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# Water Wave Polaritons

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We find that a one-dimensional groove array can be equivalent to a negative water depth and excite unidirectional surface polaritons for water waves. We explain this phenomenon through theoretical analysis, numerical simulations, and experiments. This phenomenon shows that the propagation direction of water waves can be manipulated through such simple structures, which will be very important in offshore transportation and environmental protection.

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*Introduction.*—In recent decades, electromagnetic metamaterials have developed rapidly and become a favorable tool for people to control electromagnetic (EM), elastic, acoustic, and water waves at will. Metamaterials are artificial materials composed of subwavelength structures, which can realize amazing wave functional phenomena. People have found many interesting effects and implemented optical devices with metamaterials, such as negative refraction devices [1–3], zero index devices [4], super lenses [5], invisibility cloaks [6–8], etc. In addition, surface plasmon polaritons (SPPs) are highly localized surface waves existing at the interface of two media with opposite dielectric constants at the same optical frequency. There are many applications for SPPs, such as field confinement and subwavelength resolution imaging, yet suffering from difficult fabrication and lack of low loss metals. Spoof SPPs [9] inherit the characteristics of natural SPPs, such as dispersion relationships. Therefore, they could be regarded as a metamaterial version of SPPs and have a wide application prospect, with low loss and ease of fabrication as well. The frequency of electromagnetic response for spoof SPPs can be changed by modifying the geometric parameters of the spoof structures.

Moreover, the wave equation of water waves has a similar form to that of electromagnetic waves under shallow conditions [10], where different equivalent refractive indices can be achieved by adjusting the relative depth of the water surface [11–13]. Therefore, it is possible to control water waves by using the concept of metamaterials. In recent years, people have used metamaterials to realize zero refractive index and focusing of water waves [14], negative refraction of water waves [15], energy concentrators of water waves [16], cloaking of water waves [17], etc.

Inspired by these works, is it possible for surface polaritons to be excited, and can their unidirectional propagation be realized in water waves? In this Letter, we have successfully realized the unidirectional propagation of surface polaritons for water waves. The schematic structure is shown in Fig. 1(a), where $\boldsymbol{a}$ is the groove width, $\boldsymbol{d}$ is the width of a period, $\boldsymbol{s}$ is the horizontal groove depth, and $\boldsymbol{h}$ is the static water depth (or average water depth). We will see that, under this one-dimensional groove array, the amplitude of a water wave (excited by a rotating tiny screw propeller) on one side (e.g., the left part of the water source) could be greatly weakened while the amplitude on the other side (the right part of the water source) is enhanced, forming a unidirectional surface polariton along the groove array. We will prove this exciting unidirectional effect from theoretical calculation and numerical simulations and discover the corresponding phenomena in experiments.

*Analytical analysis and numerical simulations.*—Let us first compare the shallow water equation [18] (when $kh \ll 1$) with the $z$-invariant two-dimensional wave equation for transverse magnetic (TM) EM waves. The water wave equation is written as

$$\nabla \cdot (h \nabla \eta) + \frac{\omega^2}{g} \eta = 0, \tag{1}$$

where $\eta$ is the vertical displacement of the water surface [as shown in the lower left corner in Fig. 1(a)], $\omega$ is the angular frequency, $g$ is the gravitational acceleration, and $h$ is the static water depth (or average water depth). The surface wave equation above the water wave is governed by linear dispersion $\omega = \sqrt{gh}k$. The Maxwell's equations for TM EM waves could be rewritten as

$$\nabla \cdot \left( \frac{1}{\varepsilon} \nabla H_z \right) + \frac{\omega^2}{c^2} H_z = 0, \tag{2}$$

where $H_z$ is the amplitude of the magnetic field in the $z$ direction, $\omega$ is the angular frequency, $c$ is the speed of light in vacuum, and $\varepsilon$ is the dielectric constant. Through such a comparison, we can change the TM Maxwell's equation (2) into the shallow water equation (1) by a simple mapping (see

