Black Hole Supertranslations and Hydrodynamic Enstrophy

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We study the relation between approximate horizon symmetries of AdS black branes and approximately conserved currents in their dual hydrodynamic description. We argue that the existence of an approximately conserved enstrophy current unique to $2+1$ dimensional fluid flow implies that AdS_4 black branes possess a special class of approximate supertranslations (which we identify).

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Enstrophy is a scalar quantity associated with nonrelativistic, incompressible fluid flow. In two spatial dimensions it cannot increase in time, and in the absence of dissipation it is conserved. Its properties are key ingredients in generating the unique features of turbulent flows in two spatial dimensions.

The study of enstrophy in relativistic fluids is lagging behind that of its Galilean cousin. It is possible to identify a relativistic enstrophy current which is conserved for an ideal fluid in $2+1$ dimensions [1–3]. Since dissipative effects spoil this property, we will refer to such a current as being approximately conserved.

The existence of a relativistic enstrophy current suggests, via the gauge-gravity duality [4], that AdS_4 black branes possess an approximate symmetry associated with approximate enstrophy conservation. Since the enstrophy current exists only in the hydrodynamic limit, one might expect that an associated approximate black hole symmetry will exist on the event horizon.

Indeed, it has recently been established that stationary black hole geometries are endowed with horizon symmetries, classified as supertranslations and superrotations [5,6] (see Refs. [7–10] for older work on this topic). Generic fluid flows which possess enstrophy are not stationary and imply nonstationary dual black hole configurations. While the definition of supertranslations and superrotations may be extended to nonstationary black holes, they are generally not associated with conserved currents [11].

In this Letter we will relate a subset of supertranslation transformations of AdS_4 black branes to approximate enstrophy current conservation in the dual hydrodynamic description of the field theory. Further, we clarify the role of the remaining generators of supertranslations and identify them with symmetries leading to nonlocal approximately conserved currents. Superrotations do not lead to approximately conserved currents in a dual hydrodynamic setting.

Relations between hydrodynamics and horizon symmetries have been considered in the past. In Ref. [12], the authors analyzed asymptotically AdS black brane solutions dual to a superfluid flow and identified the action of supertranslations on horizon data with the superfluid Goldstone mode. In Ref. [13], horizon symmetries in $3+1$ dimensions were shown to be in one to one correspondence with symmetries of compressible nonrelativistic fluids in $2+1$ dimensions (see also Ref. [14] for a similar relation in a different context). The authors of Ref. [15] related the dynamics of the horizon to Carrollian fluids and the horizon symmetries to Carrollian geometry. While the authors did not discuss this, their results are suggestive of a Carrollian enstrophy current [3] associated with horizon symmetries. Be that as it may, the novelty of the current work is in its explicit identification of (a subset of) horizon symmetries of asymptotically AdS black holes with the symmetries of relativistic fluid dynamics of the boundary theory.

Our exposition starts with a review of the relativistic, conformal, enstrophy current, as first discussed in Ref. [1], and its extensions [2,3]. Using the construction of Ref. [11], we then discuss the general structure of horizon preserving diffeomorphisms. Finally, we use our knowledge of enstrophy conservation in the boundary theory to show that AdS_4 black brane geometries possess special supertranslation symmetries. We end with a discussion and outlook.

The enstrophy current.—The equations of motion of an uncharged, conformal, relativistic fluid are given by

$$\nabla_\mu T^{\mu\nu} = 0, \tag{1}$$

where $T^{\mu\nu}$ is the energy momentum tensor of the fluid and is a local function of the fluid velocity u^μ (satisfying $u^\mu u_\mu = -1$) and fluid temperature T. Working in a derivative expansion we have

$$T^{\mu\nu} = P(T)[(d+1)u^\mu u^\nu + g^{\mu\nu}] + \mathcal{O}(\nabla) \tag{2}$$

where $d > 1$ is the number of spatial dimensions, $P(T) = p_0 T^{d+1}$ is the thermodynamic pressure with p_0 a positive real number, and $\mathcal{O}(\nabla)$ denotes expressions which contain one or more derivatives of the hydrodynamic variables u^μ and T.

