Editors' Suggestion

Thermodynamic Principle for Quantum Metrology

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The heat dissipation in quantum metrology represents not only an unavoidable problem towards practical applications of quantum sensing devices but also a fundamental relationship between thermodynamics and quantum metrology. However, a general thermodynamic principle which governs the rule of energy consumption in quantum metrology, similar to Landauer's principle for heat dissipation in computations, has remained elusive. Here, we establish such a physical principle for energy consumption in order to achieve a certain level of measurement precision in quantum metrology, and show that it is intrinsically determined by the erasure of quantum Fisher information. The principle provides a powerful tool to investigate the advantage of quantum resources, not only in measurement precision but also in energy efficiency. It also serves as a bridge between thermodynamics and various fundamental physical concepts related in quantum physics and quantum information theory.

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Introduction.—Deeper understanding of thermodynamic energy cost for information processing, and further reducing the heat generated by computations have always been a fundamental goal in information technology [1]. The ultimate physical limit on energy consumption of computation is set by Landauer's principle [2–13]. It states that the irreversible erasure of information would inevitably dissipate a certain amount of heat into environment, while reversible operations can in principle be implemented at no energy cost. Landauer's principle establishes a fundamental relationship between information theory and thermodynamics [14,15]. It represents the philosophy of "*information is physical*" [16], and is the key to the exorcism of Maxwell's demon [17–19].

Quantum metrology, as a fast-developing field of quantum technology, achieves highly sensitive measurements of physical parameters over classical techniques [20–23]. Understanding the quantum limits of quantum metrology, e.g., in terms of measurement precision and channel capacity, has yielded fundamental insights into its connections to quantum geometry [24], many-body entanglement [25], and Shannon-Hartley theorem [26]. Similar to computation, the energy consumption in quantum metrology would become an important issue towards potentially massive applications of quantum sensing devices, and represents a fundamental link between thermodynamics and quantum metrology. So, is there a general principle that determines the limit of energy cost in quantum metrology? The analysis of certain examples hints that work cost might be relevant in quantum metrology [27–29]. But these attempts are confined to specific models, and do not provide a complete answer. Here we take a very different approach and establish a fundamental principle for heat dissipation in quantum metrology.

As our main result, we find the physical limit on energy consumption of quantum metrology, and demonstrate that it essentially arises from the erasure of quantum Fisher information (QFI) which determines the best achievable measurement precision [24]. The result establishes a basic thermodynamic principle for quantum metrology, and clearly states the thermodynamic cost in order to achieve a certain level of measurement precision. We provide an efficient way to investigate energy efficiency of multiqubit states for quantum metrology, and point out that it is possible to achieve the Heisenberg limit of measurement precision with energy consumption that does not increase with the number of probes. Furthermore, the QFI plays important roles in various fundamental quantum phenomena, such as multipartite entanglement [30–33], quantum criticality [34], and quantum geometry [24]. Therefore, the present thermodynamics of quantum metrology would inspire us to explore the thermodynamic meaning of the related physical concepts involved in these phenomena.

Background on quantum metrology.—In quantum metrology, one generally prepares a quantum probe system in an initial state $\rho_0 = |\psi\rangle\langle\psi|$, which subsequently undergoes a parameter-dependent evolution U_λ to the final state $\rho_\lambda \equiv |\psi_\lambda\rangle\langle\psi_\lambda| = U_\lambda \rho_0 U_\lambda^\dagger$ [35–38]. In order to access information about the parameter, quantum metrology requires us to perform measurement on the parametrized state ρ_λ. Such a