 

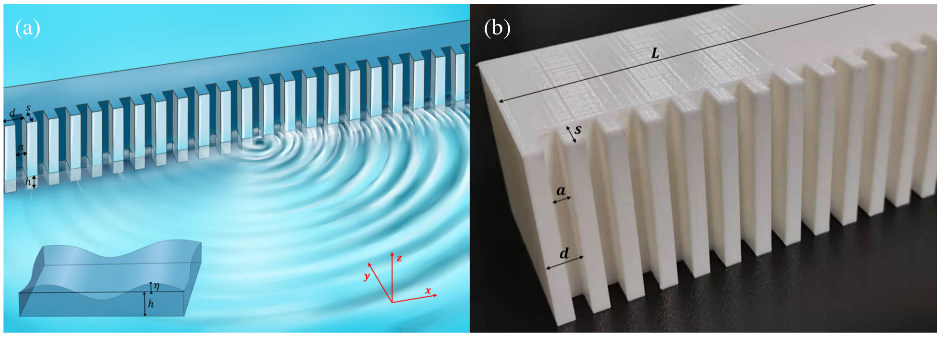


FIG. 1. Water wave unidirectional surface polaritons model and one-dimensional groove array structure. (a) Schematic diagram of unidirectional surface polaritons of water waves excited by a rotating tiny screw propeller. The picture in the lower left corner is an enlarged view of the vertical displacement of water surface $\eta$ and the static water depth $h$ (or average water depth). (b) Picture of the implemented structure and detailed size of the one-dimensional groove array used in the water wave experiment. The material is PLA plastic and made by 3D printing.

Supplemental Material, Sec. 1 [19–26]). It is noted that we have found a more general shallow water wave equation (S10) and the related boundary matching conditions (S24) for the case of anisotropic water depth and varying gravity. The hydrostatic pressure of the water surface $p$ corresponds to the magnetic field $H_z$ while $B_z \leftrightarrow \eta$, which is a more accurate mapping. It has been proved that, when the electromagnetic wave propagates on the metal surface, it is possible to stimulate unidirectional SPPs by circular polarized waves. It is also possible to excite similar unidirectional spoof surface polaritons, even in water waves with specific structures. Therefore, by analogy with the magnetic field expression for unidirectional SPPs [27], we can write the amplitude expression of the water wave in the $x-y$ plane $\eta(x,y) = H_{-1}(k_0 r)e^{-i\theta} = \int \tilde{\eta}(k_x, y)e^{ik_x x}dk_x$, where

$$\tilde{\eta}(k_x, y) = \frac{1}{\pi k_0}\left[-i\frac{k_x}{k_y} \mp 1\right]e^{ik_y|y-y_{\mathrm{source}}|}. \qquad (3)$$

$\tilde{\eta}(k_x, y)$ is the spatial Fourier transform of $\eta(x,y)$, $H_{-1}$ is the first kind of Hankel function of negative first order, $k_x$ is the wave number in the $x$ direction, $k_y = (k_0^2 - k_x^2)^{1/2}$ is the wave number in the $y$ direction, and $k_0 = 2\pi/\lambda$. $\lambda$ is the wavelength, $r$ and $\theta$ are cylindrical coordinate systems, and $r\cos\theta = x$, $r\sin\theta = y$. The minus and plus signs in Eq. (3), respectively, correspond to $y > y_{\mathrm{source}}$ and $y < y_{\mathrm{source}}$, where $y_{\mathrm{source}}$ is the location of the excited source (e.g., a rotating tiny screw propeller). We assume that the wave number of the water wave propagating along the positive direction and negative direction of the $x$ axis are $k_x > 0$ and $k_x < 0$, respectively. Because the wave source is located at $(0, y_{\mathrm{source}})$, we can use the left part and right part of the wave source to represent $k_x < 0$ and $k_x > 0$, respectively.