For any function g which satisfies

$$u^\mu \nabla_\mu g = \mathcal{O}(\nabla^{[g]+2}), \tag{3}$$

we may construct the current

$$J^\mu_g = g p_0 (d+1) T^d u^\mu \tag{4}$$

which satisfies

$$\nabla_\mu J^\mu_g = \mathcal{O}(\nabla^{[g]+2}). \tag{5}$$

In obtaining Eq. (5) we have used that $\nabla_\mu(T^d u^\mu) = \mathcal{O}(\nabla^2)$ as a result of the equations of motion (1) expanded in derivatives. The particular choice of the overall constant $p_0(d+1)$ will become clear shortly.

A naive power counting argument would suggest that $\nabla_\mu J^\mu_g$ is of the same order as g plus 1 in a derivative expansion. Instead, Eq. (5) implies that it is order g plus 2. In other words, it is conserved at least to leading order in a derivative expansion. In what follows we will refer to J^μ_g as an approximately conserved current.

We often require conserved currents to be local in the hydrodynamic variables, in this case u^μ and T. While there are many nonlocal solutions to Eq. (3), local solutions are more difficult to come by. Clearly, $g = 1$, or any constant for that matter, is a solution to Eq. (3). For such solutions we obtain

$$J^\mu_1 = s u^\mu \tag{6}$$

with $s = (\partial P/\partial T)$ the entropy density. Approximate conservation of J^μ_1 coincides with the leading order equation of motion and implies conservation of entropy in the absence of dissipation.

Another solution to Eq. (3) which is local in the hydrodynamic variables is given by

$$g = \frac{\Omega_{\alpha\beta}\Omega^{\alpha\beta}}{s^2}, \tag{7}$$

with

$$\Omega_{\mu\nu} = \partial_\mu(T u_\nu) - \partial_\nu(T u_\mu), \tag{8}$$

and is valid only in $2+1$ spacetime dimensions. The associated current

$$J^\mu_{\frac{\Omega^2}{s^2}} = \frac{\Omega_{\alpha\beta}\Omega^{\alpha\beta}}{s} u^\mu \tag{9}$$

is the relativistic enstrophy current [1–3], and its associated charge is referred to as enstrophy. Of course, if g solves Eq. (3) so do powers of g, and we find a set of conserved currents,

$$J^\mu_{(\frac{\Omega^2}{s^2})^n} = \left(\frac{\Omega_{\alpha\beta}\Omega^{\alpha\beta}}{s^2}\right)^n s u^\mu. \tag{10}$$

When $n = 0$, the above expression reduces to the entropy current [Eq. (6)]. The currents in Eqs. (6) and (10) comprise the only known local solutions to Eq. (3).

We note in passing that in a nonrelativistic, incompressible fluid the charges $\int (\omega_{ij}\omega^{ij})^n d^2x$, with $\omega_{ij} = \partial_i v_j - \partial_j v_i$ and v_i the velocity field, are conserved in the inviscid limit. The former integral with $n = 1$ is referred to as the total enstrophy. Once dissipative effects are included, the total enstrophy decreases in time. This property, together with energy conservation, leads to an inverse energy cascade in $2+1$ dimensional incompressible nonrelativistic turbulent flows whereby energy is transferred from small to large scales [16]. Whether similar statements can be made for relativistic fluids is yet an open problem.

