Quark and Lepton Compositeness: A Renormalizable Model

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(Received 30 December 2021; accepted 13 May 2022; published 15 June 2022)

In the chiral SU(15) gauge theory presented here, the quarks and leptons are bound states ("prebaryons") of massless preons. The standard model charges of the preons imply three generations of quarks and leptons, plus some vectorlike fermions lighter than the confining scale $\Lambda_{\rm pre}$. Under certain assumptions about the chiral dynamics, bound states of two prebaryons behave as Higgs fields. The QCD and electroweak groups may unify above $\Lambda_{\rm pre}$, while SU(15) prevents rapid proton decay.

DOI: 10.1103/PhysRevLett.128.241804

Composite chiral fermions.—Since the nucleons are made of quarks, it is compelling to ask whether quarks might have substructure. If they do, then leptons should also be composite particles, because the quark charges under the standard model (SM) gauge group, $SU(3)_c \times SU(2)_W \times U(1)_Y$, have gauge anomalies canceled by the lepton ones. Developing a self-consistent theory of quark and lepton compositeness is, however, a daunting task. An obstacle is the chiral nature of the SM fermions, which implies that the dynamics responsible for compositeness must also be chiral. Unfortunately, the behavior of strongly coupled chiral theories remains uncertain. Nonetheless, some consistency checks of the possible chiral dynamics have been devised [1–3], pointing to the probable spectrum of light bound states.

Even so, it remains difficult to see how the peculiar pattern of quark and lepton masses could arise from compositeness. If the quarks and leptons are made of massless fermions (traditionally called "preons" [4]) as expected in chiral gauge theories, then the quarks and leptons are likely to remain massless, or at least degenerate in mass.

A model of quark and lepton compositeness, with several realistic features, is presented here. The strong coupling dynamics is based on a preonic SU(15) gauge group. The global symmetry of preons has room for the quantum numbers of a single generation of fermions, but the chiral preonic baryons ("prebaryons") include all three SM generations. Higgs doublets arise as bound states of two prebaryons, reminiscent of the formation of deuteron within QCD. In this model, QCD loses asymptotic freedom near the SU(15) confining scale, $\Lambda_{\rm pre}$. However, the SM gauge groups may unify at a scale above $\Lambda_{\rm pre}$, recovering asymptotic freedom. Interestingly, the unification scale can be lower than usual because the SU(15) symmetry protects against rapid proton decay.

Preonic SU(15) *gauge dynamics.*—The preonic gauge group must be asymptotically free to allow confinement, and the fermion representations must be free of gauge anomalies. It has been observed [2,3,5] that SU(N) gauge theories with a fermion in the symmetric representation and $N+4$ fermions in the fundamental representation likely produce massless chiral baryons. Other gauge groups or preon representations are possible [6,7], but the dynamics is less certain.

The model proposed here has one left-handed fermion, Ω, in the symmetric two-tensor conjugate representation of the preonic $SU(15)_p$ gauge group. That representation has dimension 120, and its anomaly is canceled by 19 left-handed fermions, ψ_i, in the fundamental representation. Thus, the global symmetry, $SU(19) \times U(1)$, is large enough to embed the SM gauge group, as shown in Table I. The ψ_i preons with $i = 5, \ldots, 19$ are relabeled to indicate that they carry the charges of a SM generation.

Assuming the $SU(15)_p$ interactions are confining, the bound states made of three preons are $\psi_i \psi_j \Omega$, with $i \neq j$ and $i, j = 1, \ldots, 19$. These belong to the 171-dimensional antisymmetric representation of global SU(19). An important test that the $SU(15)_p$ dynamics leaves the 171 prebaryons massless is the 't Hooft anomaly matching [1], i.e., the condition that the global anomalies of the preons are equal to those of the prebaryons. Consider the global U(1) under which all ψ_i preons have charge z_ψ, and Ω has charge z_Ω. The $[U(1)]^3$ anomalies of the preons, and of the prebaryons are

$$A_{\rm preon} = 285 z_\psi^3 + 120 z_\Omega^3,$$
$$A_{\psi\psi\Omega} = 171 (2 z_\psi + z_\Omega)^3. \quad (1)$$

TABLE I. Preons charged under the confining gauge group $SU(15)_p$, and their SM charges.

