Fluctuations in Heat Current and Scaling Dimension

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In this work, we theoretically study the heat flow between two $1+1$D chiral gapless systems connected by a point contact. With a small temperature gradient between the two, we find that the ratio between fluctuations of the heat current and the heat current itself is proportional to the scaling dimension—a universal number that characterizes the distribution of the particles tunneling through the point contact. We adopt two different approaches, scattering theory and conformal field theory, to calculate this ratio and see that their results agree. Our findings are useful for probing not only fractional charge excitations in fractional quantum Hall states but also neutral ones.

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Introduction.—The flow of charge in an electric circuit is not continuous due to the discrete nature of the charge carriers. Similarly, we expect that the energy or the heat flow in the system will be comprised of "lumps" of energy associated with each carrier. In 1918, Schottky investigated the noncontinuous character by measuring electric current fluctuations in a vacuum tube [1]. In these tubes, the current flows by Poisson processes of independent and rare electron emission events from the tube filament. The properties of the Poisson distribution indicate that the ratio between the variance of the electric current and the average current is equal to the electric charge emitted by a single event the electron charge. Namely, $\mathcal{F}_C = S_C/2I_C = e^*$, where S_C, I_C, and e^* represents charge noise, charge current, and charge of the system, respectively.

There is a clear physical distinction between electric current and heat current: while in each random emission event the charge is fixed, the energy carried by the emitted carrier is not. Furthermore, the probability of emission itself may depend on the energy. The ratio between fluctuations in the electric current and the average electric current has long been used to infer the charge of carriers [2–5]. Analogously, it stands to reason that the ratio between the fluctuations in the heat current and the average heat current can be utilized to infer properties of the energy distribution of the emission events. In this Letter, we show that this is indeed the case. In particular, the quantum number that can be extracted from these measurements is the scaling dimension of the emitted quasiparticles, h.

We study the heat flow between two $1+1$D chiral gapless systems connected by a point contact. Canonical examples of such systems are the edges of two-dimensional systems subjected to a strong magnetic field in the fractional quantum Hall (FQH) regime. Quasiparticles which tunnel between the edges carry charge, as well as energy. Much experimental progress has been made recently in heat current measurement of FQH states, including non-Abelian phases [6–11]. Theories of FQH states predict the charges of the quasiparticles e^* as well as the scaling dimension h of the operator creating a quasiparticle [12–14]. In certain simple cases, for example at $\nu = 1/3$, there is a simple relation between h and, θ, the exchange statistics phase of the quasiparticles, via $e^{i\theta} = e^{2\pi i h}$. The scaling dimension also determines the exponent of the power law of the effective tunneling amplitude as a function of energy [15,16]. Previous attempts to measure the scaling dimension focused on measuring these power laws through the charge current [17–19].

We calculate the average heat current which tunnels through a quantum point contact (QPC), I_E, as well as the fluctuations of the heat current, S_E. When the temperature difference between the two edges, ΔT, is much smaller than the temperature of the cold edge, T, we find that the ratio between the two is

$$\mathcal{F}_E = \frac{S_E}{2I_E} = (4h+1)k_B T, \tag{1}$$

where k_B is the Boltzmann constant. This universal result is valid also for charge-neutral particles and demonstrates the importance of the scaling dimension in governing the energy distribution of excitations along the edge. We emphasize the difference between this regime and the shot-noise regime typically used to extract quasiparticle charge [2–5], which requires a *large* bias voltage $V \gg T$ between the two edges.

In the rest of the Letter we will prove Eq. (1) and show that it holds for generic interacting edge modes described by conformal field theory (CFT) [20]. We extend these results beyond the small-ΔT regime described in Eq. (1), obtaining a closed integral expression for the heat current and heat current fluctuations for general values of T_L and T_R. We focus on another limit of interest, tunneling from a hot edge $T_L = T$ to a cold edge $T_R \to 0$, in Eq. (9).