Horizon symmetries and charges.—An extensive analysis of horizon symmetries and charges was carried out in Ref. [11]. In what follows we summarize the essential ingredients of Ref. [11] required for this Letter. Consider a spacetime $\mathcal{M}$ with metric g_{ab} and event horizon $\mathcal{N}$ whose topology is $\mathcal{Z} \times \mathbb{R}$ with $\mathbb{R}$ a null direction. We refer to $\mathcal{Z}$ as the base space of $\mathcal{N}$. In the case of an asymptotically AdS_{d+2} black brane, $\mathcal{Z}$ has topology $\mathbb{R}^d$. Let us denote the pullback to the horizon by $\Pi_i{}^a$ so that $g_{ij} = \Pi_j{}^b \Pi_j{}^b g_{ab}$ is the induced metric on the event horizon. Since $\mathcal{N}$ is null, g_{ij} is not invertible. We denote the pullback from $\mathcal{N}$ to the base space $\mathcal{Z}$ by $\Pi_A{}^i$ so that the induced metric on $\mathcal{Z}$ is $g_{AB} = \Pi_A{}^a \Pi_B{}^b g_{ab}$ where $\Pi_A{}^a = \Pi_A{}^i \Pi_i{}^a$. Since $\mathcal{Z}$ is spacelike, g_{AB} is invertible. In what follows we will consistently use $a, b, \ldots$ for indices on $\mathcal{M}$, $i, j, \ldots$ for indices on $\mathcal{N}$ and $A, B, \ldots$ for indices on $\mathcal{Z}$. Later, when we will focus on asymptotically AdS spaces, we will introduce indices $\mu, \nu, \ldots$ on the asymptotic boundary of the spacetime.

We denote by ℓ^i a representative vector field generating null geodesics along the null direction $\mathbb{R}$ of $\mathcal{N}$, and by ℓ^a an extension of it to all of $\mathcal{M}$. We define the nonaffinity parameter κ via

$$\ell^a \nabla_a \ell^b|_{\mathcal{N}} = \kappa \ell^b|_{\mathcal{N}}, \tag{11}$$

where, as usual, ∇_a is the covariant derivative on $\mathcal{M}$. We also denote a null cotangent vector on $\mathcal{N}$ by n_i and normalize it and its extension to $\mathcal{M}$ such that $n_a \ell^a = -1$.

We will often need to go back and forth between tangent vectors on $\mathcal{M}$ (or $\mathcal{N}$) and tangent vectors on $\mathcal{N}$ (or $\mathcal{Z}$). For instance, suppose that $v^a \in T(\mathcal{M})$ satisfies $v^a \ell_a|_{\mathcal{N}} = 0$ (recall that ℓ_a is the normal to $\mathcal{N}$). Then, we may always define a unique $v^i \in T(\mathcal{N})$ such that $v^a w_a|_{\mathcal{N}} = v^i \Pi_i{}^a w_a$ for any $w_a \in T^*(\mathcal{M})$. To simplify our notation we will write

$$v^a \partial_a \cong v^i \partial_i. \tag{12}$$

An infinitesimal coordinate transformation $\chi = \chi^i \partial_i$ is referred to as a generator of a horizon preserving diffeomorphism if

$$\pounds_\chi \ell^i = \beta \ell^i, \qquad \pounds_\chi \kappa = (\beta \kappa + \pounds_\ell \beta), \tag{13}$$

with β a function on $\mathcal{N}$. The first equation in Eq. (13) is an infinitesimal version of a rescaling of the null vector $\ell^i \partial_i$ (which does not have a well defined length due to the fact that it is null). The second equation in Eq. (13) corresponds to a shift in the nonaffinity parameter resulting from a rescaling of $\ell^a \partial_a$, cf. Eq. (11).