Fermion	$SU(15)_p$	$SU(3)_c \times SU(2)_W$	$U(1)_Y$
ψ_Q	15	(3, 2)	+1/6
ψ_U	15	$(\bar{3}, 1)$	−2/3
ψ_D	15	$(\bar{3}, 1)$	+1/3
ψ_L	15	(1, 2)	−1/2
ψ_E	15	(1, 1)	+1
$\psi_1, \ldots, \psi_4$	$\overline{15}$	(1, 1)	0
Ω	$\overline{120}$	(1, 1)	0

The second Dynkin index for the symmetric tensor is $T_2(\Omega) = 17/2$ [8], so the $[SU(15)_p]^2 U(1)$ anomaly vanishes for $z_\Omega = -19/17 z_\psi$. As a result, $A_{\rm preon} = A_{\psi\psi\Omega} = 171(15 z_\psi/17)^3$. Likewise, the gravitational-U(1) anomalies of the preons, $285 z_\psi + 120 z_\Omega$, and of the prebaryons, $171(2 z_\psi + z_\Omega)$, are equal. This nontrivial matching makes it likely that the $SU(15)_p$ dynamics indeed generates the $\psi_i \psi_j \Omega$ chiral prebaryons. Additional evidence is provided by large-N arguments [3], and by the complementarity between the Higgs and confining phases [2].

The preons shown in Table I lead to the $SU(3)_c \times SU(2)_W \times U(1)_Y$ charges of the prebaryons listed in Table II. These composite quarks and leptons are written as left-handed fermions, using the notation $\psi_U \psi_i \Omega \equiv \Omega_{Ui}$ for $i = 1, \ldots, 4$, $\psi_Q \psi_U \Omega \equiv \Omega_{QU}$, etc. Some prebaryons have different charges even if they are made of the same preons; for example, Ω_{UD} is a color triplet while another prebaryon with the same constituents, labeled Ω'_{UD}, belongs to $\bar{6}$ of $SU(3)_c$. Fermion anticommutation prevents certain representations, e.g., Ω_{UU} cannot be a $\bar{6}$.

TABLE II. Quarks and leptons as prebaryons of $SU(15)_p$. The preon flavor index $i = 1, \ldots, 4$ implies four $\Omega_{Qi} \equiv \psi_Q \psi_i \Omega$ and six $\Omega_{ij} \equiv \psi_i \psi_j \Omega$ prebaryons. The prime on prebaryons such as Ω'_{UD} denotes a higher representation. Besides vectorlike pairs, the composite fermions form three SM generations.

Prebaryons	$SU(3)_c \times SU(2)_W$	$U(1)_Y$
Ω_{Qi}, Ω_{QE}	(3, 2)	+1/6, +7/6
Ω_{DL}, Ω_{UL}	$(\bar{3}, 2)$	−1/6, −7/6
Ω_{Ui}, $\Omega_{\rm DE}$	$(\bar{3}, 1)$	−2/3, +4/3
Ω_{DD}, Ω_{UU}	(3, 1)	+2/3, −4/3
Ω_{Di}, Ω_{UE}	$(\bar{3}, 1)$	+1/3
$\Omega_{UD} + \Omega'_{UD}$	$(3, 1) + (\bar{6}, 1)$	−1/3
$\Omega_{QQ} + \Omega'_{QQ}$	$(\bar{3}, 3) + (6, 1)$	+1/3
$\Omega_{QL} + \Omega'_{QL}$	$(3, 1) + (3, 3)$	−1/3
$\Omega_{QU} + \Omega'_{QU}$	$(1, 2) + (8, 2)$	−1/2
$\Omega_{QD} + \Omega'_{QD}$	$(1, 2) + (8, 2)$	+1/2
Ω_{Li}, Ω_{LE}	(1, 2)	−1/2, +1/2
Ω_{Ei}, Ω_{ij}, Ω_{LL}	(1, 1)	+1, 0, −1

There are four prebaryons of the type Ω_{Qi}, which transform in the $(3, 2, +1/6)$ representation of the SM gauge group, and one prebaryon, Ω_{DL}, in the conjugate representation. The latter forms a vectorlike pair with one linear combination of the four Ω_{Qi} prebaryons and acquires a mass, as discussed later on. Thus, there are three SM chiral quark doublets and one vectorlike quark doublet of hypercharge 1/6.

