PHYSICAL REVIEW LETTERS **128,** 252001 (2022)

Third-Order Fiducial Predictions for Drell-Yan Production at the LHC

Xuan Chen,[1,2] Thomas Gehrmann,[3] Nigel Glover,[4] Alexander Huss,[5] Pier Francesco Monni,[5]
Emanuele Re,[6,7] Luca Rottoli,[3] and Paolo Torrielli[8]
[1]*Institute for Theoretical Physics, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany*
[2]*Institute for Astroparticle Physics, Karlsruhe Institute of Technology, 76344 Eggenstein-Leopoldshafen, Germany*
[3]*Department of Physics, University of Zürich, CH-8057 Zürich, Switzerland*
[4]*Institute for Particle Physics Phenomenology, Physics Department, Durham University, Durham DH1 3LE, United Kingdom*
[5]*CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*
[6]*Dipartimento di Fisica G. Occhialini, U2, Università degli Studi di Milano-Bicocca and INFN, Sezione di Milano-Bicocca, Piazza della Scienza, 3, 20126 Milano, Italy*
[7]*LAPTh, Université Grenoble Alpes, Université Savoie Mont Blanc, CNRS, F-74940 Annecy, France*
[8]*Dipartimento di Fisica and Arnold-Regge Center, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125, Turin, Italy*

(Received 4 March 2022; revised 26 April 2022; accepted 23 May 2022; published 21 June 2022)

The Drell-Yan process at hadron colliders is a fundamental benchmark for the study of strong interactions and the extraction of electroweak parameters. The outstanding precision of the LHC demands very accurate theoretical predictions with a full account of fiducial experimental cuts. In this Letter we present a state-of-the-art calculation of the fiducial cross section and of differential distributions for this process at third order in the strict fixed-order expansion in the strong coupling, as well as including the all-order resummation of logarithmic corrections. Together with these results, we present a detailed study of the subtraction technique used to carry out the calculation for different sets of experimental cuts, as well as of the sensitivity of the fiducial cross section to infrared physics. We find that residual theory uncertainties are reduced to the percent level and that the robustness of the predictions can be improved by a suitable adjustment of fiducial cuts.

DOI: 10.1103/PhysRevLett.128.252001

Introduction.—The fine understanding of quantum chromodynamics (QCD) demanded by the physics programme of the Large Hadron Collider (LHC) has led to the impressive development of new computational techniques to achieve precise predictions for hadronic scattering reactions. Among these, the production of a lepton pair (the Drell-Yan process) [1] arguably constitutes the most important standard candle at hadron colliders. The precise data collected at the LHC enable a broad spectrum of high-profile applications to different areas of particle physics, such as the extraction of standard model (SM) parameters [2–6] and of the parton densities of the proton [7], and the exploration of beyond the standard model scenarios [8,9]. At present, the theoretical description of this important reaction reaches the highest-yet level of perturbative accuracy. Fixed-order perturbative predictions in QCD, obtained as an expansion in the strong coupling α_s, are known up to third order beyond the Born approximation, i.e., next-to-next-to-next-to-leading order (N^{3}LO), for the Drell-Yan cross section and rapidity distribution calculated inclusively over the phase space of QCD radiation [10–13]. Moreover, next-to-next-to-leading order (NNLO) corrections for the production of a Drell-Yan pair in association with one QCD jet have been computed in Refs. [14–23]. Similarly, electroweak (EW) corrections are known up to next-to-leading order (NLO) [24–33] and mixed QCD EW at NNLO [34–45]. The description of kinematical distributions sensitive to the emission of soft and/or collinear QCD radiation features large logarithms of the transverse momentum of the Drell-Yan pair. The presence of such logarithmic-enhanced terms at all orders in perturbation theory spoils a fixed-order description and demands in addition the resummation of radiative corrections at all orders in the strong coupling [46–52]. Currently, such calculations have been performed up to next-to-next-to-next-to-leading logarithmic (N^{3}LL) accuracy [53–57], also including the analytic constant terms up to $\mathcal{O}(\alpha_s^3)$ in Refs. [58–60], enabled by the perturbative ingredients from Refs. [61–77]. Additional sources of logarithmic corrections have been considered [78–84], and found to have a moderate numerical impact. The modeling of effects beyond collinear factorization, relevant for low-mass Drell-Yan production, has also been studied (see, e.g.,

---

Refs. [85–90]). Finally, the high accuracy of the experimental measurements of this process makes it an ideal laboratory for the development of state-of-the-art event generators [91–96].

Despite this outstanding progress, the accurate description of experimental data is challenged by the presence of fiducial selection cuts in the measurements, whose inclusion in theoretical calculations can potentially compromise the stability of the perturbative expansion [97–100]. An initial estimate of the N^{3}LO Drell-Yan cross section with an account of experimental cuts was presented in Refs. [58,101] using the q_T-subtraction formalism [102], albeit without a complete assessment of the theoretical and methodological uncertainties. The conclusions of the above study are discussed in detail in Supplemental Material to this Letter [103].