The generator χ^i can be naturally decomposed into a component parallel to ℓ^i and a component orthogonal to it,

$$\chi^i \partial_i = X^i \partial_i + f \ell^i \partial_i, \tag{14}$$

with $X^i n_i = 0$. With this decomposition Eq. (13) reads as

$$\begin{gathered} \pounds_\ell X^i \propto \ell^i, \\ \pounds_\ell(\pounds_\ell + \kappa) f + X^i \pounds_\ell(\pounds_\ell + \kappa) n_i + \pounds_X \kappa = 0. \end{gathered} \tag{15}$$

It is tempting to refer to a horizon symmetry associated with X^i as a superrotation and to a horizon symmetry associated with $f \ell^i$ as a supertranslation. Note, however, that the distinction between the two is dependent on the choice of n_i. In Ref. [11] it was shown, using an explicit construction, that there exists an n_i for which

$$\pounds_\ell(\pounds_\ell + \kappa) n_i + \partial_i \kappa = 0. \tag{16}$$

With this choice of n_i, Eq. (15) can be shown to reduce to

$$\pounds_\ell(\pounds_\ell + \kappa) f = 0. \tag{17}$$

One may refer to the supertranslations and superrotations obtained using the n_i which leads to Eq. (17) as canonical supertranslations and superrotations.

One of the results of Ref. [11] is that, in the absence of matter, we can associate to each such χ a Wald-Zoupas charge Q_χ, which is conserved whenever the horizon is stationary. Operatively, for each generator χ we define a current q^i_χ satisfying

$$q^j_\chi = (\chi^i \mathcal{K}_i{}^j - \theta \chi^j - \beta \ell^j). \tag{18}$$

Here β is associated with the scaling of ℓ^i under χ^j as in Eq. (13), $\mathcal{K}_i{}^j$ denotes the Weingarten map

$$\mathcal{K}_i{}^j \partial_j \cong \Pi_i{}^a \nabla_a \ell^b \partial_b, \tag{19}$$

and θ is the expansion associated with ℓ^i,

$$\theta = \nabla_i \ell^i = \frac{1}{\sqrt{|g_{AB}|}} \partial_i (\sqrt{|g_{AB}|} \ell^i). \tag{20}$$

It is straightforward though somewhat tedious to compute the divergence of q^i_χ. Recall that the Weingarten map satisfies

$$\ell^i \mathcal{K}_i{}^j = \kappa \ell^j, \qquad \mathcal{K}_i{}^j g_{jk} = \frac{1}{2} \pounds_\ell g_{ik}. \tag{21}$$

The expression on the right-hand side of the second equality is the second fundamental form on $\mathcal{N}$, $K_{ij} = \frac{1}{2} \pounds_\ell g_{ij}$. It is orthogonal to ℓ^i allowing us to write

$$K_{ij} = \frac{\theta}{d} g_{ij} + \Sigma_{ij} \tag{22}$$

where $\Sigma_{ij} dx^i dx^j \cong \Sigma_{AB} dx^A dx^B$ is symmetric and traceless, $\Sigma_{AB} g^{AB} = 0$. Thus, the most general expression for $\mathcal{K}_i{}^j$ satisfying Eq. (21) is

$$\mathcal{K}_i{}^j = \omega_i \ell^j + S_i{}^j, \tag{23}$$

where

$$S_i{}^j dx^i \partial_j \cong \left(\frac{\theta}{d} \delta_A{}^B + \Sigma_A{}^B \right) dx^A \partial_B, \tag{24}$$

and

$$\omega_i = -\kappa n_i dx^i + \Omega_i dx^i \tag{25}$$

is the rotation one form (sometimes also referred to as the extrinsic curvature one form) with $\Omega_i dx^i \cong \Omega_A dx^A$ the normal fundamental form on $\mathcal{Z}$.

Inserting Eq. (14) into Eq. (18), using Eq. (13) to evaluate β in Eq. (18), and also inserting the decomposition of Eq. (23) into Eq. (18), one finds

$$q^i_\chi = q_\chi \ell^i + X^j S_j{}^i - \theta X^i, \tag{26}$$

where

$$q_\chi = (\pounds_l f + kf - \theta f) + X^i(\omega_i + \pounds_l n_i). \tag{27}$$