Similarly, there are four Ω_{Ui} prebaryons, which transform as $(\bar{3}, 1, -2/3)$, and one prebaryon, Ω_{DD}, in the conjugate representation. Hence, there is an up-type vectorlike quark, and three chiral fermions identified as the SM up-type weak-singlet quarks. The down-type quark sector includes five prebaryons transforming as $(\bar{3}, 1, +1/3)$, and two prebaryons in the conjugate representation, forming two vectorlike quarks and three SM quark singlets.

In the composite lepton sector, there are two vectorlike and three chiral weak doublets, plus one vectorlike and three chiral fermions transforming as $(1, 1, +1)$. There are also six SM singlet fermions, which may acquire both Dirac and Majorana masses. All other composite fermions are in exotic vectorlike representations. Consequently, the $SU(15)_p$ model implies the existence at low energies of 3 SM generations of quarks and leptons.

Vectorlike fermion spectrum.—The gauge theory presented in Table I produces composite chiral fermions with the same quantum numbers as the SM quarks and leptons. Its composite scalar sector is more difficult to analyze. Since the $SU(15)_p$ dynamics probably preserves the chiral symmetry of the preons, it does not produce pionlike states. The only bosons that can be composed of two fields are vector "premesons," such as $\bar{\psi}_i \gamma^\mu \psi_j$ or $\bar{\Omega} \gamma^\mu \Omega$. The simplest spin-0 premesons are of the type $\bar{\psi}_i \gamma^\mu T^a \psi_j D^\nu \mathcal{G}^a_{\mu\nu}$, where $\mathcal{G}^a_{\mu\nu}$ is the $SU(15)_p$ gauge field strength. All these premesons are expected to have masses of order $\Lambda_{\rm pre}$.

Nonetheless, the low-energy theory may include composite scalars which are not premesons, but rather bound states of two prebaryons. These "diprebaryons" differ from the deuteron in QCD in several ways. While the deuteron is a nonrelativistic nucleus bound mainly by long-range pion exchange, the diprebaryons are relativistic states bound by short-range remnants of the $SU(15)_p$ dynamics. These remnant interactions can be thought of as vector premeson exchanges, although it is more accurate to view the diprebaryons as six-preon states bound by the $SU(15)_p$ gauge field. In the simplified picture of one premeson exchange with vector couplings to prebaryons, the most deeply bound diprebaryons are scalars.

The diprebaryons are more deeply bound in channels where the SM gauge interactions are attractive, with the binding potential roughly given by one-boson exchange [9]:

$$-\frac{1}{2r}(C_3 \alpha_s + C_2 \alpha_2 + C_1 \alpha_Y). \quad (2)$$

The C_k coefficients are determined by the SM charges of the two prebaryons, and α_s, α_2, and α_Y are the $SU(3)_c \times SU(2)_W \times U(1)_Y$ coupling constants at a scale of order $\Lambda_{\rm pre}$. If the sum of bindings due to premesons and SM bosons is large enough, then the composite scalar acquires a negative squared mass and develops a vacuum expectation value (VEV) [10].

Two color-octet prebaryons, Ω'_{QU} and Ω'_{QD}, form a gauge-singlet scalar ϕ_{88} with SM binding coefficients $\{C_3, C_2, C_1\} = \{6, 3/2, 1/2\}$. The large QCD binding in this channel makes it likely that ϕ_{88} develops a VEV, $\langle\phi_{88}\rangle$. Since premeson exchange interactions act at short distance and are strong but nonconfining, the effective low-energy theory includes a large Yukawa coupling between ϕ_{88} and its two constituents: $\phi^\dagger_{88}\Omega'_{QU}\Omega'_{QD}$. Hence, $\langle\phi_{88}\rangle$ induces a Dirac mass for Ω'_{QU} and Ω'_{QD}, which presumably is not much below $\Lambda_{\rm pre}$. This large mass deters the formation of deeply bound scalars involving one color-octet prebaryon and a prebaryon which is not an octet.