In this Letter, we present state-of-the-art predictions both for the fiducial Drell-Yan cross section and for differential distributions of the final-state leptons. We exploit this calculation to carry out, for the first time, a thorough study of the robustness of these theory predictions in the presence of different sets of fiducial cuts. We also present a detailed analysis of the reliability of the computational method adopted, and show that reaching a robust control over the involved systematic uncertainties requires an excellent stability of the numerical calculation in deep infrared kinematic regimes.

Methodology.—The starting point of our calculation for the production cross section $d\sigma_{\rm DY}$ of a Drell-Yan lepton pair, differential in its phase space and in the pair's transverse momentum $p_T^{\ell\ell}$, is the formula

$$d\sigma_{\rm DY}^{\rm N^3LO+N^3LL} \equiv d\sigma_{\rm DY}^{\rm N^3LL} + d\sigma_{\rm DY+jet}^{\rm NNLO} - [d\sigma_{\rm DY}^{\rm N^3LL}]_{\mathcal{O}(\alpha_s^3)}, \qquad (1)$$

where $d\sigma_{\rm DY}^{\rm N^3LL}$ represents the N^{3}LL resummed $p_T^{\ell\ell}$ distribution obtained in Ref. [59] with the computer code RadISH [52,104,105], including the analytic constant terms up to $\mathcal{O}(\alpha_s^3)$; the quantity $[d\sigma_{\rm DY}^{\rm N^3LL}]_{\mathcal{O}(\alpha_s^3)}$ is its expansion up to third order in α_s, and $d\sigma_{\rm DY+jet}^{\rm NNLO}$ is the differential $p_T^{\ell\ell}$ distribution at NNLO [i.e., $\mathcal{O}(\alpha_s^3)$], obtained with the NNLOJET code [15,19,20]. Equation (1) is finite in the limit $p_T^{\ell\ell} \to 0$: by integrating it inclusively over $p_T^{\ell\ell}$ one can obtain predictions differential in the leptonic phase space at N^{3}LO + N^{3}LL perturbative accuracy, allowing for the inclusion of fiducial cuts. An important challenge in the evaluation of the integral of Eq. (1) over $p_T^{\ell\ell}$ is given by the fact that both $d\sigma_{\rm DY+jet}^{\rm NNLO}$ and $[d\sigma_{\rm DY}^{\rm N^3LL}]_{\mathcal{O}(\alpha_s^3)}$ diverge logarithmically in the limit $p_T^{\ell\ell} \to 0$, and only their difference is finite since the large logarithmically divergent terms present in $d\sigma_{\rm DY+jet}^{\rm NNLO}$ are exactly matched by those contained in $[d\sigma_{\rm DY}^{\rm N^3LL}]_{\mathcal{O}(\alpha_s^3)}$. Guaranteeing the cancellation of such divergences requires high numerical precision in the NNLO distribution $d\sigma_{\rm DY+jet}^{\rm NNLO}$ down to very small values of $p_T^{\ell\ell}$. Setting $d\sigma_{\rm DY+jet}^{\rm NNLO} - [d\sigma_{\rm DY}^{\rm N^3LL}]_{\mathcal{O}(\alpha_s^3)} = 0$ for $p_T^{\ell\ell} \leq p_T^{\rm cut}$ introduces a slicing error of order $\mathcal{O}((p_T^{\rm cut}/m_{\ell\ell})^n)$. If one integrates inclusively over the leptonic phase space one has $n = 2$, while the presence of fiducial cuts in general leads to the appearance of linear terms with $n = 1$ [100,106–108]. Starting from order ${\alpha_s}^2$, the corrections are further enhanced by logarithms of $p_T^{\rm cut}$. The presence of these corrections introduces a systematic uncertainty which can be controlled by reducing the value of $p_T^{\rm cut}$ to a sufficiently small value. This procedure is computationally demanding especially in the presence of linear corrections, due to the smaller value of $p_T^{\rm cut}$ required to achieve the independence of the results of the slicing parameter. Such linear corrections can be resummed at all orders in Eq. (1) [56] by applying a simple recoil prescription [109] to $d\sigma_{\rm DY}^{\rm N^3LL}$, and their inclusion would in principle allow for a larger $p_T^{\rm cut}$ in the calculation. These effects are accounted for in Eq. (1), as discussed in Ref. [59]. As a consequence, our N^{3}LO + N^{3}LL fiducial predictions obtained by integrating Eq. (1) are only affected by a slicing error of order $\mathcal{O}((p_T^{\rm cut}/m_{\ell\ell})^2)$.

The perturbative expansion of the N^{3}LO + N^{3}LL fiducial cross section to third order in α_s leads to the N^{3}LO prediction as obtained according to the q_T-subtraction formalism [102]. In this case, the outlined procedure to include linear power corrections below $p_T^{\rm cut}$ in the N^{3}LO computation is analogous to that of Refs. [101,110]. Since the fiducial cross section can be computed up to NNLO using the NNLOJET code, which implements a subtraction technique [111,112] that does not require the introduction of a slicing parameter, in the fixed-order results quoted in this Letter we apply the above procedure only to the computation of the N^{3}LO correction, while retaining the $p_T^{\rm cut}$-independent result up to NNLO. This effectively suppresses the slicing error in our fiducial N^{3}LO cross section to $\mathcal{O}(\alpha_s^3(p_T^{\rm cut}/m_{\ell\ell})^2)$.