When $|k_x| > k_0$, $k_y = i\sqrt{k_x^2 - k_0^2}$, then we obtain

$$\tilde{\eta}(k_x, y) = \frac{1}{\pi k_0}\left[-\frac{k_x}{\sqrt{k_x^2 - k_0^2}} \mp 1\right]e^{-\sqrt{k_x^2-k_0^2}|y-y_{\mathrm{source}}|}. \qquad (4)$$

It can be seen that $\eta$ is an evanescent wave in the $y$ direction. And in the region of $y > y_{\mathrm{source}}$ [corresponding to minus signs in Eq. (4)], the spectral amplitude of the evanescent wave components are enhanced in the part of $k_x > 0$ (on the right part of the wave source), while for the part of $k_x < 0$ (on the left part of the wave source), the evanescent wave components are mutually eliminated. Therefore, by making the propagation wave number of the evanescent wave in the $x$ direction $|k_x| > k_0$, it is possible to excite the unidirectional water wave spoof surface polaritons [9].

The picture of metamaterial structure we implemented is shown in Fig. 1(b). It is a one-dimensional groove array. The material is polylactic acid (PLA) plastic. $\boldsymbol{a}$ is the groove width, $\boldsymbol{d}$ is the width of a period, and $\boldsymbol{s}$ is the horizontal groove depth. Through calculation, it can be concluded that the dispersion relationship of the excited surface mode of the one-dimensional groove array is (the derivation is in Supplemental Material, Sec. 1 [19]):

$$k_x = k_0\sqrt{1 + \left(\frac{a}{d}\right)^2 \tan^2(k_0 s)}. \qquad (5)$$

This structure meets the requirement of $|k_x| > k_0$, such that the unidirectional spoof surface polaritons could be excited.

We use the commercial software COMSOL Multiphysics for numerical simulations. We set $s = 0.85 * d$, $a = 0.5 * d$, and $d = \lambda/5.5$, for instance. The field patterns and the amplitude patterns for different cases are shown in Fig. 2. When the distance between the source and the one-dimensional groove array is small, e.g., $0.2 * \lambda$, the surface polariton intensity on the right side is much greater than that on the left side [Figs. 2(a) and 2(b)]. This is consistent with the theory [Eq. (3)]. When the distance between the source and the one-dimensional groove array is big, e.g., $0.4 * \lambda$, the unidirectional effect is not that obvious [Figs. 2(c) and 2(d)].