Similarly, the weak-triplet prebaryons form a scalar singlet $\phi_{33} \equiv \Omega_{QQ}\Omega'_{QL}$ with slightly weaker SM binding, $\{C_3, C_2, C_1\} = \{8/3, 4, 2/9\}$. The color sextets form a scalar $\phi_{6\bar{6}} \equiv \Omega'_{UD}\Omega'_{QQ}$ with $\{C_3, C_2, C_1\} = \{5, 0, 2/9\}$. As the three composite scalars discussed so far have the largest bindings and break the chiral symmetries of their prebaryon constituents, the vectorlike quarks of SM charges $(8, 2, 1/2)$, $(3, 3, -1/3)$, and $(6, 1, +1/3)$ are the heaviest composite fermions from Table II.

The large-N expansion indicates that theories with strongly coupled but nonconfining attractive interactions have a second order chiral phase transition [11], so the VEVs of composite scalars with weaker binding are suppressed or even 0. It is then reasonable to assume that the only other vectorlike pairs that acquire masses by coupling to the corresponding diprebaryon are the ones that carry color and either are weak doublets or have the largest hypercharge. The corresponding scalars are gauge singlets labeled by $\phi_{7/6}$, $\phi_{1/6}$, and $\phi_{4/3}$, where the index refers to the vectorlike quark hypercharge. These scalars have large Yukawa couplings to their constituents: $\phi^\dagger_{7/6}\Omega_{QE}\Omega_{UL}$, $\phi^\dagger_{1/6}\Omega_{Q4}\Omega_{DL}$, $\phi^\dagger_{4/3}\Omega_{DE}\Omega_{UU}$. Here, Ω_{Q4} represents the relabeled linear combination of the Ω_{Qi}'s that is the vectorlike partner of Ω_{DL}. Because of weaker SM binding, these scalars have smaller VEVs than ϕ_{33} or $\phi_{6\bar{6}}$. Hence, there are three vectorlike quarks of intermediate mass, of SM charges $(3, 2, +7/6)$, $(3, 2, +1/6)$, and $(3, 1, -4/3)$.

The chiral symmetries of the prebaryons are sufficiently broken by the VEVs discussed so far such that all charged vectorlike pairs formed out of prebaryons listed in Table II acquire masses. To see that, consider first the color singlets Ω_{QU} and Ω_{QD}. The interaction of the octets with ϕ_{88}, upon exchanging a gluon, induces a smaller Yukawa coupling $\phi^\dagger_{88}\Omega_{QU}\Omega_{QD}$. The color singlets Ω_{QU} and Ω_{LE} have together the same preon content as $\phi_{7/6}$, and thus a Yukawa coupling $\phi^\dagger_{7/6}\Omega_{QU}\Omega_{LE}$ arises as in Fig. 1. A coupling $\phi^\dagger_{1/6}\Omega_{QD}\Omega_{L4}$ is also induced, so that there are two vectorlike lepton doublets with a 2×2 mass matrix. One mass eigenstate is mostly $\Omega_{QU}\Omega_{QD}$, with a mass probably comparable to that of the doublet vectorlike quarks. The other mass eigenstate of charges $(1, 2, -1/2)$ is much lighter, and up to a small mixing is given by $\Omega_{LE}\Omega_{L4}$. The prebaryons of hypercharge $\pm 1/3$ form two vectorlike quarks with a 2×2 mass matrix, whose entries come from the effective Yukawa couplings

$$\phi^\dagger_{7/6}\Omega_{QL}\Omega_{UE}, \qquad \phi^\dagger_{1/6}\Omega_{D4}\Omega_{QL}, \qquad \phi^\dagger_{4/3}\Omega_{UD}\Omega_{UE}. \qquad (3)$$

At the scale $\Lambda_{\rm pre}$, four-fermion operators are induced among any two prebaryon pairs made of the same preons (e.g., see Fig. 1). The operator $(\Omega_{U4}\Omega_{DD})(\bar{\Omega}_{UD}\bar{\Omega}_{D4})$ generates a one-loop mass for $\Omega_{U4}\Omega_{DD}$ proportional to the $\Omega_{UD}\Omega_{D4}$ mass mixing, which arises from the terms (3). Another four-fermion operator generates a one-loop mass for $\Omega_{E4}\Omega_{LL}$ proportional to the mass of the lightest vectorlike lepton doublet, $\Omega_{LE}\Omega_{L4}$. As a result, the lightest vectorlike quark and charged vectorlike lepton transform as $(3, 1, +2/3)$ and $(1, 1, +1)$.

