Scaling of Acceleration Statistics in High Reynolds Number Turbulence

Dhawal Buaria[1,2,*] and Katepalli R. Sreenivasan[1,3]
[1]Tandon School of Engineering, New York University, New York, New York 11201, USA
[2]Max Planck Institute for Dynamics and Self-Organization, 37077 Göttingen, Germany
[3]Department of Physics and the Courant Institute of Mathematical Sciences, New York University, New York, New York 10012, USA

(Received 19 February 2022; accepted 21 May 2022; published 10 June 2022)

The scaling of acceleration statistics in turbulence is examined by combining data from the literature with new data from well-resolved direct numerical simulations of isotropic turbulence, significantly extending the Reynolds number range. The acceleration variance at higher Reynolds numbers departs from previous predictions based on multifractal models, which characterize Lagrangian intermittency as an extension of Eulerian intermittency. The disagreement is even more prominent for higher-order moments of the acceleration. Instead, starting from a known exact relation, we relate the scaling of acceleration variance to that of Eulerian fourth-order velocity gradient and velocity increment statistics. This prediction is in excellent agreement with the variance data. Our Letter highlights the need for models that consider Lagrangian intermittency independent of the Eulerian counterpart.

DOI: 10.1103/PhysRevLett.128.234502

Introduction.—The acceleration of a fluid element in a turbulent flow, given by the Lagrangian derivative of the velocity, resulting from the balance of forces acting on it, is arguably the simplest descriptor of its motion. This is directly reflected in the Navier-Stokes equations:

$$\boldsymbol{a} = D\boldsymbol{u}/Dt = -\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \tag{1}$$

where, $\boldsymbol{u}$ is the divergence-free velocity ($\nabla\cdot\boldsymbol{u} = 0$), p the kinematic pressure, ν is the kinematic viscosity, and $\boldsymbol{f}$ is a forcing term. Besides its fundamental role in the study of turbulence [1–4], understanding the statistics of acceleration is of paramount importance for a diverse range of applications constructed around stochastic modeling of transport phenomena in turbulence [5–8]. The application of Kolmogorov's 1941 phenomenology implies that the variance (and higher-order moments) of any acceleration component a can be solely described by the mean-dissipation rate $\langle\epsilon\rangle$ and ν [9–11]:

$$\langle a^2\rangle = \frac{1}{3}\langle|\boldsymbol{a}|^2\rangle = a_0\langle\epsilon\rangle^{3/2}\nu^{-1/2}, \tag{2}$$

where a_0 is thought to be a universal constant.

However, extensive numerical and experimental work has shown that a_0 increases with Reynolds number [12–20]. Thus, obtaining data on a_0 and modeling its R_λ variation has been a topic of considerable interest. While several theoretical works have focused on acceleration statistics [21–24], the most notable procedure, whose validity should not be taken for granted, stems from the multifractal model [25–27], which quantifies acceleration intermittency (and, in general, the intermittency of other Lagrangian quantities) by adapting to the Lagrangian viewpoint the well-known Eulerian framework, based either on the energy dissipation rate [28] or velocity increments [29]. A key result from this consideration is that $a_0 \sim R_\lambda^\chi$, $\chi \approx 0.135$, where R_λ is the Taylor-scale Reynolds number. While data from direct numerical simulations (DNS) and experiments do not directly display this power law, it was nevertheless presumed to be asymptotically correct at very large R_λ, and an empirical interpolation formula [16,19],

$$a_0 \simeq \frac{c_1 R_\lambda^\chi}{(1 + c_2/R_\lambda^{1+\chi})}, \tag{3}$$

with $\chi = 0.135$, $c_1 = 1.9$, and $c_2 = 85$, was suggested to fit the data, showing reasonable success [16]. An alternative scaling: $a_0 \sim R_\lambda^{0.25}$ was proposed by Hill [21], which was indistinguishable from Eq. (3) at low R_λ [16,20]; we discuss the veracity of this proposal later.

In this Letter, we revisit the scaling of acceleration variance (and higher-order moments) by presenting new DNS data at higher R_λ. The new variance data agrees with previous lower R_λ data, where the R_λ range overlaps, but increasing deviations from Eq. (3) occur at higher R_λ. Results for high-order moments show even stronger