In general, the presence of linear fiducial power corrections indicates an arguably undesirable sensitivity of the fiducial cross section to the infrared region in which QCD radiation has small transverse momentum, which compromises the stability of the perturbative series [100]. These issues can be avoided by modifying the definition of the fiducial cuts in such a way that the scaling of the power corrections be quadratic across most of the leptonic phase space. In the following we present a calculation of Eq. (1) and of the fiducial cross section both for the standard (*symmetric*) cuts adopted by LHC experiments [113,114], where the same cut is imposed on transverse momentum of the final state leptons, as well as for the modified (*product*) cuts proposed in Ref. [100], where a cut is instead imposed on the product of the transverse momenta of the final state leptons. This state-of-the-art calculation allows us to assess precisely the effect of different types of fiducial cuts on the

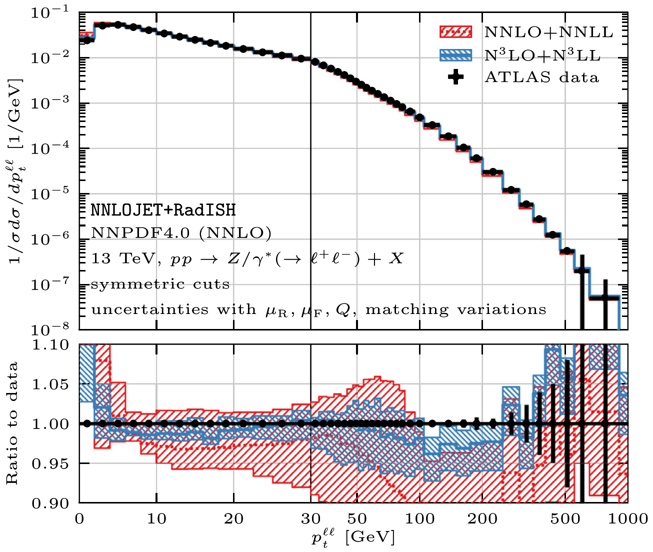

FIG. 1. Fiducial $p_T^{\ell\ell}$ distribution at N^{3}LO + N^{3}LL (blue, solid) and NNLO + NNLL (red, dotted) compared to ATLAS data from Ref. [113]. The binning is linear up to 30 GeV and logarithmic above.

theoretical prediction for the cross section, as well as on the performance of the computational approach adopted here.

Results.—We consider proton-proton collisions at a center-of-mass energy $\sqrt{s} = 13$ TeV. We adopt the NNPDF4.0 parton densities [115] at NNLO with $\alpha_s(M_Z) = 0.118$, whose scale evolution is performed with LHAPDF [116] and HOPPET [117], correctly accounting for heavy quark thresholds. We adopt the G_μ scheme with the following EW parameters taken from the particle data group [118]: $M_Z = 91.1876$ GeV, $M_W = 80.379$ GeV, $\Gamma_Z = 2.4952$ GeV, $\Gamma_W = 2.085$ GeV, and $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}. We consider two fiducial volumes, in both of which the leptonic invariant-mass window is 66 GeV $< m_{\ell\ell} <$ 116 GeV and the lepton rapidities are confined to $|\eta^{\ell^\pm}| < 2.5$. The transverse momentum of the two leptons is constrained as

$$\text{Symmetric cuts [113]}: \ |\vec{p}_T^{\,\ell^\pm}| > 27\ \text{GeV}, \tag{2a}$$

$$\text{Product cuts [100]}: \ \sqrt{|\vec{p}_T^{\,\ell^+}||\vec{p}_T^{\,\ell^-}|} > 27\ \text{GeV}, \quad \min\{|\vec{p}_T^{\,\ell^\pm}|\} > 20\ \text{GeV}. \tag{2b}$$

The central factorization and renormalization scales are chosen to be $\mu_R = \mu_F = \sqrt{m_{\ell\ell}{}^2 + p_T^{\ell\ell 2}}$ and the central resummation scale is set to $Q = m_{\ell\ell}/2$. In the results presented below, the theoretical uncertainty is estimated by varying the μ_R and μ_F scales by a factor of 2 about their central value, while keeping $1/2 \leq \mu_R/\mu_F \leq 2$. In addition, for the resummed results, for central $\mu_R = \mu_F$ scales we vary Q by a factor of 2 around its central value. Moreover, a matching-scheme uncertainty is estimated by including the full scale variation of the additive matching scheme of Ref. [59] [[27] variations that comprise the one of the central matching scale v_0 introduced in Eq. (5.2) of that article]. The final uncertainty is obtained as the envelope of all the above variations, corresponding to 7 and 36 curves for the fixed-order and resummed computations, respectively. We present results for the central member of the NNPDF4.0 set. In the fiducial cross sections quoted below at N^{3}LO and N^{3}LO + N^{3}LL, we do not consider the uncertainty related to the missing N^{3}LO parton distributions, which are currently unavailable.