Current LHC searches [12] cover vectorlike quarks of this type, setting a lower mass limit of 1.3 TeV. The presence of several other composite vectorlike fermions with larger, hierarchical masses indicates that the compositeness scale satisfies $\Lambda_{\rm pre} \gtrsim O(100)$ TeV. Labeling the lightest vectorlike lepton by $\mathcal{E}$, the main production process at the LHC is $q\bar{q} \to Z^\star/\gamma^\star \to \mathcal{E}^+\mathcal{E}^-$. Mixing of $\mathcal{E}$ with τ induces $\mathcal{E}^\pm$ decays to $\tau^\pm Z/h^0$ or $\nu W^\pm$. Dedicated searches for a vectorlike lepton of this type remain to be performed, but estimates [13] indicate a lower mass limit around 150 GeV.

Composite Higgs sector.—The $SU(15)_p$ model analyzed so far has ingredients that may lead to a potentially realistic composite Higgs sector at low energy [14]: up-type vectorlike quarks and strongly coupled four-fermion operators

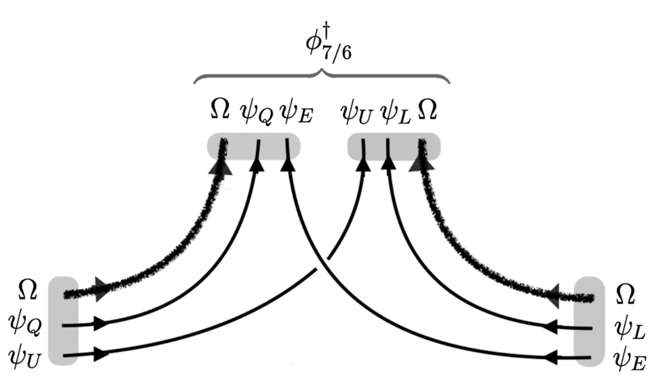

FIG. 1. Effective Yukawa coupling $\phi^\dagger_{7/6}\Omega_{QU}\Omega_{LE}$, originating from a four-prebaryon operator induced by $SU(15)_p$ dynamics.

involving both vectorlike and SM quarks. There are multiple ways in which a composite Higgs sector could be generated, and given the uncertain behavior of chiral gauge theories, it is hard to make rigorous statements. As an existence proof, a simple path toward a viable Higgs sector is sketched here.

The $SU(4)$ flavor symmetry of the ψ_i preons is broken down to $SU(3)$ by the VEV of $\phi_{1/6}$, which is an $\Omega_{Q4}\Omega_{DL}$ bound state. The linear combinations of the Ω_{Qi} prebaryons which are not the vectorlike partner of Ω_{DL} are relabeled as q_L^i, $i = 1, 2, 3$, and identified as the SM left-handed quark doublets. Similarly, u^{ci} are the linear combinations of Ω_{Ui} which remain chiral, and represent the SM up-type quark singlets, taken as left-handed fermions.

To accommodate the known quark masses, the $SU(15)_p$ model must include some interactions that break the $SU(3)$ flavor symmetry. An example is given by a scalar $\mathcal{A}$ that transforms under $SU(15)_p$ as the conjugate of the antisymmetric two tensor, i.e., in the $\overline{105}$ representation. Even for a mass $M_{\mathcal{A}} < \Lambda_{\rm pre}$, the scalar does not affect the confining dynamics of $SU(15)_p$, because the $\mathcal{A}\psi_i\psi_j$ premesons are not lighter than $\Lambda_{\rm pre}$, and the spectrum of chiral prebaryons is not modified.

The most general Yukawa couplings of $\mathcal{A}$, up to a flavor transformation of ψ_i, $i = 1, 2, 3$, can be written as

$$\mathcal{A}\psi_4(\lambda_{44}\psi_4 + \lambda_{43}\psi_3) + \mathcal{A}\sum_{i\geq j=1}^{3}\lambda_{ij}\psi_i\psi_j, \tag{4}$$

where λ_{ij} are dimensionless parameters. Integrating out $\mathcal{A}$ produces four-preon operators, such as $(\psi_4\psi_3)(\bar\psi_4\bar\psi_3)$, which upon a Fierz transformation and to leading order in $1/N$ (here $N = 15$) becomes [15]

$$-\frac{|\lambda_{43}|^2}{M_{\mathcal{A}}^2}(\bar\psi_3\sigma^\mu T^a\psi_3)(\bar\psi_4\sigma_\mu T^a\psi_4). \tag{5}$$

This attractive interaction bolsters the formation of diprebaryons, especially if $|\lambda_{43}| \gtrsim O(1)$ and $M_{\mathcal{A}} \lesssim \Lambda_{\rm pre}$.