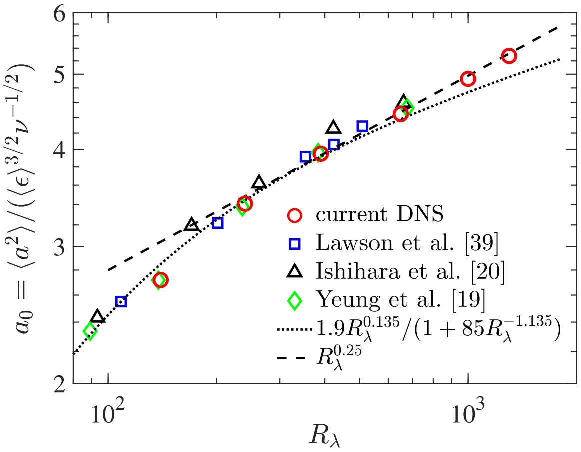

FIG. 1. Normalized acceleration variance $a_0 = \langle a^2\rangle/(\langle\epsilon\rangle^{3/2}\nu^{-1/2})$ as a function of R_λ. The data can be prescribed by a simple $R_\lambda^{0.25}$ power law at high R_λ, in contrast to the previously proposed empirical fit given by Eq. (3).

deviations from previous predictions. Further analysis shows that the extension of Eulerian multifractal models to the Lagrangian viewpoint is the source of this discrepancy. We develop a statistical model that shows excellent agreement with variance data at high R_λ, and also provide an updated interpolation fit to include low R_λ data.

Direct numerical simulations.—The DNS data used here correspond to the canonical setup of forced stationary isotropic turbulence in a periodic domain [30], allowing the use of highly accurate Fourier pseudospectral methods [31]. The novelty is that we have simultaneously achieved very high Reynolds number and the necessary grid resolution to accurately resolve the small scales [32,33]. The data correspond to the same Taylor-scale Reynolds number R_λ range of 140–1300 attained in recent studies [34–37], which have adequately established convergence with respect to resolution and statistical sampling. The grid resolution is as high as $k_{\max}\eta_K \approx 6$, which is substantially higher than $k_{\max}\eta_K \approx 1$–2, used in previous acceleration studies [16,19,20]; $k_{\max} = \sqrt{2}N/3$ is the maximum resolved wave number on a N^3 grid and $\eta_K = \nu^{3/4}\langle\epsilon\rangle^{-1/4}$ is the Kolmogorov length scale. This improved small-scale resolution is especially necessary for capturing higher-order statistics of acceleration, since acceleration is even more intermittent than spatial velocity gradients [12,19].

Acceleration variance.—Figure 1 shows the compilation of data from various sources including data from both DNS [19,20,38] and bias-corrected experiments [39]. We have also included DNS data obtained directly from Lagrangian trajectories of fluid particles [40–42], which give identical results for acceleration variance [43]. As evident, while Eq. (3) works for the previous range of R_λ, it does not fit the new data. In fact, a $R_\lambda^{0.25}$ scaling is more appropriate at higher R_λ, and as discussed later, the failure of multifractal models in fitting higher-order moments is even more conspicuous. To gain clarity on this point, it is useful to discuss the multifractal models first.

Acceleration scaling from multifractals.—The key idea in multifractal approaches is to quantify the intermittency of acceleration in terms of the intermittency of Eulerian velocity gradients or dissipation rate. Assuming a simple phenomenological equivalence between temporal and spatial derivatives, acceleration can be written in terms of dissipation rate and viscosity as $a \sim \epsilon^{3/4}\nu^{-1/4}$. Thus, the moments of acceleration are obtained as

$$\langle a^p\rangle \sim \langle\epsilon^{3p/4}\rangle\nu^{-p/4}. \tag{4}$$

Alternatively,

$$\langle a^p\rangle/\langle a_K^p\rangle \sim \langle\epsilon^{3p/4}\rangle/\langle\epsilon\rangle^{3p/4}, \tag{5}$$

where $a_K = \langle\epsilon\rangle^{3/4}\nu^{-1/4}$, i.e., acceleration based on Kolmogorov variables. Since Eulerian intermittency dictates that $\langle\epsilon^q\rangle \neq \langle\epsilon\rangle^q$ for any $q \neq 1$ [28,29], the key assumption in its extension to Lagrangian intermittency is that the pth moment of acceleration scales as the $(3p/4)$th moment of ϵ [25,27]. The scaling of $\langle\epsilon^q\rangle/\langle\epsilon\rangle^q$ can be obtained by several approaches, all leading to similar results. We briefly summarize a few approaches below, with additional details in the Supplemental Material [44].

The most direct approach is to use the multifractality of dissipation rate [25,45]. Within the multifractal framework, a scale-averaged dissipation ϵ_r, over scale r, is assumed to be Hölder continuous: $\epsilon_r/\langle\epsilon\rangle \sim (r/L)^{\alpha-1}$, where α is the local Hölder exponent, with a corresponding multifractal spectrum $F(\alpha)$ and L is the large-scale length. Note, the 1D spectrum $f(\alpha)$ is more common in the literature [45], which is simply: $f(\alpha) = F(\alpha) - 2$. Now, ϵ_r reduces to the true dissipation for a viscous cutoff defined as $r \simeq (\nu^3/\epsilon_r)^{1/4}$ or equivalently, $r/L \simeq Re^{-3/(3+\alpha)}$. Here, $Re = u'L/\nu$, u' being the large-scale velocity; we also use $Re \sim R_\lambda^2$ and $\langle\epsilon\rangle \sim u'^3/L$ from dissipation anomaly [46].