A schematic demonstration of how quantum dots may be used to probe the heat current fluctuations is given in the Supplemental Material [21] A2 for noninteracting fermionic states. Generalization to other cases requires more delicate treatment in the spirit of, for example, Ref. [22], which we leave for future works. There is subtlety when a non-Abelian anyon tunnels through the QPC due to the intrinsic entropy of each carrier. Therefore, in this work, we focus on Abelian anyon tunneling, leaving the non-Abelian cases for future works.

Scattering theory.—We begin with calculations for noninteracting fermions and bosons, using the standard tool of scattering theory. Focusing on the geometry of Fig. 1, the heat current and heat current fluctuations measured at drain 1 are given by the standard formulas [23,24]

$$I_E = \frac{1}{h}\int dER(E)[f_R^{\pm} - f_L^{\pm}]E, \tag{2}$$

$$\tilde{S}_E = \frac{2}{h}\int dEE^2\{[2-R(E)]f_R^{\pm}[1\mp f_R^{\pm}] + R(E)f_L^{\pm}[1\mp f_L^{\pm}] \pm R(E)[1-R(E)](f_R^{\pm}-f_L^{\pm})^2\}. \tag{3}$$

Here, $f_{R/L}^{\pm} \equiv [\exp(E/T_{R/L}) \pm 1]^{-1}$ is the distribution function for the right- and left-moving particles, with the $+$ sign representing the Fermi-Dirac distribution for fermions, and the—sign representing the Bose-Einstein distribution for bosons. The coefficient $R(E)$ is the probability for an incident particle of energy E to tunnel across the QPC. The heat current and heat current fluctuations at D_2 can be obtained by similar methods. We integrate Eqs. (2) and (3) for a general scatterer obeying $R(E) = R_0(E/E_c)^{\alpha}$, where $E_c \gg T_i$ is a high-energy cutoff. As will be shown in the following sections, for a standard QPC, we expect $\alpha = 4h - 2$, where h is the scaling dimension of the tunneling quasiparticle. This corresponds to $\alpha = 0$ for fermions and $\alpha = 2$ for bosons.

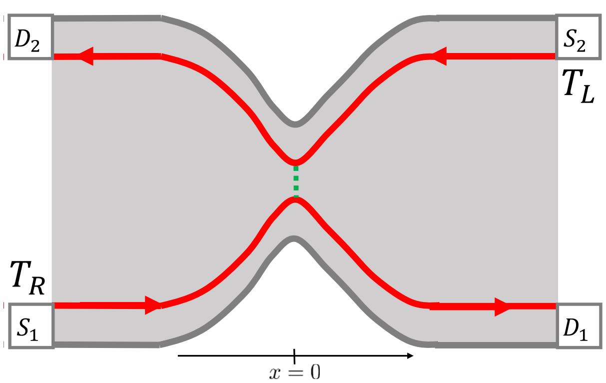

FIG. 1. A configuration of counterpropagating edge modes (red arrows) of an integer quantum Hall state. The quantum point contact (QPC), where quasiparticle tunneling occurs (green dashed line), is located at $x = 0$. The source and drain of the right- and left-moving edge mode are respectively represented by $S_{1/2}$ and $D_{1/2}$. Depending on the context, the edge modes can be replaced with the ones of a fractional quantum Hall state with filling fraction ν or with the ones of topological ordered phases described by a conformal field theory.

We wish to use these quantities to probe the quasiparticles which tunnel through the scatterer. As such, we focus only on the excess heat current fluctuations, i.e., the noise contribution that is obtained from a nonzero $R(E)$, $S_E \equiv \tilde{S}_E - \tilde{S}_E|_{R(E)=0}$. We then define the "heat Fano factor" as the ratio between the excess heat current fluctuations and the tunneling heat current, $\mathcal{F}_E \equiv S_E/2I_E$.

Calculating these integrals, we obtain the heat Fano factor for two limits of interest. For nearby temperatures, $T_R \equiv T$, $T_L = T + \Delta T$, and small reflection $R_0 \ll 1$, we obtain, for both fermions and bosons,

$$\mathcal{F}_E = (\alpha + 3)k_B T. \tag{4}$$

The limit of $T_L = T$ and $T_R = 0$ and further details are relegated to the Supplemental Material A [21]. [See also Eq. (9), where the same limit is considered in the CFT approach].

