Scattering Amplitudes: Celestial and Carrollian

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Recent attempts at the construction of holography for asymptotically flat spacetime have taken two different routes. Celestial holography, involving a two dimensional (2D) conformal field theory (CFT) dual to 4D Minkowski spacetime, has generated novel results in asymptotic symmetry and scattering amplitudes. A different formulation, using Carrollian CFTs, has been principally used to provide some evidence for flat holography in lower dimensions. Understanding of flat space scattering has been lacking in the Carroll framework. In this Letter, using ideas from Celestial holography, we show that 3D Carrollian CFTs living on the null boundary of 4D flat space can potentially compute bulk scattering amplitudes. Three-dimensional Carrollian conformal correlators have two different branches, one depending on the null time direction and one independent of it. We propose that it is the time-dependent branch that is related to bulk scattering. We construct an explicit field theoretic example of a free massless Carrollian scalar that realizes some desired properties.

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Introduction.—The holographic principle is one of our primary routes to a theory of quantum gravity, formulated in terms of a lower dimensional field theory. Although there has been a great deal of success in understanding holography in Anti–de Sitter (AdS) spacetime through AdS/CFT, a similar understanding for asymptotically flat spacetimes (AFS) is lacking. There is, however, a consorted recent effort at rectifying this situation. There are two principal avenues of addressing this problem—celestial holography and Carrollian holography. Bondi–van der Burgh–Metzner-Sachs (BMS) [1] symmetries that arise as asymptotic symmetries of AFS on the null boundary, are important to both approaches.

Celestial holography has grown out of the basic observation [2] that, in AFS, soft theorems for *S*-matrix elements can be thought of as Ward identities for asymptotic symmetries [3–11]. The fundamental claim of celestial holography is that there is a two dimensional (2D) conformal field theory (CFT) on the celestial sphere which computes the scattering amplitudes for processes taking place in 4D AFS. This computation is facilitated by writing the *S* matrix in boost eigenstates [12–16] in which the Ward identities for asymptotic symmetries take the well-known form of Ward identities in a 2D CFT. This CFT is known as the celestial CFT. This approach to flat space holography has already produced many novel results about asymptotic symmetries and 4D scattering amplitudes [17–38]. Please see Refs. [39–41] for more details.

Another school of thought has been the attempt to build duals of AFS in terms of a 1D lower field theory that enjoys BMS symmetry. These field theories are CFTs living on the null boundary of AFS and can be understood as Carroll contractions of usual relativistic CFTs, where the speed of light $c \to 0$ [42,43]. We shall call this approach Carroll holography. The success of this formulation has principally been in 3D bulk-2D field theories, where various holographic checks have been performed [44–56]. Some higher dimensional explorations include [57–59]. Crucially, the understanding of scattering processes has been lacking in this formulation.

In this Letter, we provide a bridge between the two formulations. We show that using BMS or conformal Carroll symmetries in a 3D field theory living on null infinity, one can formulate the scattering problem in 4D AFS. We further demonstrate the plausibility of our proposal by constructing an explicit realization of Carrollian CFTs in terms of a 3D massless Carroll scalar with some desired features.

BMS and Carroll CFTs.—The symmetries of interest in AFS in $d = 4$ extends beyond the Poincare group to an infinite dimensional group discovered initially by Bondi, van der Burgh, Metzner, and Sachs [1]. The BMS symmetry algebra of 4D AFS at its null boundary $\mathcal{I}^{\pm}$ is

$$[L_n, L_m] = (n-m)L_{n+m}, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m},$$
$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{n+r,s},$$
$$[\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,n+s}, \quad [M_{r,s}, M_{p,q}] = 0. \quad (1)$$

$M_{r,s}$ generate infinite dimensional angle-dependent translations at $\mathcal{I}^{\pm}$ known as supertranslations. The original BMS group was given by these infinite supertranslations on top of the Lorentz group $\{L_{0,\pm1}, \bar{L}_{0,\pm1}\}$. Following [60,61], there has been an effort to consider the full conformal group on the sphere at infinity and, hence, all modes of the L_n generators, the so-called super-rotations [62]. In 2D celestial CFT, super-rotation or local conformal transformations on the celestial sphere are generated by a stress tensor which is the shadow transform of the subleading soft graviton [5,10,64]. After shadow transformation, the subleading soft graviton theorem [64] becomes the well known Ward identity for stress tensor in a 2D CFT.