In Fig. 1, we start by showing the transverse-momentum distribution of the Drell-Yan lepton pair in the fiducial volume Eq. (2a), obtained with Eq. (1), compared to experimental data [113]. In the figure we label the distributions by the perturbative accuracy of their inclusive integral over $p_T^{\ell\ell}$. Our state-of-the-art N^{3}LO + N^{3}LL prediction provides an excellent description of the data across the spectrum, with the exception of the first bin at small $p_T^{\ell\ell}$ which is susceptible to nonperturbative corrections not included in our calculation. We point out that the term $d\sigma_{\text{DY+jet}}^{\text{NNLO}} - [d\sigma_{\text{DY}}^{\text{N}^3\text{LL}}]_{\mathcal{O}(\alpha_s^3)}$ in Eq. (1) gives a non-negligible contribution even for $p_T^{\ell\ell} \leq 15$ GeV. The residual theoretical uncertainty in the intermediate $p_T^{\ell\ell}$ region is at the few-percent level, and it increases to about 5% for $p_T^{\ell\ell} \gtrsim 50$ GeV. A more accurate description of the large-$p_T^{\ell\ell}$ region requires the inclusion of EW corrections, which we neglect in our calculation.

We now consider the fiducial cross section with symmetric cuts. In order to gain control over the slicing systematic error, we choose p_T^{cut} as low as 0.81 GeV. In the first column of Table I, denoted as N^{k}LO, we show the fixed-order results to $\mathcal{O}(\alpha_s^k)$. The second column of Table I displays the result obtained including resummation effects. In the fixed-order case, the theoretical uncertainty at N^{3}LO, estimated as discussed above, is supplemented with an estimate of the slicing uncertainty obtained by varying p_T^{cut} in the range [0.45,1.48] GeV and taking the average difference from the result with $p_T^{\text{cut}} = 0.81$ GeV. In the resummed case, we quote the total theoretical uncertainty including also the matching scheme variation. In both cases the statistical uncertainty is reported in parentheses.

We observe that the new N^{3}LO corrections decrease the fiducial cross section by about 2.5%, and the final prediction at N^{3}LO has larger theoretical errors than the NNLO counterpart, whose uncertainty band does not capture the N^{3}LO central value. This indicates a poor convergence of the fixed-order perturbative series for this process, which is consistent with what has been observed in the inclusive case in Refs. [10–12]. In the resummed case, the theoretical uncertainty is more reliable and within errors the convergence of the perturbative series is improved. The presence of linear power corrections is also responsible

TABLE I. Fiducial cross sections for the symmetric Eq. (2a) and product Eq. (2b) cuts both at fixed perturbative order and including all-order resummation. We report the theoretical uncertainty in percent and, in parentheses, the absolute value of the statistical uncertainty. The latter applies to the last significant figures displayed. At N^{3}LO we also separately indicate the slicing error, in absolute value. See the main text for details.

Order	σ (pb) Symmetric cuts		σ (pb) Product cuts	
k	N^{k}LO	N^{k}LO + N^{k}LL	N^{k}LO	N^{k}LO + N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	...	$721.16^{+12.2\%}_{-13.2\%}$	...
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

for the moderate difference between the fixed-order and the resummed prediction for the symmetric cuts, which as previously discussed indicates a sensitivity of the cross section to the infrared region of small $p_T^{\ell\ell}$. This ultimately worsens further the perturbative convergence of the fixed-order series thereby challenging the perspectives to reach percent-accurate theoretical predictions within symmetric cuts.

A possible solution to this problem [100] is to slightly modify the definition of the fiducial cuts as in Eq. (2b) in order to reduce such a sensitivity to infrared physics. We present for the first time theoretical predictions up to N^{3}LO and N^{3}LO + N^{3}LL for this set of cuts, reported in the third and fourth column of Table I. The relative difference between the fixed-order and resummed calculations for the fiducial cross section never exceeds 0.04%, which indicates that the predictions with product cuts can be computed accurately with fixed-order perturbation theory. Nevertheless, we still observe a more reliable estimate of the theoretical uncertainties when resummation is included.

In order to study the stability of our predictions against variations of the infrared parameter p_T^{cut}, in Fig. 2 we show the dependence of the N^{k}LO correction [i.e., the $\mathcal{O}(\alpha_s^k)$ term in the expansion of the fiducial cross section] on p_T^{cut} down to $p_T^{\text{cut}} \simeq 0.4$ GeV. In the case of symmetric cuts Eq. (2a), we observe that the inclusion of the linear power corrections is essential to reach a plateau at small p_T^{cut}, achieving the necessary independence of the result on the slicing parameter. We thus obtain an excellent control over the estimate of the slicing error quoted in Table I. Furthermore, Fig. 2 clearly shows that the omission of such linear corrections leads to an incorrect result for the fiducial cross section computed with the q_T-subtraction method, unless $d\sigma^{\text{NNLO}}_{\text{DY+jet}}$ can be computed precisely down to $p_T^{\text{cut}} \ll 1$ GeV. Conversely, in the case of the product cuts, we observe a much milder dependence of the N^{k}LO correction on p_T^{cut}, and the further inclusion of power corrections does not lead to any visible difference, consistent with the fact that such corrections are quadratic in most of the phase space [100]. As an additional sanity check, we have repeated the test of Fig. 2 for each individual flavour channel contributing to the N^{3}LO Drell-Yan cross section. The results are collected in Supplemental Material [103], together with a discussion on alternative approaches to q_T subtraction employing a fitting procedure [119], and a comparison to the literature [58,101].