Taking the divergence of Eq. (26) we find

$$\begin{aligned}\nabla_i q^i_\chi = {}& \pounds_\ell(\pounds_\ell + \kappa) f + X^i \pounds_\ell(\pounds_\ell + \kappa) n_i + \pounds_X \kappa \\ & + X^i(\pounds_\ell \omega_i - \partial_i \kappa) + \nabla_j (X^i \Sigma_i{}^j) - \theta(f(\theta - \kappa) \\ & - X^i(\omega_i + \pounds_\ell n_i)) - f \pounds_\ell \theta - \nabla_i(\theta X^i)\left(1 - \frac{1}{d}\right).\end{aligned} \tag{28}$$

Note that the first line on the right-hand side of Eq. (28) vanishes if χ^i is a generator of a horizon preserving diffeomoprhism, cf. Eq. (15).

If the event horizon is stationary, that is, there exists an α such that $\tau^a = e^\alpha \ell^a$ is a Killing vector near the horizon

$$\pounds_\tau g_{ab}|_{\mathcal{N}} = 0, \qquad \nabla_c \pounds_\tau g_{ab}|_{\mathcal{N}} = 0, \tag{29}$$

then

$$K_{ij} = \pounds_\ell g_{ij} = e^{-\alpha} \pounds_\tau g_{ij} = 0, \tag{30}$$

so that $\Sigma_{ij} = 0$ and $\theta = 0$. In addition, using

$$\nabla_a \nabla_b \tau_c|_{\mathcal{N}} = -R_{bca}{}^d \tau_d|_{\mathcal{N}} \tag{31}$$

[which results from Eq. (29)], we find that $\pounds_\tau \nabla_a \tau^b|_{\mathcal{N}} = 0$ implying

$$\pounds_{e^\alpha \ell}(\omega_i + \nabla_i \alpha) = 0. \tag{32}$$

Further, using the zeroth law of black holes (for stationary horizons) [17], we have

$$\partial_i[e^\alpha(\kappa + \pounds_\ell \alpha)] = 0. \tag{33}$$

Putting together Eqs. (32) and (33) we find

$$\pounds_\ell \omega_i - \partial_i \kappa = 0, \tag{34}$$

independent on α. Thus, $\nabla_i q^i_\chi = 0$ for stationary horizons as long as χ is a supertranslation or superrotation (or a combination thereof).

Enstrophy and supertranslations.—In an asymptotically AdS geometry, there exist stationary black brane solutions characterized by a uniform Hawking temperature T and a constant center of mass velocity u^μ relative to an observer at infinity. Here greek indices μ, ν, denote coordinates on the boundary of AdS space. As discussed in detail in Ref. [4], it is possible to perturb these black brane solutions in a derivative expansion where one assumes that derivatives of T and u^μ are small relative to T. The perturbative solution to the Einstein equations takes the form

$$\begin{aligned}g_{ab}dx^a dx^b = {}& -r^2 h(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \\ & - 2u_\mu dx^\mu dr + \mathcal{O}(\nabla),\end{aligned} \tag{35}$$

where T and u^μ are constrained to satisfy

$$\nabla_\mu(T^d u^\mu) = \mathcal{O}(\nabla^2), \quad T u^\nu \nabla_\nu u_\mu - P^\nu_\mu \nabla_\nu T = \mathcal{O}(\nabla^2) \tag{36}$$

with ∇_μ a covariant derivative in Minkowski space, $P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$, and

$$h = 1 - \left(\frac{4\pi T}{(d+1)r}\right)^{d+1}. \tag{37}$$

Incidentally, Eq. (36) corresponds to the hydrodynamic Eq. (1) (see Ref. [4]).