CFT approach.—Now we turn to *interacting* cases by considering the edge modes described by a CFT with central charge c. We remark that, one can obtain the same results by means of the standard bosonization formalism instead of adapting the CFT approach. This is illustrated in detail in the Supplemental Material B [21]. For simplicity, in the following calculations, we will set $\hbar = k_B = 1$. We envisage the same geometry as the scattering theory portrayed in Fig. 1, where there is a pair of counterpropagating edge modes of a CFT, connected by a single QPC. We denote the temperature of the right and left-moving edge by $T_{R/L}$, and introduce the QPC at the coordinate $x = 0$. Notice that we do not impose a potential gradient between the two edges.

The Hamiltonian of this system is given by

$$H = H_0 + H_T,$$
$$H_0 = \frac{v}{2\pi}\int_{-\infty}^{+\infty} dx[\mathcal{T} + \bar{\mathcal{T}}]$$
$$H_T = \Gamma_0 O_R(0) O_L(0). \tag{5}$$

Here, H_0 describes kinetic terms consisting of the stress-energy tensor $\mathcal{T}(z)[\bar{\mathcal{T}}(\bar{z})]$ in the right [left] moving sector with $z = i(vt - x)[\bar{z} = i(vt + x)]$ and v is the velocity of the edge mode [20]. The term H_T represents tunneling at the QPC, and the tunneling entity is described by the operator $O_{R/L}(z)$ with a tunneling amplitude Γ_0. This operator is a primary field of the CFT, with a scaling dimension of $h_{O_R} = h_{O_L} \equiv h_O$ [25].

We evaluate the heat current at the drain D_1, which we place at coordinate $x = d$, at time t. The unperturbed heat current is defined by $I_E^{(0)}(t) = (v^2/2\pi)\mathcal{T}(d, t)$ [26]. To find the correction to the heat current induced by the tunneling at the QPC, we resort to linear response theory, assuming the tunneling H_T in Eq. (5) is perturbatively small and the perturbation H_T is turned on at time $t = -\infty$. One obtains the perturbative expansion of the heat current operator up to the second order of Γ_0:

$$I_E(t) = I_E^{(0)}(t) + I_E^{(1)}(t) + I_E^{(2)}(t) + \mathcal{O}(\Gamma_0^3), \tag{6}$$

where

$$I_E^{(1)}(t) = i\int_{-\infty}^{t} dt' \left[H_T(t'), \frac{v^2}{2\pi}\mathcal{T}(d,t)\right],$$
$$I_E^{(2)}(t) = i^2\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \left[H_T(t''), \left[H_T(t'), \frac{v^2}{2\pi}\mathcal{T}(d,t)\right]\right].$$

See the Supplemental Material C [21] for more details.

The unperturbed heat current $I_E^{(0)}(t)$ is related to the heat conductance via $\kappa = \{[\partial\langle I_E^{(0)}(t)\rangle]/[\partial T_R]\} = (\pi c/6)T_R$, from which we can extract the central charge c of the edge mode [7,8,26–28]. Since we are interested in excess heat fluctuations driven by the quasiparticle tunneling, throughout this work, we concentrate on the perturbative corrections to the heat current and noise, i.e., the second and third term in Eq. (6). Taking the expectation value of these terms gives

$$\langle I_E^{(1)}(t)\rangle = 0,$$
$$\langle I_E^{(2)}(t)\rangle = -i\Gamma_0^2 \int_{-\infty}^{+\infty} d\tau G_R(\tau)\partial_\tau G_L(\tau), \tag{7}$$

where we have introduced the correlator of the primary field in the right- and left-moving sector at temperature $T_{R/L}$ as $G_{R/L}(\tau) = \langle O_{R/L}(\tau)O_{R/L}(0)\rangle$. The scaling dimension enters as the power law of the correlator. At zero temperature, this can be seen as $G_{R/L}(\tau) \sim \tau^{-2h_{O_{R/L}}}$.