The above framework leads to the result:

$$\langle\epsilon^q\rangle/\langle\epsilon\rangle^q \sim R_\lambda^{\tau_q}, \qquad \tau_q = \sup_\alpha \frac{6[q(1-\alpha) - 3 + F(\alpha)]}{3+\alpha}. \tag{6}$$

An approximation for $F(\alpha)$, such as the p-model [45,47], can be used to obtain τ_q. The pth moment of acceleration can then be simply obtained as [48]

$$\langle a^p\rangle/a_K^p \sim R_\lambda^{\zeta_p}, \quad \text{with} \quad \zeta_p = \tau_{3p/4}. \tag{7}$$

Instead of dissipation, one can also start by taking the velocity increment δu_r over scale r to be Hölder continuous: $\delta u_r/u' \sim (r/L)^h$, where h is the local Hölder exponent

TABLE I. Scaling exponents ζ for R_λ scaling of acceleration moments $M_a \sim R_\lambda^\zeta$, as predicted from intermittency models, compared with current DNS results (see Figs. 1 and 2).

M_a	p-model	She-Lévêque	K62 log-normal	DNS result
$\langle a^2 \rangle / a_K^2$	0.135	0.140	0.140	0.25
$\langle a^4 \rangle / a_K^4$	0.943	1.00	1.13	1.60
$\langle a^6 \rangle / a_K^6$	2.06	2.30	2.95	3.95
$\langle a^4 \rangle / \langle a^2 \rangle^2$	0.673	0.720	0.850	1.10
$\langle a^6 \rangle / \langle a^2 \rangle^3$	1.66	1.88	2.53	3.20

and $D(h)$ is the corresponding multifractal spectrum. A scale-dependent dissipation rate ϵ_r can then be defined as $\epsilon_r \sim (\delta u_r)^3/r$, which reduces to the true dissipation for the viscous cutoff defined by the condition $\delta u_r r/\nu \simeq 1$. This framework leads to the same result as in Eq. (6), corresponding to $\alpha = 3h$ and $F(\alpha) = D(h)$. A well-known approximation for $D(h)$ is given by the She-Lévêque model [49]. Finally, we can also use the Kolmogorov (1962) log-normal model [50], which gives $\tau_q = 3\mu q(q-1)/4$, even though it is untenable for large q [29]. Here, μ is the intermittency exponent, with experiments and DNS suggesting $\mu \approx 0.25$ [51,52].

The scaling of acceleration moments obtained from these three approaches and also from DNS data is listed in Table I, up to sixth-order. All approaches give essentially the same result for the acceleration variance, with the exponent of about 0.135 used in Eq. (3). However, the high-R_λ DNS data clearly do not conform to any of the power laws shown in Table I. The results for normalized fourth- and sixth-order moments, also plotted in Fig. 2, clearly show that the power laws increasingly differ from multifractal predictions.

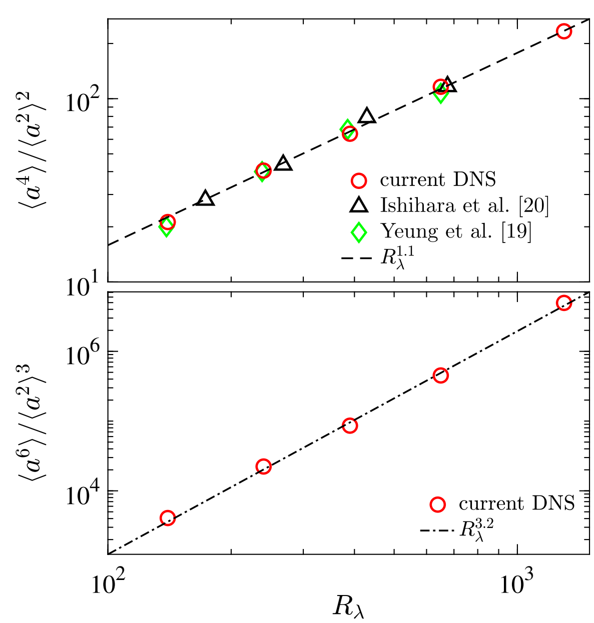

FIG. 2. Normalized fourth-order (top) and sixth-order (bottom) moments of acceleration as a function of R_λ.