Finally, the computation presented in this Letter allows us to obtain, for the first time, N^{3}LO + N^{3}LL predictions

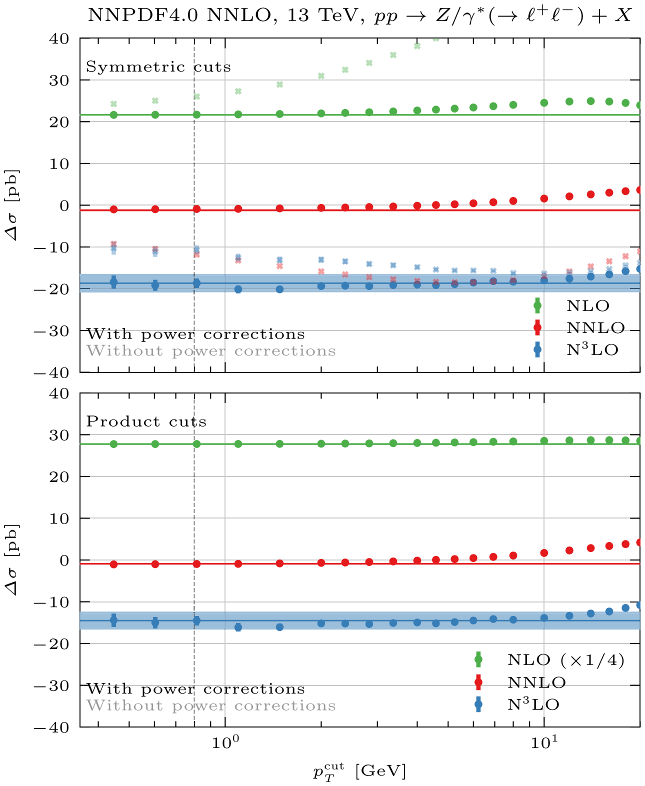

FIG. 2. Dependence of the extracted N^{k}LO corrections to the fiducial cross sections shown in Table I on the p_T^{cut} infrared parameter, both for the symmetric and product cuts. In the latter case, the NLO correction has been rescaled by a factor 1/4. The dashed vertical line indicates our default value $p_T^{\text{cut}} = 0.81$ GeV. The blue band is obtained by combining linearly the statistical and slicing errors.

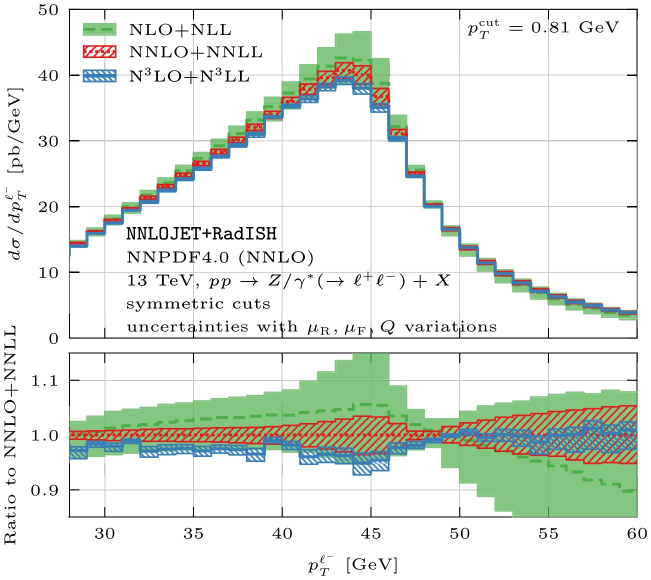

FIG. 3. Lepton transverse momentum distribution up to N^{3}LO + N^{3}LL order in the fiducial phase space Eq. (2a). The labels indicate the order in the fiducial cross section.

for the kinematical distributions of the final-state leptons. A particularly relevant distribution is the leptonic transverse momentum, which plays a central role in the precise extraction of the W-boson mass at the LHC [2,6]. Figure 3 shows the differential distribution of the negatively charged lepton at three different orders, for our default value $p_T^{\rm cut} = 0.81$ GeV. Unlike for the fiducial cross section, the inclusion of $p_T^{\ell\ell}$ resummation in this observable is crucial to cure local (integrable) divergences in the spectrum due to the presence of a Sudakov shoulder [120] at $p_T^{\ell^-} \sim m_{\ell\ell}/2$. The figure shows an excellent convergence of the perturbative prediction, with residual uncertainties at N^{3}LO + N^{3}LL of the order of a few percent across the entire range.