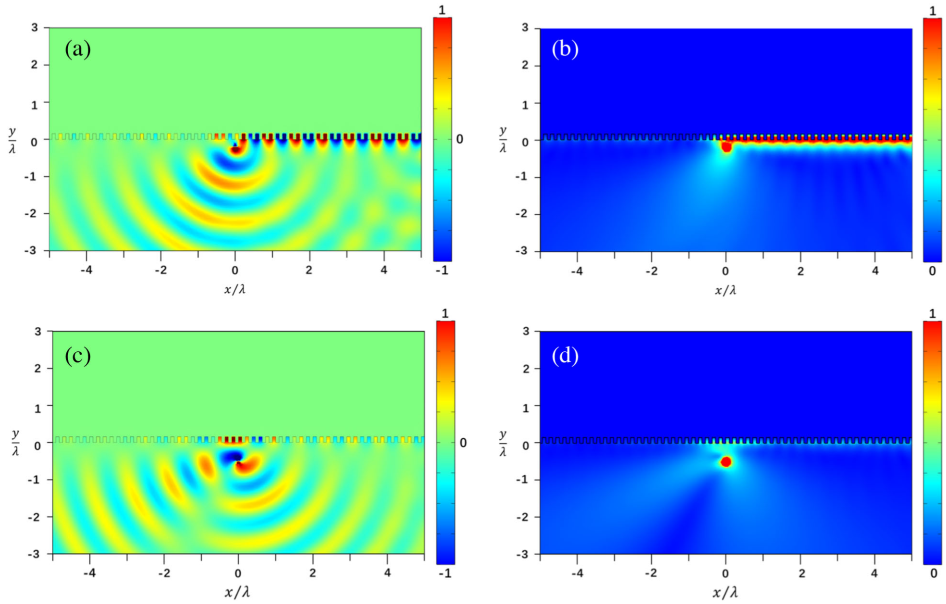


FIG. 2. Field patterns and amplitude patterns of water waves excited by a rotating tiny screw propeller on the one-dimensional groove array. (a) Field pattern; (b) amplitude pattern for the distance between the source and the one-dimensional groove array with $O = 0.2 * \lambda$; (c) field pattern; (d) amplitude pattern for the distance between the source and the one-dimensional groove array with $O = 0.4 * \lambda$. Here, $O$ is the distance between the source and the one-dimensional groove array, $y_{\mathrm{source}} = -O$, and $\lambda$ is the wavelength.

Following Ref. [10], the water wave equation in isotropy [Eq. (1)] could be extended to the anisotropy:

$$\nabla \cdot (\overleftrightarrow{h} \cdot \nabla \eta) + \frac{\omega^2}{g} \eta = 0, \tag{6}$$

where $\overleftrightarrow{h}$ is a tensor with $\overleftrightarrow{h} = [\begin{smallmatrix} h_x & 0 \\ 0 & h_y \end{smallmatrix}]$. The one-dimensional groove array can be equivalent to a water layer with an anisotropic water depth layer with a thickness of $s$ in front of a perfect impermeable rigid body, as shown in Supplemental Fig. S2 [19]. Here, we assume that the water depth is $h_1 = h_0$, $g_1 = g_0$ for $y < 0$, while the water depth at the anisotropic layer $(0 < y < s)$ is $h_{2x} = 0$, $h_{2y} = a/d * h_0$, and $g_2 = d/a * g_0$. By combining the isotropic transfer matrix method [28] and the wave equation of a water wave in an anisotropic water layer, we deduce the anisotropic transfer matrix method. Then we use the anisotropic transfer matrix method and the shallow water wave equation to obtain the reflection coefficient of the equivalent water layer (the derivation is in Supplemental Material, Sec. 1 [19]):

$$R = \frac{(h_0/h_{2y} * k_{1y} - k_0) + (h_0/h_{2y} * k_{1y} + k_0)e^{i2sk_0}}{(h_0/h_{2y} * k_{1y} + k_0) + (h_0/h_{2y} * k_{1y} - k_0)e^{i2sk_0}}. \tag{7}$$

By making the denominator of Eq. (7) equal to 0 [25], the dispersion relationship of the equivalent water layer can be obtained as $k_x = k_0\sqrt{1 + (a/d)^2 \tan^2(k_0 s)}$. This is exactly the same as the previous dispersion relation expression of the one-dimensional groove array [Eq. (5)], which verifies the above equivalence. By bringing the equivalent model in Supplemental Fig. S2 into numerical simulations [19], we plot the field pattern in Fig. 3(c). It can be seen that the excited source (a rotating tiny screw propeller) generates a rightward unidirectional surface polariton in the equivalent anisotropic water layer. We also plot the water wave displacement at $y = -0.1 * \lambda$ in Fig. 3(d). The water wave displacement on the right side is significantly greater than that on the left side. For comparison, we also plot the field pattern in the case of a one-dimensional groove array in Fig. 3(a) and the related water wave displacement at $y = -0.1 * \lambda$ in Fig. 3(b).