As noted earlier, the use of multifractals is primarily motivated by Eq. (4). To get better insight, in Fig. 3(a), we plot a_0 and $\langle \epsilon^{3/2} \rangle / \langle \epsilon \rangle^{3/2}$ versus R_λ. While the latter shows a clear $R_\lambda^{0.14}$ scaling as anticipated from multifractals (and also the log-normal model), the former shows a steeper scaling of $R_\lambda^{0.25}$. An even more general and direct test is presented in Fig. 3(b), by checking the validity of Eq. (4) for different p values. The data clearly suggest that the acceleration intermittency, being stronger, cannot be described by extending the Eulerian intermittency of the dissipation rate. In fact, a similar observation has been made for Lagrangian velocity structure functions, where extensions of the p-model and the She-Lévêque model severely underpredict their intermittency (i.e., overpredict the inertial-range exponents) [53].

It is worth considering if one might describe the scaling of acceleration moments in terms of enstrophy $\Omega = |\boldsymbol{\omega}|^2$ ($\boldsymbol{\omega}$ being the vorticity), instead of dissipation. This change addresses the likelihood that acceleration is influenced more by transverse velocity gradients than by longitudinal ones [54,55]. In isotropic turbulence $\langle \Omega \rangle = \langle \epsilon \rangle / \nu$, but the higher moments differ, enstrophy being more

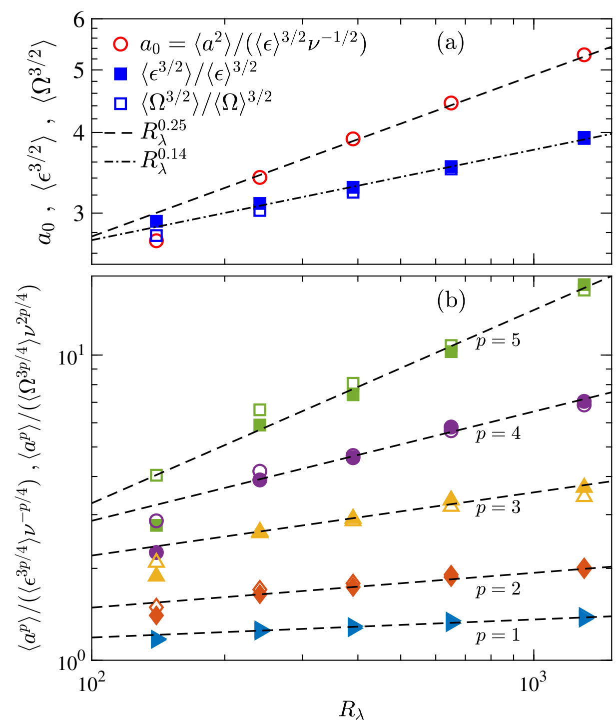

FIG. 3. (a) Scaling of a_0, $\langle \epsilon^{3/2} \rangle$, and $\langle \Omega^{3/2} \rangle$. For clarity, data for ϵ and Ω are shifted up by factors of 2 and 1.5, respectively. (b) Scaling of pth moments of acceleration normalized by moments of $\epsilon^{3p/4}$ (filled symbols) and $\Omega^{3p/4}$ (open symbols). For clarity, data for $p = 1$–5 are respectively shifted by factors of 1, 0.75, 0.5, 0.25, 0.12 for ϵ and 0.94, 1.01, 1.2, 1.35, 1.6 for Ω.

intermittent [32,56]. The resulting modification to Eq. (4) is $\langle a^p\rangle \sim \langle \Omega^{3p/4}\nu^{2p/4}\rangle$. However, as tested in Figs. 3(a) and (b), the differences arise only for large p; even then, it is not sufficient to explain the stronger intermittency of acceleration (also see Supplemental Material [44]).

Acceleration variance from fourth-order structure function.—A statistical model for acceleration variance is now obtained using a methodology similar to that proposed by [21], but differing in some crucial aspects. From Eq. (1), acceleration variance can be obtained directly as [57]

$$\langle|\boldsymbol{a}|^2\rangle = \langle|\nabla p|^2\rangle + \nu^2\langle|\nabla^2 u|^2\rangle. \quad (8)$$

The viscous contribution is known to be small and can be ignored [13]. An exact relation for variance of pressure gradient is also known [58,59]:

$$\langle|\nabla p|^2\rangle = \int_r r^{-3}[D_{1111}(r) + D_{\alpha\alpha\alpha\alpha}(r) - 6D_{11\beta\beta}(r)]dr, \quad (9)$$

where the Ds are the fourth-order longitudinal, transverse, and mixed structure functions, in order. The above results can be rewritten as [21]

$$\langle|\boldsymbol{a}|^2\rangle \simeq 4H_\chi \int_r r^{-3}D_{1111}(r)dr, \quad (10)$$

where H_χ is a constant defined by Eqs. (8), (9). At sufficiently high R_λ ($\gtrsim$200), DNS data [13,20] confirm that $H_\chi \approx 0.65$ (also see Supplemental Material [44]). We can normalize both sides by Kolmogorov scales to write

$$a_0 \simeq \frac{4H_\chi}{3}\int_r \left(\frac{r}{\eta_K}\right)^{-3}\frac{D_{1111}(r)}{u_K^4} d\left(\frac{r}{\eta_K}\right). \quad (11)$$