The Ω_{U4} component of the lightest vectorlike quark may form a diprebaryon with Ω_{Q3}, due to premeson exchange plus SM binding, which has coefficients $\{8/3, 0, 2/9\}$. The additional attraction provided by $\mathcal{A}$ exchange may be sufficient for the squared mass of this scalar to turn negative. The SM charges of the $\Omega_{U4}\Omega_{Q3}$ scalar are $(1, 2, -1/2)$, so it is appropriate to call it the up-type Higgs doublet, H_u. Several other scalars with the same quantum numbers may form as $\Omega_{Ui}\Omega_{Qj}$ bound states, but their squared masses may be positive and large if $|\lambda_{ij}|^2 \ll |\lambda_{43}|^2$. The H_u doublet has a large Yukawa coupling to q_L^3 and Ω_{U4}. In addition, $\mathcal{A}$ exchange leads to effective Yukawa couplings of the type $H_u^\dagger u^{ci} q_L^j$, with coefficients proportional to $\lambda_{ij}\lambda_{43}^\dagger$. Therefore, a mass matrix for the SM up-type quarks is generated. The observed charm to top quark mass ratio requires $|\lambda_{22}| \ll |\lambda_{33}|$.

Operator (5) together with premeson and SM boson exchanges also produces a scalar $H_d \equiv \Omega_{D4}\Omega_{Q3}$, which induces a mass matrix for the SM down-type quarks. Given that hypercharge is repulsive within H_d, and also the vectorlike quarks of charge $-1/3$ are heavier than the vectorlike quark of charge $2/3$, the H_d VEV is suppressed compared to the H_u VEV. Subleading contributions to the down-type quark mass matrix, such as those arising from the operators $|\phi_{1/6}/\Lambda_{\rm pre}|^2 H_d^\dagger\Omega_{Di}\Omega_{Qj}$, may sufficiently contribute to the strange quark mass to account for the relation between observed quark mass ratios at the TeV scale: $m_s/m_b \approx 5m_c/m_t$ [16]. The subleading contributions also lead to a slight misalignment of the up- and down-type quark mass matrices, resulting in off-diagonal elements of the Cabibbo-Kobayashi-Maskawa matrix.

Four-preon operators mediated by the $\mathcal{A}$ scalar, such as $\lambda_{11}^\dagger\lambda_{22}/M_{\mathcal{A}}^2(\psi_1\psi_1)(\bar\psi_2\bar\psi_2)$, lead to four-quark operators that contribute to flavor-changing processes. Experimental constraints on the four-quark operators are satisfied given that $M_{\mathcal{A}} \sim \Lambda_{\rm pre} \gtrsim O(100)$ TeV and $\lambda_{11} \ll \lambda_{22} \ll 1$.

SM lepton masses arise from effective Higgs Yukawa couplings whose origin is more intricate. Dimension-eight operator $\phi_{7/6}\phi_{88}^\dagger(\psi_D\psi_Q)(\bar\psi_L\bar\psi_E)$, generated by $SU(15)_p$ dynamics at the $\Lambda_{\rm pre}$ scale, produces four-prebaryon operators $(\bar\Omega_{D4}\bar\Omega_{Q3})(\Omega_{L4}\Omega_{E3} + \Omega_{L3}\Omega_{E4})$, $(\bar\Omega_{D3}\bar\Omega_{Q3})(\Omega_{L3}\Omega_{E3})$, and $(\bar\Omega_{D4}\bar\Omega_{Q4})(\Omega_{L4}\Omega_{E4})$, which via $\mathcal{A}$ exchange lead to the τ Yukawa coupling $H_d^\dagger e^{c3}\ell_L^3$. Replacing ϕ_{88} by $\phi_{6\bar6}$ gives additional contributions. Therefore, m_τ is suppressed by $\langle\phi_{7/6}\rangle\langle\phi_{88}\rangle/\Lambda_{\rm pre}^2$ compared to m_b, but this is partially compensated by the multiplicity of contributions. Moreover, a Yukawa coupling of τ to H_u is produced by a $\phi_{7/6}^\dagger\phi_{\rm M}^\dagger(\Omega_{U4}\Omega_{Q3})(\Omega_{L3}\Omega_{E3})$ operator, where $\phi_{\rm M}$ is a singlet scalar discussed below.