When there is no temperature gradient between the edges, $\langle I_E^{(2)}(t)\rangle$ vanishes [29]. In the presence of a small temperature gradient between the two edge modes, i.e., $T_R = T$, $T_L = T + \Delta T$ ($|\Delta T/T| \ll 1$), the heat current (7) behaves as $\langle I_E^{(2)}(t)\rangle \sim T^{4h_O - 1}\Delta T$.

We proceed to calculate the heat current fluctuations. We define these as

$$\tilde{S}_E(\omega) = \int_{-\infty}^{+\infty} dt_{12} e^{i\omega t_{12}} \langle\{\Delta I_E(t_1), I_E(t_2)\}\rangle,$$

with $\Delta I_E(t) = I_E(t) - \langle I_E(t)\rangle$ and $\{\cdots,\cdots\}$ representing an anticommutator. Expanding the heat current up to second order in Γ_0, as demonstrated in Eq. (6), yields [30]

$$\tilde{S}(\omega) = S_E^{(00)}(\omega) + S_E^{(11)}(\omega) + S_E^{(02)}(\omega) + S_E^{(20)}(\omega) + \mathcal{O}(\Gamma_0^3).$$

Here, we have defined $(i,j = 0,1,2)$

$$S_E^{(ij)}(\omega) = \int_{-\infty}^{+\infty} dt_{12} e^{i\omega t_{12}} \langle\{\Delta I_E^{(i)}(t_1), I_E^{(j)}(t_2)\}\rangle.$$

The term $S_E^{(00)}(\omega)$ gives the unperturbed heat current fluctuations, and is an equilibrium property. Known also as the so-called Johnson-Nyquist (JN) noise of the heat current [31,32], it is related to the heat conductance κ in the dc limit by $S_E^{(00)}(0) = 2\kappa T_R^2$, which is addressed for non-interacting cases in Ref. [24]. This relation is in line with the one between the charge JN noise S_c and the charge conductance G via $S_c = 2GT$ in the thermal noise limit.

We are interested only in the perturbative contributions to the heat current fluctuations. We hence focus on the excess heat current fluctuations, defined as $S_E(\omega) = \tilde{S}_E(\omega) - S_E^{(00)}(\omega)$. The excess heat current fluctuations are comprised of three terms. For $S_E^{(11)}(\omega)$, which represents the autocorrelations of the heat current which tunnels at the QPC, it is straightforward to derive the form

$$S_E^{(11)}(\omega) = -2i\Gamma_0^2 \int_{-\infty}^{+\infty} d\tau \cos(\omega\tau) G_R(\tau)\partial_\tau^2 G_L(\tau).$$

To evaluate the "cross terms" $S_E^{(02)}(\omega) + S_E^{(20)}(\omega)$, corresponding to the correlation between the unperturbed and excess heat current, one has to calculate the correlator involving the stress-energy tensor and the primary fields. Such a task can be accomplished by exploiting the conformal Ward identity [20], as outlined in the Supplemental Material C [21]. Overall, the excess heat current fluctuations in the dc limit $\omega \to 0$ are given by

$$S_E(0) = 4T_R(2h_O - 1)\langle I_E^{(2)}\rangle + S_E^{(11)}(0) + 2iT_R \frac{\partial S_E^{(11)}(\omega)}{\partial\omega}\bigg|_{\omega\to 0}. \tag{8}$$

We are now are in a good place to study the heat Fano factor, defined by

$$\mathcal{F}_E = \frac{S_E(0)}{2\langle I_E^{(2)}(t)\rangle}.$$

The first term of Eq. (8) includes $\langle I_E^{(2)}(t)\rangle$, therefore, the heat Fano factor has the term proportional to the scaling dimension which is a universal number. What remains to do is to calculate the ratio between the last two terms of Eq. (8) and $\langle I_E^{(2)}(t)\rangle$.

It is challenging to evaluate this ratio analytically for generic values of $T_{R/L}$. Instead of doing this, we focus on the case of a small temperature gradient, $T_R = T$, $T_L = T + \Delta T$ ($|\Delta T/T| \ll 1$), and expand the last two terms in Eq. (8) up to first order of $\Delta T/T$. Relegating the details to the

Supplemental Material C [21], and retrieving the Boltzmann constant k_B, the heat Fano factor becomes $\mathcal{F}_E = (4h_O + 1)k_B T$, which completes the proof of Eq. (1).