Now, let us discuss 3D Carrollian CFT. This CFT lives on $\mathcal{I}^+$, which is topologically $\mathbb{R}_u \times \mathbb{S}^2$, where $\mathbb{R}_u$ is a null line and $\mathbb{S}^2$ is the sphere at infinity. The induced metric of $\mathcal{I}^+$ is degenerate and Carrollian structures emerge on the intrinsic geometry of the hypersurface. These theories are naturally expected to be invariant under Carrollian conformal isometries. Rather intriguingly, conformal Carrollian symmetries are isomorphic to BMS symmetries in one higher dimension [42,53]: $\mathfrak{CCarr}_d = \mathfrak{bms}_{d+1}$. Hence, a 3D Carrollian CFT naturally realizes the extended infinite-dimensional BMS_4 symmetry. We will show that these field theories are potential candidates for a holographic description of scattering amplitudes in 4D AFS.

For these 3D theories, a useful representation of vector fields is [57]

$$L_n = -z^{n+1}\partial_z - \frac{1}{2}(n+1)z^n u\partial_u, \quad M_{r,s} = z^r \bar{z}^s \partial_u. \quad (2)$$

$\bar{L}_n$ is defined analogously. z, $\bar{z}$ are stereographic coordinates on the sphere. We will label the Carroll conformal fields Φ living on $\mathcal{I}^+$ with their weights under L_0, $\bar{L}_0$

$$[L_0, \Phi(0)] = h\Phi(0), \qquad [\bar{L}_0, \Phi(0)] = \bar{h}\Phi(0). \quad (3)$$

We will assume the existence of Carrollian primary fields living on $\mathcal{I}^+$. The primary conditions are [57,65]

$$[L_n, \Phi(0)] = 0, \qquad [\bar{L}_n, \Phi(0)] = 0, \quad \forall \ n > 0, \quad (4a)$$

$$[M_{r,s}, \Phi(0)] = 0, \quad \forall \ r, s > 0. \quad (4b)$$

The last condition is an additional requirement on these fields unlike a 2D CFT. The transformation rules of the 3D Carrollian primary fields $\Phi_{h,\bar{h}}(u, z, \bar{z})$ at an arbitrary point on $\mathcal{I}^+$ under the infinitesimal BMS transformations are

$$\delta_{L_n}\Phi_{h,\bar{h}} = \epsilon\left[z^{n+1}\partial_z + (n+1)z^n\left(h + \frac{1}{2}u\partial_u\right)\right]\Phi_{h,\bar{h}}, \quad (5a)$$

$$\delta_{M_{r,s}}\Phi_{h,\bar{h}} = \epsilon z^r \bar{z}^s \partial_u \Phi_{h,\bar{h}}. \quad (5b)$$

Above $\Phi_{h,\bar{h}} = \Phi_{h,\bar{h}}(u, z, \bar{z})$. There is a similar relation for the antiholomorphic piece.

Relation to 4D scattering via celestial holography.—One of the main reasons for studying Carrollian CFTs is that their symmetries are the same as the extended BMS algebra. So potentially Carroll CFTs can be a holographic dual of a quantum theory of gravity in AFS. From general considerations, the only observables in a quantum theory of gravity in AFS are the S-matrix elements. Therefore, given a holographic dual, one should be able to compute the spacetime S matrix from this. Moreover, if the dual is a field theory, or at least looks like one, then, presumably, the S-matrix elements should be somehow related to the correlation functions of the field theory. This is the point of view that we adopt in this Letter.

Below, we study correlation functions of Carroll CFTs and find two distinct types of solutions or branches. In one branch, the correlation functions are independent of the null time direction [66] and resemble those of a 2D CFT, while the correlators of the other branch have explicit time dependence and are very different from 2D CFT. For example, unlike 2D CFT, the two-point function in this branch is ultralocal in the spatial directions and nonzero even when the scaling dimensions of the operators are different. Similarly, one can show, using 4D Poincare or global conformal Carroll invariance of the Carrollian CFT, that the time-dependent three-point function is zero. This problem can be solved if we treat z and $\bar{z}$ as independent complex coordinates rather than complex conjugates of each other. These are reminiscent of the properties of scattering amplitudes of massless particles in 4D AFS. So what is the relation of Carroll CFT correlations to scattering amplitudes? In this Letter, we propose an answer using ideas from celestial holography.