The location of the event horizon for the geometry [Eq. (35)] is given by

$$r = \frac{4\pi T}{d+1} + \mathcal{O}(\nabla). \tag{38}$$

Here and in the remainder of this section we will use the coordinates x^μ to parametrize the horizon. It now follows that

$$\ell^i \partial_i = u^\mu \partial_\mu + \mathcal{O}(\nabla) \tag{39}$$

and also

$$\kappa = 2\pi T + \mathcal{O}(\nabla). \tag{40}$$

We will also choose

$$n_i dx^i = u_\mu dx^\mu + \mathcal{O}(\nabla). \tag{41}$$

(Note that this choice of n_i satisfies Eq. (16) to leading order in a derivative expansion.) With this parametrization we find that $\Omega_A = \mathcal{O}(\nabla)$ and that

$$\Sigma_i{}^j dx^i \partial_j = \frac{1}{2}\sigma_\mu{}^\nu dx^\mu \partial_\nu + \mathcal{O}(\nabla^2), \tag{42}$$

where $\sigma_{\mu\nu}$ is the shear tensor for a fluid with velocity u^μ,

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta}(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) - \frac{2}{d} P^{\mu\nu} \nabla_\alpha u^\alpha. \tag{43}$$

With our choice of parametrization we find that the Einstein equations take the form

$$\theta = \mathcal{O}(\nabla^2), \qquad (\pounds_\ell \omega_i - \partial_i \kappa)\Pi^i_A = \mathcal{O}(\nabla^2). \tag{44}$$

Suppose that χ^i is a supertranslation ($X^i = 0$) and that f is order $\mathcal{O}(\nabla^{[f]})$ in a derivative expansion. In this case q^i_χ is also $\mathcal{O}(\nabla^{[f]})$ but

$$\nabla_i q^i_{fu} = \mathcal{O}(\nabla^{[f]+2}) \tag{45}$$

on account of the equation of motion [Eq. (44)]. Thus, supertranslations lead to an approximately conserved current in the derivative expansion. Superrotations will not lead to approximately conserved currents owing to the fact that $\nabla_i q^i_X = \mathcal{O}(\nabla^{[X]+2})$ but $q^i_X = \mathcal{O}(\nabla^{[X]+1})$.

In a derivative expansion the supertranslation constraint, Eq. (15), reads as

$$\pounds_u(\kappa f) = \mathcal{O}(\nabla^{[f]+2}) \tag{46}$$

and the resulting current is

$$q^i_{fu}\partial_i = \kappa f u^\mu \partial_\mu. \tag{47}$$

The divergence of q^i_{fu} is given, in our current coordinate system, by

$$\nabla_i q^i_{fu} = \frac{1}{T^d}\partial_i(T^d q^i_{fu}), \tag{48}$$

where the factors of T^d come from the measure on the spatial section of the horizon. We are guaranteed that q^i_{fu} is approximately conserved provided that Eq. (46) is satisfied which implies that the divergence of

$$J^\mu_f = T^d \kappa f u^\mu \tag{49}$$

in Minkowski space will approximately vanish.

Equation (49) and its approximate conservation reproduces Eq. (5) once we identify J^μ_f in Eq. (49) with J^μ_g in Eq. (4). Thus, approximate enstrophy conservation is a result of a particular set of supertranslations on the horizon of AdS_4 black branes which are local in the black hole temperature and center of mass velocity, T and u^μ. Likewise, approximate entropy conservation is a result of horizon supertranslations of AdS_{d+2} black branes with $f = c/\kappa$, (with c a constant).

Discussion and outlook.—In this Letter we have shown that horizon supertranslation generators of AdS black brane geometries, which are conserved in the stationary limit, are associated with approximately conserved currents in a dual fluid description of the geometry.

In asymptotically AdS_4 geometries there exists a small subset of supertranslation generators which are local functions of the Hawking temperature and black brane null generators which are dual to the approximately conserved enstrophy and its various moments. Horizon superrotations are not associated with conserved charges of the dual fluid since conservation of superrotations in the stationary limit is not enhanced to an approximate symmetry once stationarity is only approximate.

The analysis carried out in this Letter pertains to asymptotically AdS black branes in the absence of matter, dual to uncharged conformal fluids. We expect our main result to be applicable more broadly to charged nonconformal fluids. More generally, relying on the membrane paradigm [18], one might expect a similar construction for black holes in asymptotically flat space, or even black holes in general.

