substrate, as we discuss in the following [36].

Significance of the bounds within quantum Darwinism.—A physical state is *classically objective* if independent observers agree about its properties. *Quantum Darwinism* describes the origin of classical objectivity within quantum theory [22,36]. Different observers access information about a system $\mathcal{S}$ by eavesdropping on different parts of the environment (Fig. 2). Because of decoherence, only information about "pointer observables" $\{\hat{\mathbf{M}}\coloneqq|\hat{\alpha}\rangle\langle\hat{\alpha}|\}$ is communicated through the environment [64], e.g., by photons that interact with a central system and then carry information about it. Such scattered light [59,65] is then intercepted by rod cells or artificial photoreceptors. Crucially, only classical information survives decoherence and becomes available to observers [22,36]. The statement is formalized by a characteristic scaling of classical correlations:

$$J(\check{\mathcal{S}}:\mathcal{F}_k)\geq(1-\delta)H(\mathcal{S}),\quad\forall\ \mathcal{F}_k,k\geq k_\delta, \quad (11)$$

in which $k_\delta\ll N$ is determined by the information deficit δ [22,24–26,36]. That is, any fragment $\mathcal{F}_k$ carries the same large amount of classical information about $\mathcal{S}$. However, we have recently established that quantum Darwinism is better formalized by the scaling of classical correlations with respect to measurements on $\mathcal{F}_k$ [58],

$$J(\mathcal{S}:\check{\mathcal{F}}_k)\geq(1-\delta)H(\mathcal{S}),\quad\forall\ \mathcal{F}_k,k\geq k_\delta. \quad (12)$$

We stress that $J(\mathcal{S}:\check{\mathcal{F}}_k)$ is the maximal information about $\mathcal{S}$ one extracts by measuring on $\mathcal{F}_k$ [66].

Our result [Eq. (7)] corroborates quantum Darwinism's central tenet. Recognizing the information deficit δ as a measure of (lack of) classical objectivity elucidates how redundancy of classical information suppresses quantum correlations. In particular, for pure states, quantum Darwinism [Eq. (12)] implies [56]

$$D(\mathcal{S}:\check{\mathcal{F}}_k)\leq 2\delta H(\mathcal{S}),\quad\forall\ \mathcal{F}_k,k\in[k_\delta,N-k_\delta], \quad (13)$$

certifying that quantum information is not concurrently accessible to multiple independent observers.

Further, we prove that Eq. (12), and therefore our bound on quantum discord, signify uniqueness of the pointer observable [56].

Result 3. For any disjoint fragments $\mathcal{F}_k$, $\mathcal{F}_l$ and any state $\mathbf{U}_{\mathcal{SE}_{/k+l}}\mathbf{V}_{\mathcal{SF}_l}\mathbf{W}_{\mathcal{SF}_k}|\psi\rangle_{\mathcal{S}}|\phi\rangle_{\mathcal{F}_k}|\varphi\rangle_{\mathcal{F}_l}|\chi\rangle_{\mathcal{E}_{/k+l}}$, $k,l\geq k_\delta$, if Eq. (12) holds, then

$$(1-2\delta)H(\mathcal{S}) \le I(\mathcal{F}_{k,\hat{\mathbf{M}}_k}:\mathcal{F}_{l,\hat{\mathbf{M}}_l}),$$
$$I(\mathcal{F}_{k,\hat{\mathbf{M}}_k}:\mathcal{F}_{l,\hat{\mathbf{M}}_l}) \le \begin{cases} (1+\delta)H(\mathcal{S}), \text{ if } \Delta_{\mathbf{U}_{\mathcal{S}\mathcal{E}_{/k+l}}\mathbf{V}_{\mathcal{S}\mathcal{F}_l}}H(\mathcal{S}) \ge 0, \\ \log_2 d_{\mathcal{S}} + \delta H(\mathcal{S}), \text{ otherwise,} \end{cases} \tag{14}$$

where $\Delta_{\mathbf{U}_{\mathcal{S}\mathcal{E}_{/k+l}}\mathbf{V}_{\mathcal{S}\mathcal{F}_l}}H(\mathcal{S})$ is the entropy variation due to $\mathbf{U}_{\mathcal{S}\mathcal{E}_{/k+l}}\mathbf{V}_{\mathcal{S}\mathcal{F}_l}$. The lower limit holds, in fact, for any pure state $|\psi\rangle_{\mathcal{S}\mathcal{E}}$, while the restriction on the dynamics is necessary to establish the upper bound, as it ensures that classical correlations between fragments are strictly information about $\mathcal{S}$. These stringent bounds show that the maximally informative observables in disjoint fragments are highly correlated, $\hat{\mathbf{M}}_k \approx \hat{\mathbf{M}}_l \approx \hat{\mathbf{M}}$. When $\mathcal{S}$ and small fragments $\mathcal{F}_k, k \ge k_\delta$, already share maximal classical correlations, the maximally informative measurement for any observer is inevitably the projection on the pointer basis $\{\hat{\alpha}\}$. While similar statements were proven for the ideal case of $\delta = 0$ [36–40], the generalization of the result, as suggested by model-dependent studies [37], allows for verifying quantum Darwinism in realistic, imperfect ($\delta \neq 0$) scenarios.