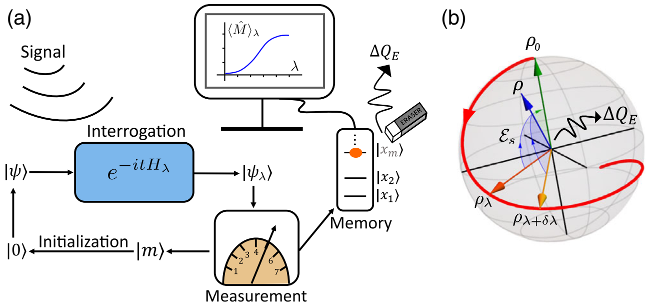

FIG. 1. Heat dissipation in quantum metrology. (a) In standard quantum metrology, after the interrogation for time t, the parametrized final state $|\psi_\lambda\rangle$ is measured, which provides information on the unknown parameter λ. To realize a closed metrological cycle, both the quantum probe system and the memory need to be recovered to their initial states. A unitary operation can evolve the system into its initial state $|0\rangle$ without heat dissipation; while the erasure of information stored in the memory about the measurement outcomes inevitably results in heat dissipation. (b) The QFI characterizes the distinguishability between neighboring parametrized quantum states ρ_λ and $\rho_{\lambda+\delta\lambda}$, which is erased as they are transformed into a parameter-independent state ρ via the map $\mathcal{E}_s$ (blue line). The erasure of QFI during this process will cause heat dissipation or entropy increase, see Eq. (9). A possible subsequent operation (green line) that transforms ρ back to the initial pure state may dissipate additional heat into environment according to Landauer's principle.

measurement in the basis of $\{|m\rangle\langle m|\}_{m=1}^{d}$ (d is the dimension of the system) can be described as [18,39]

$$\mathcal{M}: \rho_\lambda \otimes |x\rangle\langle x| \to \sum_m \alpha_m^\lambda |m\rangle\langle m| \otimes |x_m\rangle\langle x_m|, \tag{1}$$

where $\alpha_m^\lambda = \langle m|\rho_\lambda|m\rangle$, and $\{|x_m\rangle\}$ represents the internal structure of a memory, interacting with the system and storing the measurement outcomes, see Fig. 1(a). The ultimate precision of parameter estimation, achieved by performing optimal measurements on ρ_λ, is determined by the QFI,

$$F_Q[\rho_\lambda] = 4[\langle \partial_\lambda \psi_\lambda | \partial_\lambda \psi_\lambda \rangle - |\langle \psi | \partial_\lambda \psi_\lambda \rangle|^2], \tag{2}$$

that represents a measure of how much information a parametrized quantum state contains about the unknown parameter [24].

Main result.—In the standard framework of quantum metrology, see Fig. 1(a), the parameter information is transferred to the memory after the measurement. A unitary operation conditional on the memory's state brings the system back to the initial state without heat dissipation into environment because of its reversibility [40,41]. In order to realize a closed metrological cycle, the memory must be recovered to its original standard state $|x\rangle$, leading to a heat dissipation ΔQ_E^λ into environment according to Landauer's principle. Given an optimal measurement basis $\{|m_\lambda\rangle\langle m_\lambda|\}_{m=1}^{d}$ defined by the symmetric logarithmic derivative (SLD) [23,24], the Shannon entropy, $\mathcal{S} = -\sum_m \alpha_m^\lambda \log(\alpha_m^\lambda)$ with $\alpha_m^\lambda = |\langle m_\lambda|\psi_\lambda\rangle|^2$, which quantifies the amount of information associated with the measurement outcomes, is found to be determined by the QFI [41], namely,

$$\mathcal{S} \geq \log(2)\|h_\lambda\|^{-2} F_Q[\rho_\lambda], \tag{3}$$

where $h_\lambda = iU_\lambda^\dagger \partial_\lambda U_\lambda$ represents the local generator of parametric translation, and $\|h_\lambda\|$ denotes the operator seminorm [35]. The result in Eq. (3) relates two information-theoretic quantities of particular interest in different fields, i.e., the QFI in quantum metrology and the Shannon entropy in information theory. It opens the possibility of establishing a relationship connecting quantum metrology and thermodynamics, as illustrated by the following theorem [41].

Theorem 1.—Given an environment of temperature T, the heat dissipation to extract the parameter information via an optimal measurement protocol determined by the SLD on the parametrized state $\rho_\lambda = U_\lambda \rho_0 U_\lambda^\dagger$ (with $\rho_0 = |\psi\rangle\langle\psi|$) is lower bounded by

$$\Delta Q_E^\lambda \geq \log(2) k_B T \|h_\lambda\|^{-2} F_Q[\rho_\lambda], \tag{4}$$

where k_B is the Boltzmann constant, and h_λ is the local generator of parametric translation.