In celestial holography, the dual theory is conjectured to be a 2D (relativistic) CFT living on the celestial sphere. The important point, for our purpose, is that the correlation functions of the celestial CFT are given by the Mellin transform of 4D scattering amplitudes [12–16]. Let us briefly describe this. For simplicity, we consider only massless particles whose four-momenta are parametrized as

$$p^\mu = \omega(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}), \qquad p^\mu p_\mu = 0. \quad (6)$$

We also introduce a symbol $\epsilon = \pm 1$ if the particle is (outgoing) incoming. Using this parametrization, the Mellin transformation can be written as [13,14],

$$\mathcal{M}(\{z_i,\bar{z}_i,h_i,\bar{h}_i,\epsilon_i\})=\prod_{i=1}^{n}\int_0^\infty d\omega_i\omega_i^{\Delta_i-1}S(\{\epsilon_i\omega_i,z_i,\bar{z}_i,\sigma_i\}), \tag{7}$$

where S is the S-matrix element for n massless particle scattering and $\Delta\in\mathbb{C}$, $\sigma\in(\mathbb{Z}/2)$ are the dilatation weight and spin of the particle. Here, we have also defined $h=[(\Delta+\sigma)/2]$, $\bar{h}=[(\Delta-\sigma)/2]$. One can show [13,14] using the Lorentz transformation property of S matrix that the object $\mathcal{M}$ on the lhs, indeed, transforms like the correlation function of n primary operators of weight $(h,\bar{h})$ in a 2D CFT [67]. After Mellin transformation, $(z,\bar{z})$ can be interpreted as the stereographic coordinates of the celestial sphere and physically represent the direction of motion of the massless particle. For our purpose, however, we will use a modification [15,16] of (7) such that the correlation function $\mathcal{M}$ is now defined on a 3D space with coordinates $(u,z,\bar{z})$. This space can be interpreted as the (future) null infinity with u as the retarded time and $(z,\bar{z})$ as the stereographic coordinates of the celestial sphere. One can show [15–17,65] that under supertranslation,

$$u\to u'=u+f(z,\bar{z}),\quad z\to z'=z,\quad \bar{z}\to\bar{z}'=\bar{z}. \tag{8}$$

Similarly, under super-rotation or local conformal transformations,

$$u\to u'=\left(\frac{dw}{dz}\right)^{\frac{1}{2}}\left(\frac{d\bar{w}}{d\bar{z}}\right)^{\frac{1}{2}}u,\qquad z\to z'=w(z),$$
$$\bar{z}\to\bar{z}'=\bar{w}(\bar{z}). \tag{9}$$

The modified transformation [15,16] has the form

$$\tilde{\mathcal{M}}(\{u_i,z_i,\bar{z}_i,h_i,\bar{h}_i,\epsilon_i\})=\prod_{i=1}^{n}\int_0^\infty d\omega_i\omega_i^{\Delta_i-1}e^{-i\epsilon_i\omega_iu_i}\times S(\{\epsilon_i\omega_i,z_i,\bar{z}_i,\sigma_i\}),\quad \Delta\in\mathbb{C}. \tag{10}$$

One can show [15–17,65] using the celebrated soft theorem-Ward identity correspondence [2–10] that $\tilde{\mathcal{M}}$ transforms covariantly under the extended BMS_4 transformations. In celestial holography, the modified Mellin transformation (10) is used to compute the graviton celestial amplitudes in general relativity because the original Mellin transformation integral (7) is not convergent due to bad uv behavior of graviton scattering amplitudes in general relativity. It turns out that, instead, when (10) is used, the time coordinate u acts as a uv regulator, and as a result, $\tilde{\mathcal{M}}$ is finite. For more details, see Refs. [17,65,68].