We observe that Eq. (12) holds when $\mathcal{S}$ and *all* fragments of a certain size $k \ge k_\delta$ share a certain amount of classical correlations. The criterion can be relaxed by replacing $J(\mathcal{S}:\check{\mathcal{F}}_k)$ with its average value over all $\mathcal{F}_k$. Under this less strong condition, bounds like Eq. (14) exist for the average $I(\mathcal{F}_{k,\hat{\mathbf{M}}_k}:\mathcal{F}_{l,\hat{\mathbf{M}}_l})$. Also, adopting Eq. (11) as quantum Darwinism signature is justifiable *a posteriori*. The quantity $J(\check{\mathcal{S}}:\mathcal{F}_k)$ displays the same scaling with k of $J(\mathcal{S}:\check{\mathcal{F}}_k)$ [and $I(\mathcal{S}:\mathcal{F}_k)$] in the widely applicable "c-maybe" model in Fig. 3 [58].

Finally, we show how to certify the emergence of classical objectivity when the universe is in a certain state $|\psi\rangle_{\mathcal{S}\mathcal{E}}$. Verifying Eq. (12) is computationally hard, requiring an optimization over all possible measurements on $\mathcal{F}_k$ [10,11,13,41,67]. The problem is bypassed by linking quantum Darwinism to the scaling of an analytical function. Consider the conditional mutual information $I(\mathcal{S}:\mathcal{F}_l|\mathcal{F}_k) \coloneqq I(\mathcal{S}:\mathcal{F}_{k+l}) - I(\mathcal{S}:\mathcal{F}_k)$, which is the supplemental information one acquires about $\mathcal{S}$ by enlarging the monitored fragment [42,68]. If and only if such information is vanishing, then independent observers access maximal classical information about $\mathcal{S}$ [56].

Result 4. For any state $|\psi\rangle_{\mathcal{S}\mathcal{E}}$, given $k_\delta \le N/2$,

$$\begin{aligned} J(\mathcal{S}:\check{\mathcal{F}}_k) = H(\mathcal{S}), \quad &\forall\ \mathcal{F}_k, \quad k \ge k_\delta \\ \Rightarrow I(\mathcal{S}:\mathcal{F}_l|\mathcal{F}_k) = 0, \quad &\forall\ \mathcal{F}_k, \mathcal{F}_l, \quad k \ge k_\delta, \quad k+l \le N-k_\delta \\ \Rightarrow J(\mathcal{S}:\check{\mathcal{F}}_k) = H(\mathcal{S}), \quad &\forall\ \mathcal{F}_k, \quad k \ge 2k_\delta. \end{aligned} \tag{15}$$

Therefore, the quantum Darwinism condition [Eq. (12)] can be verified, in the strongest form ($\delta = 0$), without explicit calculation of classical and quantum correlations. A more general one-way implication reads [56]

$$\begin{aligned} J(\mathcal{S}:\check{\mathcal{F}}_k) \ge (1-\delta)H(\mathcal{S}), \quad &\forall\ \mathcal{F}_k, \quad \forall\ k \ge k_\delta \\ \Rightarrow I(\mathcal{S}:\mathcal{F}_l|\mathcal{F}_k) \le 2\delta H(\mathcal{S}), \quad &\forall\ \mathcal{F}_k, \mathcal{F}_l, \quad k \ge k_\delta, \quad k+l \le N-k_\delta. \end{aligned} \tag{16}$$

Redundancy of classical information allows the mutual information to increase rapidly only for $k > N - k_\delta$. Quantum correlations, and therefore quantum information about $\mathcal{S}$, significantly build up only in large fragments.

Conclusion.—We have established universal, quantitative bounds on quantum correlations in multipartite systems. Independent observers can simultaneously access classical information about a quantum system that redundantly spreads into the environment, but quantum information is out of reach. Hence, classical reality is not only consistent with quantum laws, but an emergent byproduct of decoherence and quantum Darwinism. We conjecture that stronger bounds might exist when the environment state is mixed [69,70], and for multipartite correlations [42,71]. Also, the analytical witness of quantum Darwinism may enable its experimental verification in large dimensional systems.

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