Conclusions.—In this Letter, we have presented state-of-the-art predictions for the fiducial cross section and differential distributions in the Drell-Yan process at the LHC, through both N^{3}LO and N^{3}LO + N^{3}LL in QCD. These new predictions are obtained through the combination of an accurate NNLO calculation for the production of a Drell-Yan pair in association with one jet, and the N^{3}LL resummation of logarithmic corrections arising at small $p_T^{\ell\ell}$. The high quality of these results allowed us to carry out a thorough study of the performance of the computational method adopted, reaching an excellent control over all systematic uncertainties involved. We presented predictions for two different definitions of the fiducial volumes, relying either on symmetric cuts Eq. (2a) on the transverse momentum of the leptons, or on a recently proposed product cuts Eq. (2b) which is shown to improve the stability of the perturbative series. Our results display residual theoretical uncertainties at the $\mathcal{O}(1\%)$ level in the fiducial cross section, and at the few-percent level in differential distributions. These predictions will play an important role in the comparison of experimental data with an accurate theoretical description of the Drell-Yan process at the LHC.

We are grateful to Luca Buonocore, Massimiliano Grazzini, and Gavin Salam for discussions and constructive comments on the manuscript, and to Aude Gehrmann–De Ridder, Tom Morgan, and Duncan Walker for their contributions to the V + jet process in the NNLOJET code. This work has received funding from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Grant No. 396021762-TRR 257, from the Swiss National Science Foundation (SNF) under Contracts No. PZ00P2_201878, No. 200020_188464, and No. 200020_204200, from the U.K. Science and Technology Facilities Council (STFC) through Grant No. ST/T001011/1, from the Italian Ministry of University and Research (MIUR) through Grant No. PRIN 20172LNEEZ, and from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme grant agreement 101019620 (ERC Advanced Grant TOPUP).

[1] S. D. Drell and T.-M. Yan, Phys. Rev. Lett. **25**, 316 (1970); **25**, 902(E) (1970).
[2] M. Aaboud *et al.* (ATLAS Collaboration), Eur. Phys. J. C **78**, 110 (2018); **78**, 898(E) (2018).
[3] R. D. Ball, S. Carrazza, L. Del Debbio, S. Forte, Z. Kassabov, J. Rojo, E. Slade, and M. Ubiali (NNPDF Collaboration), Eur. Phys. J. C **78**, 408 (2018).
[4] V. Bertacchi, S. Roy Chowdhury, L. Bianchini, E. Manca, and G. Rolandi, Eur. Phys. J. C **80**, 328 (2020).
[5] E. Bagnaschi and A. Vicini, Phys. Rev. Lett. **126**, 041801 (2021).
[6] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 01 (2022) 036.
[7] R. Boughezal, A. Guffanti, F. Petriello, and M. Ubiali, J. High Energy Phys. 07 (2017) 130.
[8] A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 07 (2021) 208.
[9] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. Lett. **127**, 141801 (2021).
[10] C. Duhr, F. Dulat, and B. Mistlberger, J. High Energy Phys. 11 (2020) 143.
[11] C. Duhr, F. Dulat, and B. Mistlberger, Phys. Rev. Lett. **125**, 172001 (2020).
[12] C. Duhr and B. Mistlberger, J. High Energy Phys. 03 (2022) 116.
[13] X. Chen, T. Gehrmann, N. Glover, A. Huss, T.-Z. Yang, and H. X. Zhu, Phys. Rev. Lett. **128**, 052001 (2022).
[14] R. Boughezal, C. Focke, X. Liu, and F. Petriello, Phys. Rev. Lett. **115**, 062002 (2015).
[15] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, Phys. Rev. Lett. **117**, 022001 (2016).

[16] R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu, and F. Petriello, Phys. Rev. Lett. **116**, 152001 (2016).

[17] R. Boughezal, X. Liu, and F. Petriello, Phys. Rev. D **94**, 113009 (2016).

[18] R. Boughezal, X. Liu, and F. Petriello, Phys. Rev. D **94**, 074015 (2016).

[19] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, J. High Energy Phys. 07 (2016) 133.

[20] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, J. High Energy Phys. 11 (2016) 094; 10 (2018) 126(E).

[21] R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, and A. Huss, J. High Energy Phys. 11 (2017) 003.

[22] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and D. M. Walker, Phys. Rev. Lett. **120**, 122001 (2018).

[23] R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, I. Majer, and A. Rodriguez Garcia, Phys. Lett. B **829**, 137111 (2022).

[24] S. Dittmaier and M. Krämer, Phys. Rev. D **65**, 073007 (2002).

[25] U. Baur, O. Brein, W. Hollik, C. Schappacher, and D. Wackeroth, Phys. Rev. D **65**, 033007 (2002).

[26] U. Baur and D. Wackeroth, Phys. Rev. D **70**, 073015 (2004).

[27] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, Eur. Phys. J. C **46**, 407 (2006); **50**, 505(E) (2007).

[28] V. A. Zykunov, Phys. Rev. D **75**, 073019 (2007).

[29] V. A. Zykunov, Phys. At. Nucl. **69**, 1522 (2006).

[30] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, J. High Energy Phys. 12 (2006) 016.

[31] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, J. High Energy Phys. 10 (2007) 109.