Assuming standard scaling regimes [29], we can write

$$\frac{D_{1111}(r)}{u_K^4} = \begin{cases} \frac{F}{225}\left(\frac{r}{\eta_K}\right)^4 & r < \ell, \\ C_4\left(\frac{r}{\eta_K}\right)^{\xi_4} & \ell < r < L, \\ C & r > L, \end{cases} \quad (12)$$

where F is the flatness of $\partial u/\partial x$, ξ_4 is the inertial-range exponent, and C_4, C are constants that depend on R_λ; ℓ is a crossover scale between the viscous and inertial range and is determined by matching the two regimes as

$$(F/225)(\ell/\eta_K)^4 = C_4(\ell/\eta_K)^{\xi_4}. \quad (13)$$

Now, taking

$$F \sim R_\lambda^\alpha, \qquad C_4 \sim R_\lambda^\beta, \quad (14)$$

we have

$$\ell/\eta_K \sim R_\lambda^{(\beta-\alpha)/(4-\xi_4)}. \quad (15)$$

Finally, from piecewise integration of Eq. (11), it can be shown that (see Supplemental Material [44] for intermediate steps):

$$a_0 \sim F(\ell/\eta_K)^2. \quad (16)$$

Substituting the R_λ dependencies, we get

$$a_0 \sim R_\lambda^{(2\alpha-\alpha\xi_4+2\beta)/(4-\xi_4)}. \quad (17)$$

The values of α, β, and ξ_4 are in principle obtainable from Eulerian intermittency models. The exponent α simply corresponds to τ_2 in Eq. (6), since $F \sim \langle\epsilon^2\rangle/\langle\epsilon\rangle^2$. Multifractal and log-normal models predict $\alpha = \tau_2 \approx 0.38$. The DNS data for F are shown in Fig. 4(a), giving $\alpha \approx 0.387$, in excellent agreement with the prediction, and also with previous experimental and DNS results in literature [18,20].

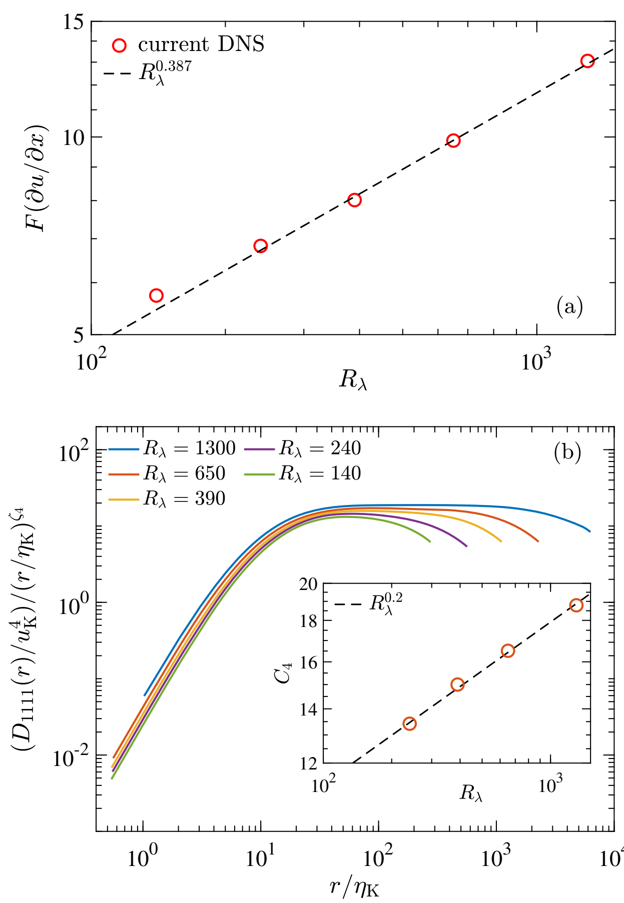

FIG. 4. (a) Flatness of longitudinal velocity gradient as a function of R_λ. (b) Fourth-order structure function compensated by its inertial-range scaling. The inset shows the variation of coefficient C_4 in Eq. (12) as a function of R_λ.