Theorem 1 straightforwardly leads to some important implications in quantum metrology. In the traditional scenario of quantum metrology, one typically considers interrogation Hamiltonians of the form [35], which have been implemented in a number of systems [20–23],

$$H_\lambda = \lambda h. \tag{5}$$

The local generator of the above Hamiltonian is given by $h_\lambda = ht$ with t the interrogation time. Without loss of generality, we assume a normalized energy scale by requiring the operator seminorm $\|h\| = 1$ [35]. Stated by the so-called quantum Cramér-Rao bound (CRB) [24], the measurement precision as quantified by the variance of unbiased estimators [46,47] for the parametrized state ρ_λ is lower bounded by $(\delta\lambda)^2 \geq F_Q^{-1}[\rho_\lambda]$. According to Theorem 1, we find that the ultimate measurement precision is limited by the interrogation time t and heat dissipation ΔQ_E^λ as

$$(\delta\lambda)^2 \geq \frac{\log(2)}{t^2} \frac{k_B T}{\Delta Q_E^\lambda}. \tag{6}$$

This result implies that a certain level of measurement precision would set a constraint on the interrogation time or the heat dissipation.

To characterize the overall thermodynamic performance of a typical quantum metrology setting, we average the heat dissipation $\Delta Q_E = \langle \Delta Q_E^\lambda \rangle$ over all possible values of the unknown parameter λ. In the metrological framework based on the interrogation Hamiltonian in Eq. (5), starting from a pure initial state $|\psi\rangle$, the parametrized final state is given by $\rho_\lambda = |\psi_\lambda\rangle\langle\psi_\lambda|$. Here $|\psi_\lambda\rangle = e^{-iH_\lambda t}|\psi\rangle$ takes the simple form of a linear superposition $|\psi_\lambda\rangle = \sum_a c_a e^{-ia\lambda t}|a\rangle$ in the eigenbasis defined by h (i.e., $h|a\rangle = a|a\rangle$) with coefficients c_a. Note that the QFI $F_Q[\rho_\lambda] \equiv F_Q$ is independent of the parameter λ in this scenario [41]. Since the parameter λ is unknown, we shall introduce a density matrix to describe the ensemble of λ parametrized quantum states $\{\rho_\lambda\}$, which is denoted as $\rho_s = \langle \rho_\lambda \rangle$. For a general measurement protocol which is not necessarily optimal, we obtain that [41]

$$\mathcal{S} \geq S(\rho_s) \geq \log(2) t^{-2} F_Q. \tag{7}$$

Here, $\mathcal{S}$ is the Shannon entropy of the measurement outcomes stored in the memory [see Fig. 1(a)], and $S(\rho_s) = -\mathrm{tr}(\rho_s \log \rho_s)$ represents the von Neumann entropy of the density matrix ρ_s. As $\Delta Q_E \geq k_B T \mathcal{S}$ stated by Landauer's principle, Eq. (7) straightforwardly leads to $\Delta Q_E \geq k_B T S(\rho_s)$, namely,

$$\Delta Q_E \geq \log(2) k_B T t^{-2} F_Q. \tag{8}$$

Erasure of quantum Fisher information.—The physical limit on energy consumption in the above quantum metrological framework is derived from the perspective of measurement. Similar to Landauer's principle, which relates the energy cost in computations with the erasure of information, we demonstrate that the heat dissipation in quantum metrology is essentially determined by the erasure of QFI. The measurement process shown in Fig. 1(a) erases the QFI of the entity including the probe system and the memory, which can be represented by a "many-to-one" map $\mathcal{E}_s : |\psi_\lambda\rangle\langle\psi_\lambda| \otimes |x\rangle\langle x| \to |0\rangle\langle 0| \otimes |x\rangle\langle x|$ for arbitrary λ. The erasure of QFI can be realized by a map of the more general form $\mathcal{E}_s : |\psi_\lambda\rangle\langle\psi_\lambda| \to \rho$ via the interaction with environment. Such a map transforms the system into a parameter-independent state ρ that contains no information on the unknown parameter λ. Following Landauer's principle in the quantum regime [44] and using the inequality in Eq. (7), we find that [41]

$$\Delta Q_E + k_B T S(\rho) \geq \log(2) k_B T t^{-2} F_Q, \tag{9}$$

where ΔQ_E represents the heat dissipation induced by the QFI-erasure map $\mathcal{E}_s$ averaged over all possible values of the unknown parameter λ, and $S(\rho)$ denotes the von Neumann entropy of the parameter-independent state ρ.