Now, it is useful to write the modified celestial amplitude $\tilde{\mathcal{M}}$ as a correlation function of fields defined on null infinity. So, following [15], we define

$$\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})=\int_0^\infty d\omega\omega^{\Delta-1}e^{-i\epsilon\omega u}a(\epsilon\omega,z,\bar{z},\sigma), \tag{11}$$

where $a(\epsilon\omega,z,\bar{z},\sigma)$ is the momentum space (creation) annihilation operator of a massless particle with helicity σ when ($\epsilon=-1$) $\epsilon=1$. In terms of these fields, we can write

$$\tilde{\mathcal{M}}(\{u_i,z_i,\bar{z}_i,h_i,\bar{h}_i,\epsilon_i\})=\left\langle\prod_{i=1}^{n}\phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i,z_i,\bar{z}_i)\right\rangle. \tag{12}$$

Now, the field $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})$ transforms covariantly under the extended BMS_4 transformation. Under super-rotation [15–17,65],

$$\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})\to\left(\frac{dw}{dz}\right)^{h}\left(\frac{d\bar{w}}{d\bar{z}}\right)^{\bar{h}}\phi^{\epsilon}_{h,\bar{h}}(u',z',\bar{z}'), \tag{13}$$

where the primed coordinates are defined in (9). Similarly, under supertranslation,

$$\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})\to\phi^{\epsilon}_{h,\bar{h}}(u+f(z,\bar{z}),z,\bar{z}). \tag{14}$$

It is easy to see that, for infinitesimal BMS_4 transformations, (13) and (14) reduce to the equations (5) written in terms of the primaries of a Carrollian CFT.

Therefore, it is not unreasonable to wonder whether one can identify the Carrollian primaries with the primaries $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})$ of celestial holography. If this is true, then, this will open the road toward connecting the Carrollian CFT correlation functions with bulk scattering amplitudes because the field $\phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z})$ is directly related to standard creation-annihilation operators by (11).

The proposal.—Our central claim in this Letter is the following. It is natural to identify the time-dependent correlation functions of primaries in a Carrollian CFT with the modified Mellin amplitude

$$\tilde{\mathcal{M}}(\{u_i,z_i,\bar{z}_i,h_i,\bar{h}_i,\epsilon_i\})=\prod_i\langle\phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i,z_i,\bar{z}_i)\rangle.$$

In other words, the time-dependent correlators of a 3D Carrollian CFT compute the 4D scattering amplitudes in the Mellin basis.

We would like to emphasize that we are not saying that every Carrollian CFT computes spacetime scattering amplitude. But, if a specific Carrollian CFT does so, then it does it in the modified Mellin basis (10).

One may think that this identification is kinematical because both the objects transform in the same way under relevant symmetries. However, the dynamics enters nontrivially when we choose one branch of the Carrollian CFT correlators.

Celestial holography, as it stands, requires the existence of an infinite number of conformal primary fields with complex scaling dimensions. So, any Carrollian CFT that computes 4D scattering amplitudes should also have this feature. Over the last few years, study of tree level massless scattering amplitudes using the framework of celestial holography has revealed a much larger asymptotic symmetry group than the extended BMS_4. For example, the SL_2 current algebra at level zero turns out to be a symmetry algebra [17] of tree level graviton scattering amplitudes. In fact, it has been shown that the $w_{1+\infty}$ is a symmetry algebra [19–23] for massless scattering amplitudes. This also holds at the loop level in some special cases. Therefore, the asymptotic symmetry algebra for AFS is expected to be far richer than the extended BMS_4 algebra. The current Carrollian framework has to be extended in order to accommodate these additional symmetries.

Different branches of Carroll CFT correlators.—Now, we show the existence of two different branches of correlation functions for 3D Carroll CFTs by computing the two-point vacuum correlation functions of primary fields. Like relativistic CFTs, it is possible to completely determine the two- and three-point functions here using symmetry arguments. We demand that the correlation functions are invariant under the Poincare subalgebra ($\{M_{l,m}, L_n\}$ with l, $m = 0$, 1 and $n = 0, \pm 1$) of (1). Consider the two-point function

$$G(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2) = \langle 0|\Phi(u_1, z_1, \bar{z}_1)\Phi'(u_2, z_2, \bar{z}_2)|0\rangle. \tag{15}$$

Here, $\Phi(u_1, z_1, \bar{z}_1)$ and $\Phi'(u_2, z_2, \bar{z}_2)$ are primaries with weight (h_1, h_2) and $(\bar{h}_1, \bar{h}_2)$, respectively. Invariance under Carroll time translations leads to

$$\left(\frac{\partial}{\partial u_1} + \frac{\partial}{\partial u_2}\right) G(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2) = 0. \tag{16}$$