On the other hand, intermittency models predict $\xi_4 \approx 1.28$ [49]. Our DNS shows $\xi_4 \approx 1.3$, which is well within statistical error bounds. Finally, the prediction for β from multifractal model is $\beta \approx (4 - 3\xi_4)/2$, which reduces to $\beta \approx \mu/3$ for log-normal model; both predictions give $\beta \approx 0.08$ (also see Supplemental Material [44]). Figure 4(b) shows the normalized fourth-order structure function from our DNS data, using $\xi_4 \approx 1.3$. Note, as expected, the inertial-range increases with R_λ. The inset of the bottom panel shows C_4, giving $\beta \approx 0.2$. This observed β is substantially larger than 0.08 anticipated from multifractal and log-normal models.

The use of $\alpha = 0.387$, $\beta = 0.2$, and $\xi_4 = 1.3$ in Eq. (17) leads to

$$a_0 \sim R_\lambda^\chi, \qquad \chi = \zeta_2 \approx 0.25, \tag{18}$$

which is in excellent agreement with the high-R_λ data shown in Figs. 1 and 3(a). The exponent 0.25 is virtually insensitive to a small variation in ξ_4, but is significantly impacted by the choice of $\beta = 0.2$ (instead of 0.08). Moreover, the use of $\beta \approx 0.08$ in Eq. (17) gives $a_0 \sim R_\lambda^{0.15}$, which is essentially the same as the exponent 0.14 obtained earlier in Table I. This shows the robustness of piecewise integration leading to the result in Eq. (17) and also suggests that the discrepancy from multifractal prediction is due to the exponent β (and hence the proportionality constant C_4). In this regard, the role of β needs to be further explored, especially in relation to the inadequacy of Eq. (4).

We note that the exponent 0.25 was also suggested by Hill [21]. However, Hill arrived at this result by deriving that $a_0 \sim F^{0.79}$ and $F \sim R_\lambda^{0.31}$ based on [60]; evidently, the current data do not agree with both of these results. It appears that the two errors fortuitously cancelled out each other to give the 0.25 exponent. Finally, we point out that the exponent 0.25 describes the data for $R_\lambda \gtrsim 200$. To describe the data at lower R_λ, an empirical interpolation formula in the spirit of Eq. (3) can be devised with $\chi = 0.25$. Least-square fit gives $c_1 \approx 0.89$, $c_2 \approx 40$ (also see Supplemental Material [44]).

Conclusions.—The moments of Lagrangian acceleration are known to deviate from classical K41 phenomenology due to intermittency. Attempts were made to quantify these deviations by extending the Eulerian multifractal models to the Lagrangian viewpoint and devising an *ad hoc* interpolation formula to agree with available data from DNS and experiments. The first contribution of this article is to present new, very well resolved DNS data on Lagrangian acceleration at higher R_λ, and show that they disagree with the results from multifractal models, and the interpolation formula. The disagreement gets increasingly stronger with the moment order. As a second contribution, the article devises a statistical model that is able to correctly capture the scaling of acceleration variance. While this framework does not seem amenable for generalization to higher-order moments, our results show that the intermittency of Lagrangian quantities remains an open problem, even more compellingly than before.

We thank P. K. Yeung and Luca Biferale for providing helpful comments on an earlier draft of the manuscript. We gratefully acknowledge the Gauss Centre for Supercomputing e.V. for providing computing time on the supercomputers JUQUEEN and JUWELS at Jülich Supercomputing Centre (JSC), where the simulations reported in this Letter were performed.

*Corresponding author.
dhawal.buaria@nyu.edu

[1] A. La Porta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz, Fluid particle accelerations in fully developed turbulence, Nature (London) **409**, 1017 (2001).
[2] F. Toschi and E. Bodenschatz, Lagrangian properties of particles in turbulence, Annu. Rev. Fluid Mech. **41**, 375 (2009).
[3] N. Stelzenmuller, J. I. Polanco, L. Vignal, I. Vinkovic, and N. Mordant, Lagrangian acceleration statistics in a turbulent channel flow, Phys. Rev. Fluids **2**, 054602 (2017).
[4] D. Buaria, A. Pumir, F. Feraco, R. Marino, A. Pouquet, D. Rosenberg, and L. Primavera, Single-particle Lagrangian statistics from direct numerical simulations of rotating-stratified turbulence, Phys. Rev. Fluids **5**, 064801 (2020).
[5] B. L. Sawford, Reynolds number effects in Lagrangian stochastic models of turbulent dispersion, Phys. Fluids A **3**, 1577 (1991).
[6] J. C. Wyngaard, Atmospheric turbulence, Annu. Rev. Fluid Mech. **24**, 205 (1992).
[7] S B Pope, Lagrangian pdf methods for turbulent flows, Annu. Rev. Fluid Mech. **26**, 23 (1994).
[8] J. D. Wilson and B. L. Sawford, Review of Lagrangian stochastic models for trajectories in the turbulent atmosphere, Boundary-Layer Meteorol. **78**, 191 (1996).
[9] A. N. Kolmogorov, The local structure of turbulence in an incompressible fluid for very large Reynolds numbers, Dokl. Akad. Nauk SSSR **30**, 299 (1941).
[10] W Heisenberg, Zur statistichen theorie der turbulenz, Z. Phys. **124**, 628 (1948).
[11] A. M. Yaglom, On the acceleration field in a turbulent flow, C. R. Akad. Nauk. URSS **67**, 795 (1949).
[12] P. K. Yeung and S. B. Pope, Lagrangian statistics from direct numerical simulations of isotropic turbulence, J. Fluid Mech. **207**, 531 (1989).
[13] P. Vedula and P. K. Yeung, Similarity scaling of acceleration and pressure statistics in numerical simulations of isotropic turbulence, Phys. Fluids **11**, 1208 (1999).
[14] T. Gotoh and R. S. Rogallo, Intermittency and scaling of pressure at small scales in forced isotropic turbulence, J. Fluid Mech. **396**, 257 (1999).
[15] G. A. Voth, A. La Porta, A. M. Crawford, J. Alexander, and E. Bodenschatz, Measurement of particle accelerations in fully developed turbulence, J. Fluid Mech. **469**, 121 (2002).