As is well known, the dielectric constant of metal $\varepsilon$ is negative in a specific frequency band, such that unidirectional SPPs can be excited on its surface [26], How about to achieve the similar effect in water waves? By observing Eqs. (1) and (2), if we can achieve a negative depth $h$, it would be possible to achieve this unidirectional surface polariton effect, which sounds quite weird but definitely deserves to be explored. In Supplemental Material, Sec. 1 and Supplemental Fig. S6 [19], we prove that the one-dimensional groove array can also be equivalent to an isotropic uniform water layer with a negative depth. We assume that the water depth is $h_1 = h_0$ for $y < 0$. Through calculation, we obtain the water depth at the isotropic water layer $(y > 0)$ as $h_2 = -0.35 * h_0$. At the same time, the dispersion relation of the equivalent water layer can be calculated as (the derivation is in Supplemental Material, Sec. 1 [19])

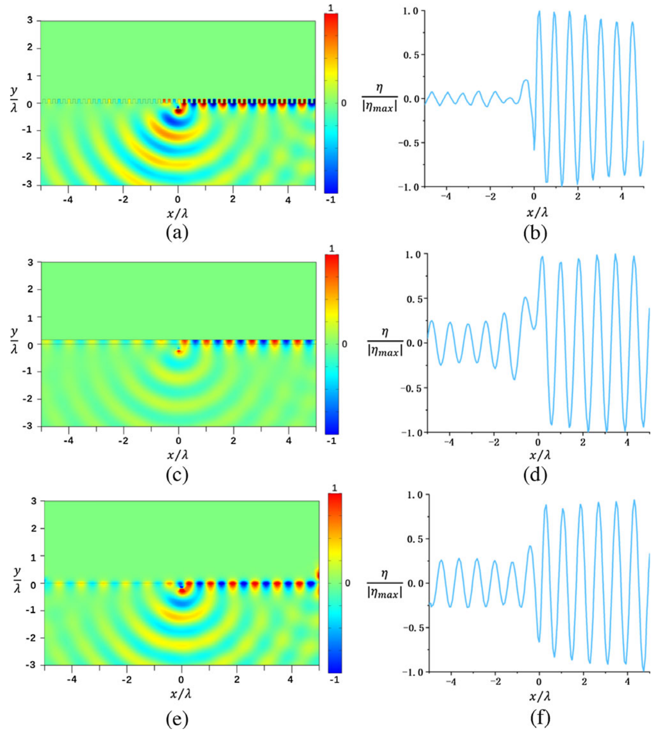


FIG. 3. Simulation of equivalent model in different water depths. (a) Field pattern of a water wave excited by a one-dimensional groove array. (b) Water wave displacement excited by a one-dimensional groove array at $y = -0.1 * \lambda$. (c) Field pattern of a water wave excited by an equivalent anisotropic water layer. (d) Water wave displacement excited by an equivalent anisotropic water layer at $y = -0.1 * \lambda$. (e) Field patterns of a water wave excited by an equivalent isotropic water layer with effective negative depth. (f) Water wave displacement excited by an equivalent isotropic water layer with effective negative depth at $y = -0.1 * \lambda$.

$$k_x = k_0\sqrt{\frac{h_0/h_1 * h_0/h_2}{h_0/h_1 + h_0/h_2}}. \tag{8}$$

By taking $h_1 = h_0$, $h_2 = -0.35 * h_0$ into Eq. (8), we obtain $k_x = 1.24k_0$, which is exactly the same as the calculation result of the dispersion relation expression [Eq. (5)] of the previous one-dimensional groove array. We also plot the field pattern in the case of negative depth in Fig. 3(e) and the related water wave displacement at $y = -0.1 * \lambda$ in Fig. 3(f) to verify this equivalence. By comparing Figs. 3(a), 3(c), and 3(e), we find that the case of a one-dimensional groove array is equivalent to both the anisotropic equivalent water layer and the isotropic water layer with negative depth. And from Figs. 3(b), 3(d), and 3(f), we see that the strength of a unidirectional surface polariton excited by the one-dimensional groove array is basically similar to that of the anisotropic equivalent water layer and isotropic equivalent water layer with negative water depth, which verifies again the theoretical calculation [Eqs. (3), (5), (7), and (8)].