The mechanisms for fermion mass generation sketched above rely on the existence of two Higgs doublets. Other composite scalar doublets, such as $\Omega_{Qi}\Omega_{Dj}$ or $\Omega_{L4}\Omega_{E3}$, may also form; even if these have large positive squared masses, they might mediate additional effective couplings of the fermions to H_d and H_u. As the chiral phase transition is of second order, it is difficult to estimate the composite scalar masses.

The attractive interaction (5) and the analogous one proportional to $|\lambda_{44}|^2$ are amplified by a factor of 3 or 4 in the self-interaction of Ω_{43}, leading to a gauge-singlet scalar $\phi_{\rm M}$ with negative squared mass. Its VEV induces a Majorana mass for Ω_{43}, which may be near $\Lambda_{\rm pre}$, and after an $\mathcal{A}$ exchange, gives masses to the other Ω_{ij} prebaryons. These composite gauge-singlet fermions participate in a seesaw mechanism responsible for neutrino masses.

$SO(10)$ *unification.*—The preon fields include 60 color triplets, so the QCD coupling is not asymptotically free above $\Lambda_{\rm pre}$. Even below that, the sextet and octet quarks turn the $SU(3)_c$ β function positive, indicating that the SM group must be embedded in a larger gauge group. It turns out that the preon field content from Table I allows SO(10) unification. Moreover, the SM gauge couplings evolve roughly toward a common value at a scale $\Lambda_{10} > \Lambda_{\rm pre}$, albeit establishing coupling unification is difficult due to nonperturbative effects.

The field content of the theory above Λ_{10} is the following: $SU(15)_p \times SO(10)$ gauge group, with fermions

$$\Psi \in (15,16), \quad \psi_2,\psi_3,\psi_4 \in (15,1), \quad \Omega \in (\overline{120},1), \quad (6)$$

a scalar $\mathcal{A} \in (\overline{105},1)$, and two scalars whose VEVs break SO(10) and transform, e.g., as (1, 16) and (1, 45). Both $SU(15)_p$ and SO(10) are asymptotically free. The prebaryons listed in Table II form an SO(10) representation: $3 \times (16+1) + 120$, where 120 contains all $\Psi\Psi\Omega$ states, and the three 16's contain the $\Psi\psi_{2,3,4}\Omega$ states. Note that an $SU(N)_p$ group leads to three generations only for $N = 15$.

The SM-singlet component of Ψ is ψ_1, so $\lambda_{i1} = 0$ due to the SO(10) symmetry, implying that the first generation fermions are naturally the lightest. First generation masses emerge from dimension-five couplings of $\mathcal{A}\psi_i\Psi$ to the (1, 16) scalar, which arise from renormalizable interactions of a $(15+\overline{15},1)$ fermion heavier than $\Lambda_{\rm pre}$.

Interestingly, proton decay operators are not generated at the unification scale. The dimension-six operator $qqq\ell$ is a four-prebaryon operator $(\Omega_{Qi})^3\Omega_{Li}$, which could arise from the dimension-18 operator $(\psi_Q\psi_i\Omega)^3\psi_L\psi_i\Omega$. The latter is not mediated by SO(10) gauge bosons. Same argument applies to the $u^c u^c d^c e^c$ operator. Thus, Λ_{10} can be substantially below the usual [17] unification scale.

To conclude, the $SU(15)_p \times SO(10)$ gauge theory with the preons shown in (6) produces light prebaryons transforming as three SM generations of fermions. Several composite vectorlike fermions have a hierarchical mass spectrum. A key observation is that a potentially viable Higgs sector arises from bound states of two prebaryons. Further studies of this renormalizable theory of composite quarks and leptons are warranted. Particularly useful would be an improved understanding of strongly coupled chiral gauge theories (e.g., see Refs. [18,19]), and experimental searches for the composite scalars and vectorlike fermions predicted here.

I thank B. Assi, Y. Bai, S. Chivukula, P. Fox, and K. Kong for insightful comments. This work is supported by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy.

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