Carroll boost invariance ($u \to u + bz + \bar{b}\,\bar{z}$) [69] gives us

$$\left(z_1 \frac{\partial}{\partial u_1} + z_2 \frac{\partial}{\partial u_2}\right) G = \left(\bar{z}_1 \frac{\partial}{\partial u_1} + \bar{z}_2 \frac{\partial}{\partial u_2}\right) G = 0. \tag{17}$$

Combining the above equations, we get

$$z_{12} \frac{\partial}{\partial u_1} G(u_{12}, z_{12}, \bar{z}_{12}) = \bar{z}_{12} \frac{\partial}{\partial u_1} G(u_{12}, z_{12}, \bar{z}_{12}) = 0. \tag{18}$$

These equations have two independent solutions that give rise to two different classes of correlators [70]. The first class of correlators corresponds to the choice of solution

$$\frac{\partial}{\partial u_1} G(u_{12}, z_{12}, \bar{z}_{12}) = 0. \tag{19}$$

Using invariance under the subalgebra $\{L_{0,\pm 1}, \bar{L}_{0,\pm 1}\}$ of BMS_4, the above gives rise to a standard 2D CFT two-point correlation function [57]

$$G(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2) = \delta_{h_1, h_2} \delta_{\bar{h}_1, \bar{h}_2} z_{12}^{-2h} \bar{z}_{12}^{-2\bar{h}}. \tag{20}$$

For our discussions in this Letter, we will not be interested in this particular branch. The second class of solution corresponds to the choice

$$\frac{\partial}{\partial u_1} G(u_{12}, z_{12}, \bar{z}_{12}) \propto \delta^2(z_{12}). \tag{21}$$

This gives rise to correlation functions in which we are interested [15]. Thus,

$$G(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2) = f(u_{12}) \delta^2(z_{12}). \tag{22}$$

We will call this the delta-function branch [73]. By demanding invariance under the subalgebra $\{L_{0,\pm 1}, \bar{L}_{0,\pm 1}\}$ of BMS_4, it is straightforward to show

$$(\Delta + \Delta' - 2) f(u_{12}) + u_{12} \partial_{u_1} f(u_{12}) = 0, \tag{23a}$$

$$(\sigma + \sigma') f(u_{12}) = 0. \tag{23b}$$

Here, $\Delta = (h + \bar{h})$ is the scaling dimension and $\sigma = (h - \bar{h})$ is spin. The solution of the above equations is

$$f(u_{12}) = C \delta_{\sigma+\sigma', 0} u_{12}^{-(\Delta+\Delta'-2)}, \tag{24}$$

where C is a constant factor. Hence,

$$G(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2) = C u_{12}^{-\Delta-\Delta'+2)} \delta^2(z_{12}) \delta_{\sigma+\sigma', 0}. \tag{25}$$

Once this correlator has this form (25), the equation which imposes M_{11} or the transformation $u \to u + \epsilon z \bar{z}$ is trivially satisfied.

Notice that, very unlike a relativistic CFT two-point function, here, one does not have to have equal scaling dimensions for the fields to get a nonzero answer. Thus, this branch cannot be accessed by taking a limit from relativistic CFT correlation functions.

Now, let us discuss how one can obtain the same two-point function by modified Mellin transformation (10) of scattering amplitudes [15]. Of course, in the case of a two-point function, the scattering amplitude is trivial and is given by the inner product

$$\langle p_1, \sigma_1 | p_2, \sigma_2 \rangle = (2\pi)^3 2E_{p_1} \delta^3(\vec{p}_1 - \vec{p}_2) \delta_{\sigma_1+\sigma_2, 0}. \tag{26}$$

Here, the notation is standard except that we label the helicity of an external particle as if it were an outgoing particle. Using (6), we write

$$\langle p_1, \sigma_1 | p_2, \sigma_2 \rangle = \frac{4\pi^3}{\omega_1} \delta(\omega_{12}) \delta^2(z_{12}) \delta_{\sigma_1+\sigma_2,0}. \quad (27)$$