Nonrelativistic enstrophy is not only conserved in the absence of dissipation; it cannot increase in time. Whether the same can be said regarding the relativistic enstrophy current is yet an open problem. Another hydrodynamic quantity which we know must not decrease in time (in a relativistic setting or not) is the entropy. That entropy cannot decrease translates into the well known area increase theorem of black holes [19]. Thus, one cannot help but wonder whether there is an enstrophy decrease theorem for black holes valid, at the very least, at low velocities. While the present work has not dealt with a full description of black hole enstrophy and its dynamics, we hope it will provide a stepping stone toward it.

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[1] F. Carrasco, L. Lehner, R. C. Myers, O. Reula, and A. Singh, Turbulent flows for relativistic conformal fluids in 2 + 1 dimensions, Phys. Rev. D **86**, 126006 (2012).
[2] R. Marjieh, N. Pinzani-Fokeeva, and A. Yarom, Enstrophy from symmetry, SciPost Phys. **12**, 085 (2022).
[3] N. Pinzani-Fokeeva and A. Yarom, Enstrophy without boost symmetry, SciPost Phys. **12**, 136 (2022).
[4] S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, Nonlinear fluid dynamics from gravity, J. High Energy Phys. 02 (2008) 045.
[5] L. Donnay, G. Giribet, H. A. Gonzalez, and M. Pino, Supertranslations and Superrotations at the Black Hole Horizon, Phys. Rev. Lett. **116**, 091101 (2016).
[6] L. Donnay, G. Giribet, H. A. González, and M. Pino, Extended symmetries at the black hole horizon, J. High Energy Phys. 09 (2016) 100.

[7] S. Carlip, Entropy from conformal field theory at Killing horizons, Classical Quantum Gravity **16**, 3327 (1999).
[8] M. Hotta, K. Sasaki, and T. Sasaki, Diffeomorphism on horizon as an asymptotic isometry of Schwarzschild black hole, Classical Quantum Gravity **18**, 1823 (2001).
[9] J.-i. Koga, Asymptotic symmetries on Killing horizons, Phys. Rev. D **64**, 124012 (2001).
[10] M. Hotta, Holographic charge excitations on horizontal boundary, Phys. Rev. D **66**, 124021 (2002).
[11] V. Chandrasekaran, E. E. Flanagan, and K. Prabhu, Symmetries and charges of general relativity at null boundaries, J. High Energy Phys. 11 (2018) 125.
[12] C. Eling and Y. Oz, On the membrane paradigm and spontaneous breaking of horizon BMS symmetries, J. High Energy Phys. 07 (2016) 065.
[13] R. F. Penna, Near-horizon BMS symmetries as fluid symmetries, J. High Energy Phys. 10 (2017) 049.
[14] W. Donnelly, L. Freidel, S. F. Moosavian, and A. J. Speranza, Gravitational edge modes, coadjoint orbits, and hydrodynamics, J. High Energy Phys. 09 (2021) 008.
[15] L. Donnay and C. Marteau, Carrollian physics at the black hole horizon, Classical Quantum Gravity **36**, 165002 (2019).
[16] R. H. Kraichnan, Inertial ranges in two-dimensional turbulence, Phys. Fluids **10**, 1417 (1967).
[17] J. M. Bardeen, B. Carter, and S. W. Hawking, The four laws of black hole mechanics, Commun. Math. Phys. **31**, 161 (1973).
[18] T. Damour, Quelques propriétés mécaniques, électromagnétiques, thermodynamiques et quantiques des trous noirs, Doctoral Thesis, Universite Pierre et Marie Curie Paris 6, (1979), https://www.ihes.fr/~damour/Articles/these1.pdf.
[19] J. D. Bekenstein, Black holes and the second law, Lett. Nuovo Cimento **4**, 737 (1972).