*Experimental qualitative verifications.*—We then finish several experiments to qualitatively observe the unidirectional surface polaritons excited in water waves. In EM waves, groove perfect electric conductors can be used to achieve an artificial metal. Likewise, a groove impervious rigid body can also be applied to obtain an effective "water wave metal." We fabricate the structure shown in Fig. 1(b) with a 3D printer, with its material PLA plastic. The reference wavelength here is 5 cm, and the structural size of the corresponding one-dimensional groove array is $d = \lambda/5.5 = 0.909$ cm, $a = 0.5 * d = 0.455$ cm, and $s = 0.85 * d = 0.773$ cm. Considering the shallow water approximation of the water wave equation, the water depth $h$ in our experiment is about 1.5 cm.

The equipment used in the experiment is shown in Fig. 4(a). A rotating tiny screw propeller is used to excite water waves with angular momentum (like a circular light for EM waves). The impermeable rigid body is a one-dimensional groove array (in white), and the black sponges on the left and right sides are used to eliminate the reflection of water waves at boundaries (detailed structure could be seen in Supplemental Material, Sec. 2 [19]).

The experimental qualitative observations are shown in Figs. 4(b)–4(f). Figure 4(b) shows that the wave source (a clockwise rotating propeller) is far (7 cm) from the surface of the one-dimensional groove array (wavelength 5 cm). A point source with angular momentum can be seen, while there is no unidirectional surface polariton effect. When the source is moving close, about 0.2 wavelength (1 cm) from the surface, the amplitude of the water wave in a sector near the surface of the spoof structure on the left is significantly reduced, forming a unidirectional surface polariton propagating to the right side [see Fig. 4(c)]. By changing the rotation direction of the screw propeller, we can change the propagation direction of the unidirectional surface polariton. For example, Fig. 4(d) shows the case of a counterclockwise rotating propeller, where the amplitude of the water wave in a sector near the surface of the spoof structure on the right is significantly reduced, forming a unidirectional surface polariton propagating to the left side. As the surface polariton is not clearly shown above, we put two pairs of small balls on both sides of the source. Figure 4(e) shows an amazing transport effect, which is the combination of spoof-surface polaritons and flow nonlinearity of hydrodynamics [the lighting comes from the top, which produces the pattern of light and shadow observed in Fig. 4(e)]. As the surface polariton is propagating along the left side from Fig. 4(d), the pair on the left is driven to move quickly to the left end (see Supplemental Material, Video 3 [19]).

Meanwhile, the pair on the right stays on its position, showing that the surface polariton on the right side is very weak. By reducing the rotating speed of the propeller, i.e., for a longer wavelength (6 cm), the angular momentum of the source and the unidirectional surface polariton effect disappear, as shown in Fig. 4(f). We also put the demonstration