Now, the Mellin transformed two point function is

$$\begin{aligned}
&\tilde{\mathcal{M}}(u_1, z_1, \bar{z}_1, u_2, z_2, \bar{z}_2, h_1, \bar{h}_1, h_2, \bar{h}_2, \epsilon_1 = 1, \epsilon_2 = -1) \\
&= 4\pi^3 \delta_{\sigma_1+\sigma_2,0}, \\
&\int_0^\infty d\omega_1 d\omega_2 \omega_1^{\Delta_1-1} \omega_2^{\Delta_2-1} e^{-i\omega_1 u_1} e^{i\omega_2 u_2} \frac{\delta(\omega_{12}) \delta^2(z_{12})}{\omega_1} \\
&= 4\pi^3 \Gamma(\Delta_1 + \Delta_2 - 2) \frac{\delta^2(z_{12})}{[i(u_{12})]^{\Delta_1+\Delta_2-2}} \delta_{\sigma_1+\sigma_2,0}. \quad (28)
\end{aligned}$$

We see that, modulo the constant normalization, this has the same structure as the time dependent two-point function of the Carrollian CFT. More importantly, the presence of the spatial delta function $\delta^2(z_{12})$ in the Carrollian two-point function has the (dual) physical interpretation that the momentum direction of a free particle in the bulk spacetime does not change.

In the same way, following [15], one can show that the time dependent three-point function in the Carrollian CFT is zero. This has the physical interpretation that, in Minkowski signature, the scattering amplitude of three massless particles vanish due to energy momentum conservation.

Therefore, the peculiarities of the time dependent correlators of a Carroll CFT are precisely what we need to connect to the spacetime scattering amplitudes of massless particles. This is the main message of our Letter.

Massless Carrollian scalar field.—Now, we will focus on a particular simple example, that of a massless Carroll scalar field, to show that this gives us the correlation functions in the delta-function branch. The minimally coupled massless scalar field on a flat Carrollian background is described by

$$\mathcal{S} = \int du d^2 x^i \frac{1}{2} (\partial_u \Phi)^2. \quad (29)$$

The 2D analog of this action (29) has been extensively used to study the tensionless limit of string theory (see, e.g., Ref. [74]). We will see that, here, this simple 3D action carries the seeds of a potential dual formulation of 4D gravity in AFS.

Now, we compute the two-point correlation function of the massless Carroll scalar first by computing the Green's functions. The Green's function equation, here, is

$$\partial_u^2 G(u_{12}, z_{12}^i) = \delta^3(u_{12}, z_{12}^i). \quad (30)$$

This can be solved in the usual way by going to Fourier space where we get $\tilde{G}(k_u, k_i) = -(1/k_u^2)$. Transforming back into position space yields

$$\begin{aligned}
G(u_{12}, z_{12}^i) &= -\int \frac{dk_u}{k_u^2 + \mu^2} e^{ik_u u_{12}} \int d^2 z e^{ik_i z_{12}^i} \\
&= \frac{i}{2} (\mu^{-1} - u_{12}) \delta^2(z_{12}). \quad (31)
\end{aligned}$$

As the equation of motion has no spatial derivatives, this integral diverges. We regulate it by throwing away the troublesome infinite piece

$$G(u_{12}, z_{12}^i) = -\frac{i}{2} u_{12} \delta^2(z_{12}). \quad (32)$$

For scalar fields, the spin $\sigma = 0$ and conformal weights of $\Phi(u, z, \bar{z})$ are [e.g., from (29)] $h = \bar{h} = \frac{1}{4}$. Hence, (32) is in perfect agreement with the answer previously derived from symmetry arguments.

Now, we rederive this two-point correlator through canonical quantization. We put the free scalar theory on the round sphere and, then, take its radius to infinity to recover our answer in the plane coordinates. The scalar field action on a manifold with topology $\mathbb{R} \times \mathbb{S}^2$ is

$$\mathcal{S} = \int du d^2 z \sqrt{q} \left[\frac{1}{2} (\partial_u \Phi)^2 - k^2 \Phi^2 \right]. \quad (33)$$