[16] B. L. Sawford, P. K. Yeung, M. S. Borgas, P. Vedula, A. La Porta, A. M. Crawford, and E. Bodenschatz, Conditional and unconditional acceleration statistics in turbulence, Phys. Fluids **15**, 3478 (2003).

[17] N. Mordant, E. Lévêque, and J.-F. Pinton, Experimental and numerical study of the Lagrangian dynamics of high reynolds turbulence, New J. Phys. **6**, 116 (2004).

[18] A. Gylfason, S. Ayyalasomayajula, and Z. Warhaft, Intermittency, pressure and acceleration statistics from hot-wire measurements in wind-tunnel turbulence, J. Fluid Mech. **501**, 213 (2004).

[19] P. K. Yeung, S. B. Pope, A. G. Lamorgese, and D. A. Donzis, Acceleration and dissipation statistics of numerically simulated isotropic turbulence, Phys. Fluids **18**, 065103 (2006).

[20] T. Ishihara, Y. Kaneda, M. Yokokawa, K. Itakura, and A. Uno, Small-scale statistics in high resolution of numerically isotropic turbulence, J. Fluid Mech. **592**, 335 (2007).

[21] R. J. Hill, Scaling of acceleration in locally isotropic turbulence, J. Fluid Mech. **452**, 361 (2002).

[22] A. M. Reynolds, Superstatistical Mechanics of Tracer-Particle Motions in Turbulence, Phys. Rev. Lett. **91**, 084503 (2003).

[23] C. Beck, Statistics of Three-Dimensional Lagrangian Turbulence, Phys. Rev. Lett. **98**, 064502 (2007).

[24] L. Bentkamp, C. C. Lalescu, and M. Wilczek, Persistent accelerations disentangle Lagrangian turbulence, Nat. Commun. **10**, 3550 (2019).

[25] M. S. Borgas, The multifractal Lagrangian nature of turbulence, Phil. Trans. R. Soc. A **342**, 379 (1993).

[26] L. Chevillard, S. G. Roux, E. Lévêque, N. Mordant, J.-F. Pinton, and A. Arnéodo, Lagrangian Velocity Statistics in Turbulent Flows: Effects of Dissipation, Phys. Rev. Lett. **91**, 214502 (2003).

[27] L. Biferale, G. Boffetta, A. Celani, B. J. Devenish, A. Lanotte, and F. Toschi, Multifractal Statistics of Lagrangian Velocity and Acceleration in Turbulence, Phys. Rev. Lett. **93**, 064502 (2004).

[28] K. R. Sreenivasan and R. A. Antonia, The phenomenology of small-scale turbulence, Annu. Rev. Fluid Mech. **29**, 435 (1997).

[29] U. Frisch, *Turbulence: The Legacy of Kolmogorov* (Cambridge University Press, Cambridge, England, 1995).

[30] T. Ishihara, T. Gotoh, and Y. Kaneda, Study of high-Reynolds number isotropic turbulence by direct numerical simulations, Annu. Rev. Fluid Mech. **41**, 165 (2009).

[31] R. S. Rogallo, Numerical experiments in homogeneous turbulence, NASA Technical Memo, 1981.

[32] D. Buaria, A. Pumir, E. Bodenschatz, and P. K. Yeung, Extreme velocity gradients in turbulent flows, New J. Phys. **21**, 043004 (2019).