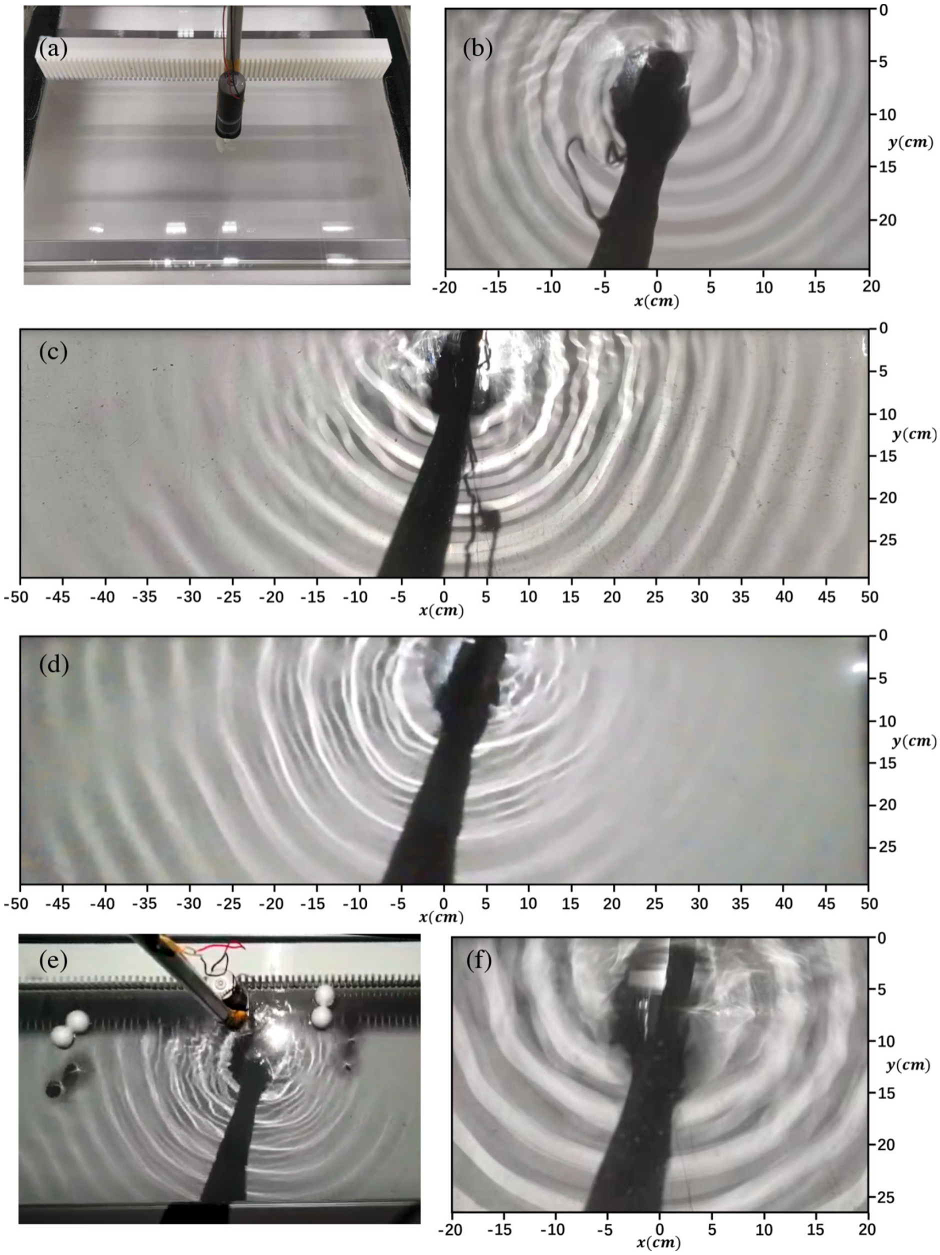


FIG. 4. Experimental observations. (a) Picture of the experimental setup. (b) Field pattern for the case when the rotating tiny screw propeller is far (7 cm) from the one-dimensional groove array. (c) Field pattern for the case when the rotating propeller is close to the one-dimensional groove array, 0.2 wavelength (1 cm). The propeller is clockwise rotating. (d) Field pattern for the case when the rotating propeller is close to the one-dimensional groove array, 0.2 wavelength (1 cm). The propeller is counterclockwise rotating. (e) The "transport" effect of surface polaritons to drive the balls to the left. (f) Field pattern for the case when the wavelength is 6 cm.

videos corresponding to Figs. 4(c)–4(e) in Supplemental Material, Sec. 2 [19].

*Discussion.*—Inspired by spoof SPPs in EM waves, we found that a rotating screw propeller can excite the unidirectional surface polaritons on the surface of a one-dimensional groove array for water waves. From theoretical analysis and numerical simulations, the one-dimensional groove array is equivalent to both a layer of anisotropic depth and, in particular, a layer of isotropic negative depth. In other words, we found that the groove array acts like a metal for water waves. We qualitatively observed the phenomena similar to simulations from experiments. Our work is of great value for understanding the mechanism of unidirectional surface polariton effect in water waves. Meanwhile, the unidirectional effect would be very important in cargo transportation near the port if further considering the nonlinear effect and high amplitudes, which might therefore be very useful for environmental protection in the future.

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