We extend our result to an additional regime of interest with a large temperature gradient, by setting $T_R = 0$ and $T_L = T$. In this case, the heat Fano factor is given by

$$\mathcal{F}_E = (2h_O + 1)(\pi k_B T)\frac{J(2h_O, 2)}{J(2h_O, 1)}, \tag{9}$$

where we define the integral (see the Supplemental Material C [21])

$$J(a, b) = \int_{-\infty}^{+\infty} dz[\cosh(z)]^{-a}\left[i\left(z - i\frac{\pi}{2}\right)\right]^{-a-b}.$$

When h_O takes integer or half-integer values, the integrals of Eq. (9) can be explicitly calculated via contour integration. This includes two canonical cases: when $h_O = 1/2$, corresponding to fermion tunneling, the heat Fano factor is given by $\mathcal{F}_E = 3k_B T\{[\zeta(3)]/[\zeta(2)]\}$; when $h_O = 1$, which corresponds to density-density interactions, we have $\mathcal{F}_E = 4k_B T\{[\zeta(5)]/[\zeta(4)]\}$. Here, $\zeta(s)$ is the Zeta function. Interestingly, this result coincides with the one obtained from scattering theory. The more general case is relegated to Supplemental Material C [21].

Discussion.—We identify several advantages to focusing on heat vs alternate approaches that have been proposed to extract the scaling dimension from charge fluctuations [33–36]. First, a crucial feature of the heat current measurement is that it may probe not only charged excitations but also *neutral* ones, as both of them carry heat. Heat current measurement thus opens a new possibility to provide us with smoking gun evidence for neutral topological order phases [37–39]. Indeed, experiments probing the quantized heat conductance $\kappa = c(\pi k_B/6\hbar)T$, with c being the central charge of the FQH edge mode, have been successfully realized for both integer [6,7] and half-integer [8,9] central charges, the latter hosted by non-Abelian phases [40].

Second, charge current based measurements often require bias voltages that are *larger* than the base temperature. This, combined with the requirement that voltages not exceed the bulk gap, limits the available temperature range for measurements. Large bias voltages may furthermore lead to nonuniversal effects, such as changing the confining potential at the edge and the electrostatic properties of the QPC, or enabling longitudinal conductance through the bulk. Conversely, our method requires only small temperature gradients, such that effects that may mask the power law behavior of the tunneling are minimized. Finally, probing power laws directly requires bias voltages which span several decades, an issue which we outright avoid.

Summary.—Recent decades have seen considerable interest in noise as a means for extracting physical insight, as opposed to an unfortunate byproduct of an imperfect measuring apparatus. In the condensed matter community, the ratio between the electric current fluctuations and the average electric current has been used to extract the quantized charge of carriers [2–5]. In this Letter, we examine the ratio between the heat current fluctuations and the average heat current, which we dub the heat Fano factor. We refer the reader to further works discussing heat current fluctuations in theoretical [41–44] and experimental [45,46] contexts, as well as the references therein.

We demonstrate that the heat Fano factor for tunneling between two gapless one-dimensional modes yields a universal number. In particular, in the presence of a small temperature gradient between the two edges, $\Delta T \ll T$, the scaling dimension h of the tunneling quasiparticle is immediately obtained from the heat Fano factor given in Eq. (1). This property plays a crucial role in governing the edge dynamics of the gapless edge excitations [14]. For certain simple cases, such as Abelian theories with no counterpropagating modes, the scaling dimension is directly related to the statistical phase obtained by braiding two quasiparticles, via $\theta = 2\pi h$.

For completeness, we also obtain closed integral expressions for the heat Fano factor for two general edge temperatures, which solely depends on the scaling dimension. If $2h$ is an integer, and the temperature of the cold edge is zero, then this expression is reduced to fractions of Zeta functions (see the Supplemental Materials B and C for more details [21]).

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