[33] D. Buaria and K. R. Sreenivasan, Dissipation range of the energy spectrum in high Reynolds number turbulence, Phys. Rev. Fluids **5**, 092601(R) (2020).

[34] D. Buaria, E. Bodenschatz, and A. Pumir, Vortex stretching and enstrophy production in high Reynolds number turbulence, Phys. Rev. Fluids **5**, 104602 (2020).

[35] D. Buaria, A. Pumir, and E. Bodenschatz, Self-attenuation of extreme events in Navier-Stokes turbulence, Nat. Commun. **11**, 5852 (2020).

[36] D. Buaria and A. Pumir, Nonlocal amplification of intense vorticity in turbulent flows, Phys. Rev. Research **3**, L042020 (2021).

[37] D. Buaria, A. Pumir, and E. Bodenschatz, Generation of intense dissipation in high Reynolds number turbulence, Phil. Trans. R. Soc. A **380**, 20210088 (2022).

[38] Because of resolution concerns, we only use series 2 data from [20], corresponding to $k_{\max}\eta_K \approx 2$.

[39] J. M. Lawson, E. Bodenschatz, C. C. Lalescu, and M. Wilczek, Bias in particle tracking acceleration measurement, Exp. Fluids **59**, 172 (2018).

[40] D. Buaria, B. L. Sawford, and P. K. Yeung, Characteristics of backward and forward two-particle relative dispersion in turbulence at different Reynolds numbers, Phys. Fluids **27**, 105101 (2015).

[41] D. Buaria, P. K. Yeung, and B. L. Sawford, A Lagrangian study of turbulent mixing: Forward and backward dispersion of molecular trajectories in isotropic turbulence, J. Fluid Mech. **799**, 352 (2016).

[42] D. Buaria and P. K. Yeung, A highly scalable particle tracking algorithm using partitioned global address space (PGAS) programming for extreme-scale turbulence simulations, Comput. Phys. Commun. **221**, 246 (2017).

[43] This data is at somewhat lower resolution of $k_{\max}\eta_K \approx 1.5$, which does not effect the variance, but errors for higher-order moments are significant.

[44] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.234502 for additional details.

[45] C. Meneveau and K. R. Sreenivasan, The multifractal nature of turbulent energy dissipation, J. Fluid Mech. **224**, 429 (1991).

[46] K. R. Sreenivasan, On the scaling of the turbulence energy dissipation rate, Phys. Fluids **27**, 1048 (1984).

[47] C. Meneveau and K. R. Sreenivasan, Simple Multifractal Cascade Model for Fully Developed Turbulence, Phys. Rev. Lett. **59**, 1424 (1987).

[48] Note, this result corresponds to absolute moments, since we only considered the magnitude, but the odd moments of acceleration components are identically zero from symmetry.

[49] Z.-S. She and E. Leveque, Universal Scaling Laws in Fully Developed Turbulence, Phys. Rev. Lett. **72**, 336 (1994).

[50] A. N. Kolmogorov, A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high Reynolds number, J. Fluid Mech. **13**, 82 (1962).

[51] K. R. Sreenivasan and P. Kailasnath, An update on the intermittency exponent in turbulence, Phys. Fluids A **5**, 512 (1993).

[52] D. Buaria and K. R. Sreenivasan, Intermittency of turbulent velocity and scalar fields using 3D local averaging, arXiv: 2204.08132.

[53] B. L. Sawford and P. K. Yeung, Direct numerical simulation studies of Lagrangian intermittency in turbulence, Phys. Fluids **27**, 065109 (2015).

[54] A. Arnéodo, R. Benzi, J. Berg, L. Biferale, E. Bodenschatz *et al.*, Universal Intermittent Properties of Particle Trajectories in Highly Turbulent Flows, Phys. Rev. Lett. **100**, 254504 (2008).

[55] A. S. Lanotte, L. Biferale, G. Boffetta, and F. Toschi, A new assessment of the second-order moment of lagrangian velocity increments in turbulence, J. Turb. **14**, 34 (2013).

[56] D. Buaria and A. Pumir, Vorticity-Strain Rate Dynamics and the Smallest Scales of Turbulence, Phys. Rev. Lett. **128**, 094501 (2022).

[57] The forcing term has negligible contribution to acceleration moments. This is also reaffirmed by collapse of data in Fig. 1 from various sources that use different forcings.

[58] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* (MIT Press, Cambridge, MA, 1975), Vol. 2.

[59] R. J. Hill and J. M. Wilczak, Pressure structure functions and spectra for locally isotropic turbulence, J. Fluid Mech. **296**, 247 (1995).

[60] R. A. Antonia, A. J. Chambers, and B. R. Satyaprakash, Reynolds number dependence of high-order moments of the streamwise turbulent velocity derivative, Bound.-Layer Meteorol. **21**, 159 (1981).