Here, $k = (1/2R)$ where R is the radius of the sphere and q_{ij} is its metric. The equation of motion is

$$\ddot{\Phi} + k^2 \Phi^2 = 0. \quad (34)$$

Generic real solutions are given by

$$\Phi(u, z, \bar{z}) = \frac{1}{\sqrt{k}} [C^\dagger(z, \bar{z}) e^{iku} + C(z, \bar{z}) e^{-iku}]. \quad (35)$$

Canonical commutation relation between the C fields and the Hamiltonian are, respectively,

$$[C(z, \bar{z}), C^\dagger(z', \bar{z}')] = \frac{1}{2} \delta^2(z - z'), \quad (36)$$

$$H = k \int d^2 z \sqrt{q} \left(2C^\dagger(z, \bar{z}) C(z, \bar{z}) + \frac{1}{2} \delta^2(0) \right). \quad (37)$$

The physical part of the Hamiltonian [neglecting the unphysical zero point energy $\delta^2(0)$] implies that the time translation symmetric ground state is annihilated by C

$$C(z, \bar{z}) |0\rangle = 0, \quad \text{for } (z, \bar{z}) \in \mathbb{S}^2. \quad (38)$$

It is straightforward to calculate the two-point function

$$G(u_1, u_2, z_1^i, z_2^i) = \langle 0 | T \Phi(u_1, z_1, \bar{z}_1) \Phi(u_2, z_2, \bar{z}_2) | 0 \rangle.$$

Taking $u_1 > u_2$, we obtain

$$G(u_{12}, z_{12}^i) = -\frac{1}{2k}[\cos ku_{12} + i \sin ku_{12}]\delta^2(z_{12}). \quad (39)$$

In the limit $R \to \infty$, or $k \to 0$

$$G(u_1, u_2, z_1^i, z_2^i) = -\left(\frac{1}{2k} + \frac{i}{2}u_{12}\right)\delta^2(z_{12}). \quad (40)$$

We read off the scaling dimension of Φ as $\Delta = 1/2$. The physical part of the two-point function (40) matches exactly with the one computed earlier.

Conclusions.—In this Letter, we have provided evidence that the correlation functions of 3D Carrollian CFTs encode scattering amplitudes for 4D AFS, specifically in the Mellin basis. The Carroll correlators have two distinct branches, and one of these, the one with explicit Carroll time dependence which we called the delta-function branch, was the one relevant for the connection to flat space scattering.

Carrollian theories can be of two types, electric and magnetic. The electric type theories generically exhibit ultralocal correlation functions, while the magnetic ones do not. This can be seen, e.g., in the scalar theory as well. In [71], two distinct scalar theories with Carroll invariance were constructed and their two-point functions computed, and it was shown that the electric theory has ultralocal behavior while the magnetic theory gives a different branch. Massless versions of these theories are conformal Carroll invariant. So, of the types of scalar theories, the one that captures scattering information of a higher dimensional asymptotically flat spacetime is the electric one, which we have discussed in the Letter. It is clear that this is not fixed by just kinematic considerations and contains dynamics. Although the electric leg is connected to higher dimensional scattering, we don't yet understand what determines this choice.

There are a number of intriguing questions that arise from our considerations in this Letter. Originally, the version of flat space holography envisioned with connections with Carroll CFTs was one that emerged as a systematic limit from AdS/CFT. It is clear that the correlation functions that we focused on in this Letter cannot emerge as a Carroll limit from standard relativistic 3D CFT correlators in position space, since, e.g., the CFT two-point function would vanish for unequal weight primaries, and in the time-dependent Carroll branch, this does not happen. Hence, it would seem that the formulation of Carroll holography we require for connections to scattering amplitudes is disconnected from AdS/CFT. While this makes sense because AFS and AdS are fundamentally different, how this fits in with, e.g., the program of attempting to find flat space correlations from AdS/CFT (see, e.g., Refs. [75–77]), remains to be seen.

Our construction and, specifically, the emergence of two different branches of correlation functions is also reminiscent of recent advances in the tensionless regime of string theory where three distinct quantum theories appear from a single classical theory [78,79]. Recent findings of different correlation functions in these theories [80,81] is an indication that perhaps there is an interesting nontrivial quantum vacuum structure underlying the Carrollian theories we have discussed in our Letter.

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Note added.—Recently, Ref. [82] appeared on the arXiv. Although both articles attempt to link Carroll and celestial holography, our approaches are complementary.

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