[32] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, and R. Sadykov, Eur. Phys. J. C **54**, 451 (2008).

[33] S. Dittmaier and M. Huber, J. High Energy Phys. 01 (2010) 060.

[34] S. Dittmaier, A. Huss, and C. Schwinn, Nucl. Phys. **B885**, 318 (2014).

[35] S. Dittmaier, A. Huss, and C. Schwinn, Nucl. Phys. **B904**, 216 (2016).

[36] R. Bonciani, F. Buccioni, R. Mondini, and A. Vicini, Eur. Phys. J. C **77**, 187 (2017).

[37] D. de Florian, M. Der, and I. Fabre, Phys. Rev. D **98**, 094008 (2018).

[38] R. Bonciani, F. Buccioni, N. Rana, I. Triscari, and A. Vicini, Phys. Rev. D **101**, 031301(R) (2020).

[39] M. Delto, M. Jaquier, K. Melnikov, and R. Röntsch, J. High Energy Phys. 01 (2020) 043.

[40] L. Cieri, D. de Florian, M. Der, and J. Mazzitelli, J. High Energy Phys. 09 (2020) 155.

[41] R. Bonciani, F. Buccioni, N. Rana, and A. Vicini, Phys. Rev. Lett. **125**, 232004 (2020).

[42] F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov, and R. Röntsch, Phys. Lett. B **811**, 135969 (2020).

[43] A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov, and R. Röntsch, Phys. Rev. D **103**, 013008 (2021).

[44] L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini, and F. Tramontano, Phys. Rev. D **103**, 114012 (2021).

[45] R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, and A. Vicini, Phys. Rev. Lett. **128**, 012002 (2022).

[46] G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979).

[47] J. C. Collins, D. E. Soper, and G. F. Sterman, Nucl. Phys. **B250**, 199 (1985).

[48] C. Balazs and C. P. Yuan, Phys. Rev. D **56**, 5558 (1997).

[49] S. Catani, D. de Florian, and M. Grazzini, Nucl. Phys. **B596**, 299 (2001).

[50] G. Bozzi, S. Catani, G. Ferrera, D. de Florian, and M. Grazzini, Phys. Lett. B **696**, 207 (2011).

[51] T. Becher and M. Neubert, Eur. Phys. J. C **71**, 1665 (2011).

[52] W. Bizon, P. F. Monni, E. Re, L. Rottoli, and P. Torrielli, J. High Energy Phys. 02 (2018) 108.

[53] W. Bizoń, X. Chen, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and P. Torrielli, J. High Energy Phys. 12 (2018) 132.

[54] T. Becher and M. Hager, Eur. Phys. J. C **79**, 665 (2019).

[55] W. Bizon, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, and D. M. Walker, Eur. Phys. J. C **79**, 868 (2019).

[56] M. A. Ebert, J. K. L. Michel, I. W. Stewart, and F. J. Tackmann, J. High Energy Phys. 04 (2021) 102.

[57] T. Becher and T. Neumann, J. High Energy Phys. 03 (2021) 199.

[58] S. Camarda, L. Cieri, and G. Ferrera, Phys. Rev. D **104**, L111503 (2021).

[59] E. Re, L. Rottoli, and P. Torrielli, J. High Energy Phys. 09 (2021) 108.

[60] W.-L. Ju and M. Schönherr, J. High Energy Phys. 10 (2021) 088.

[61] T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, and C. Studerus, J. High Energy Phys. 06 (2010) 094.

[62] S. Catani, L. Cieri, D. de Florian, G. Ferrera, and M. Grazzini, Eur. Phys. J. C **72**, 2195 (2012).

[63] T. Gehrmann, T. Luebbert, and L. L. Yang, J. High Energy Phys. 06 (2014) 155.

[64] T. Lübbert, J. Oredsson, and M. Stahlhofen, J. High Energy Phys. 03 (2016) 168.

[65] M. G. Echevarria, I. Scimemi, and A. Vladimirov, J. High Energy Phys. 09 (2016) 004.

[66] Y. Li and H. X. Zhu, Phys. Rev. Lett. **118**, 022004 (2017).

[67] A. A. Vladimirov, Phys. Rev. Lett. **118**, 062001 (2017).

[68] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, J. High Energy Phys. 10 (2017) 041.

[69] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, Phys. Lett. B **782**, 627 (2018).

[70] R. N. Lee, A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, J. High Energy Phys. 02 (2019) 172.

[71] J. M. Henn, G. P. Korchemsky, and B. Mistlberger, J. High Energy Phys. 04 (2020) 018.

[72] R. Brüser, A. Grozin, J. M. Henn, and M. Stahlhofen, J. High Energy Phys. 05 (2019) 186.

[73] J. M. Henn, T. Peraro, M. Stahlhofen, and P. Wasser, Phys. Rev. Lett. **122**, 201602 (2019).

[74] A. von Manteuffel, E. Panzer, and R. M. Schabinger, Phys. Rev. Lett. **124**, 162001 (2020).

[75] M.-x. Luo, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, Phys. Rev. Lett. **124**, 092001 (2020).

[76] M. A. Ebert, B. Mistlberger, and G. Vita, J. High Energy Phys. 09 (2020) 146.

[77] M.-x. Luo, T.-Z. Yang, H. X. Zhu, and Y. J. Zhu, J. High Energy Phys. 06 (2021) 115.

[78] E. Laenen, G. F. Sterman, and W. Vogelsang, Phys. Rev. D **63**, 114018 (2001).

[79] S. Berge, P. M. Nadolsky, and F. I. Olness, Phys. Rev. D **73**, 013002 (2006).

[80] S. Marzani, Phys. Rev. D **93**, 054047 (2016).

[81] G. Lustermans, W. J. Waalewijn, and L. Zeune, Phys. Lett. B **762**, 447 (2016).

[82] P. Pietrulewicz, D. Samitz, A. Spiering, and F. J. Tackmann, J. High Energy Phys. 08 (2017) 114.

[83] L. Cieri, G. Ferrera, and G. F. R. Sborlini, J. High Energy Phys. 08 (2018) 165.

[84] T. R. Rabemananjara, J. High Energy Phys. 12 (2020) 073.

[85] I. Scimemi and A. Vladimirov, J. High Energy Phys. 06 (2020) 137.

[86] V. Bertone, I. Scimemi, and A. Vladimirov, J. High Energy Phys. 06 (2019) 028.

[87] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, and M. Radici, J. High Energy Phys. 07 (2020) 117.

[88] F. Hautmann, I. Scimemi, and A. Vladimirov, Phys. Lett. B **806**, 135478 (2020).

[89] A. Bermudez Martinez *et al.*, Eur. Phys. J. C **80**, 598 (2020).

[90] A. B. Martinez, F. Hautmann, and M. L. Mangano, Phys. Lett. B **822**, 136700 (2021).

[91] S. Höche, Y. Li, and S. Prestel, Phys. Rev. D **91**, 074015 (2015).

[92] A. Karlberg, E. Re, and G. Zanderighi, J. High Energy Phys. 09 (2014) 134.

[93] S. Alioli, C. W. Bauer, C. Berggren, F. J. Tackmann, and J. R. Walsh, Phys. Rev. D **92**, 094020 (2015).

[94] P. F. Monni, P. Nason, E. Re, M. Wiesemann, and G. Zanderighi, J. High Energy Phys. 05 (2020) 143.

[95] P. F. Monni, E. Re, and M. Wiesemann, Eur. Phys. J. C **80**, 1075 (2020).

[96] S. Alioli, A. Broggio, A. Gavardi, S. Kallweit, M. A. Lim, R. Nagar, D. Napoletano, C. W. Bauer, and L. Rottoli, Phys. Rev. D **104**, 094020 (2021).

[97] M. Klasen and G. Kramer, Phys. Lett. B **366**, 385 (1996).

[98] B. W. Harris and J. F. Owens, Phys. Rev. D **56**, 4007 (1997).

[99] S. Frixione and G. Ridolfi, Nucl. Phys. **B507**, 315 (1997).

[100] G. P. Salam and E. Slade, J. High Energy Phys. 11 (2021) 220.

[101] S. Camarda, L. Cieri, and G. Ferrera, arXiv:2111.14509.

[102] S. Catani and M. Grazzini, Phys. Rev. Lett. **98**, 222002 (2007).

[103] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.252001 for containing consistency checks and comparison to the literature.

[104] P. F. Monni, E. Re, and P. Torrielli, Phys. Rev. Lett. **116**, 242001 (2016).

[105] P. F. Monni, L. Rottoli, and P. Torrielli, Phys. Rev. Lett. **124**, 252001 (2020).

[106] M. Grazzini, S. Kallweit, and M. Wiesemann, Eur. Phys. J. C **78**, 537 (2018).

[107] M. A. Ebert and F. J. Tackmann, J. High Energy Phys. 03 (2020) 158.

[108] S. Alekhin, A. Kardos, S. Moch, and Z. Trócsányi, Eur. Phys. J. C **81**, 573 (2021).

[109] S. Catani, D. de Florian, G. Ferrera, and M. Grazzini, J. High Energy Phys. 12 (2015) 047.

[110] L. Buonocore, S. Kallweit, L. Rottoli, and M. Wiesemann, Phys. Lett. B **829**, 137118 (2022).

[111] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, J. High Energy Phys. 09 (2005) 056.

[112] J. Currie, E. W. N. Glover, and S. Wells, J. High Energy Phys. 04 (2013) 066.

[113] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **80**, 616 (2020).

[114] A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 12 (2019) 061.

[115] R. D. Ball *et al.*, Eur. Phys. J. C **82**, 428 (2022).

[116] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, Eur. Phys. J. C **75**, 132 (2015).

[117] G. P. Salam and J. Rojo, Comput. Phys. Commun. **180**, 120 (2009).

[118] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).

[119] G. Billis, B. Dehnadi, M. A. Ebert, J. K. L. Michel, and F. J. Tackmann, Phys. Rev. Lett. **127**, 072001 (2021).

[120] S. Catani and B. R. Webber, J. High Energy Phys. 10 (1997) 005.