The result in Eq. (9) implies that the erasure of QFI leads to either heat dissipation into environment (i.e., ΔQ_E), or entropy increase of the system from the pure state $|\psi_\lambda\rangle$ to a possible mixed state ρ. One notes that a subsequent transformation of the parameter-independent state ρ into the initial state ρ_0 may also dissipate heat according to Landauer's principle. Therefore, the overall heat dissipation will be lower bounded by the right-hand side of Eq. (9), which is consistent with the result in Eq. (8). We remark that the thermodynamic bound for the erasure of QFI can be further improved to $\mathcal{Q}_s(\Delta_F)$ [41] with $\Delta_F = \log(2) t^{-2} F_Q - S(\rho)$ via the low-temperature correction [45]. Below we illustrate the behavior of the bound in Eq. (9) using two well-known examples, i.e., the quantum Rabi model and the pure dephasing process.

In the first example of quantum Rabi model, a qubit-probe system in the parametrized state $|\psi_\lambda\rangle$ interacts with environment, i.e., the bosonic mode initially in a thermal equilibrium state. The global dynamics is governed by the Hamiltonian $H = (\Omega/2)\sigma_z + \omega a^\dagger a - g(a + a^\dagger)\sigma_x$, where a is the bosonic mode and $\sigma_{x,z}$ are Pauli matrices. After a certain evolution time, the qubit probe evolves into an approximately identical final state for different values of the parameter λ [41]. In Fig. 2, we show the heat dissipation into the bosonic mode ΔQ_E and the corresponding bound from initial states that give rise to different QFI. The results confirm that the heat dissipation is lower bounded by the QFI and the entropy increase of the qubit-probe system.

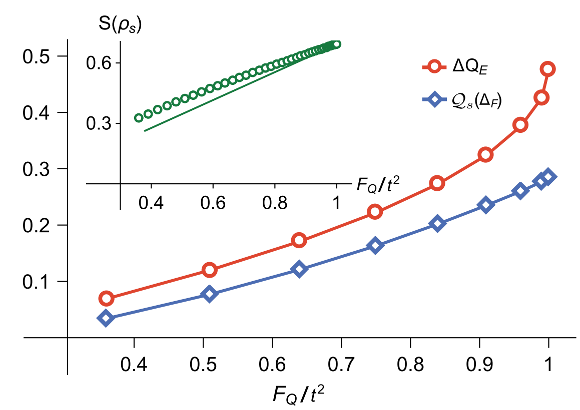

FIG. 2. Erasure of quantum Fisher information via single bosonic mode environment. Starting from the parametrized states $|\psi_\lambda\rangle = \exp[-i(\lambda\sigma_z/2)t](c_0|0\rangle + c_1|1\rangle)$ with the QFI $F_Q = 4|c_0|^2|c_1|^2 t^2$, the qubit-probe system approximately evolves into the ground state $|0\rangle$ for different values of λ after a certain time interacting with the environment of a bosonic mode. The blue diamonds and the red circles correspond to the thermodynamic bound $\mathcal{Q}_s(\Delta_F)$ and the exact heat dissipation, respectively, caused by the erasure of QFI. In addition, the inset shows the exact value of the von Neumann entropy $S(\rho_s)$ (circles) and the bound $\log(2)t^{-2}F_Q$ (solid line) given in Eq. (7) for different values of QFI. The parameters are $\omega = \Omega = 1$, $g = 0.05$, and the erasure time is $\tau_0 = 30.8$. The bosonic mode is initially in a thermal state at a temperature $T = 0.3$.

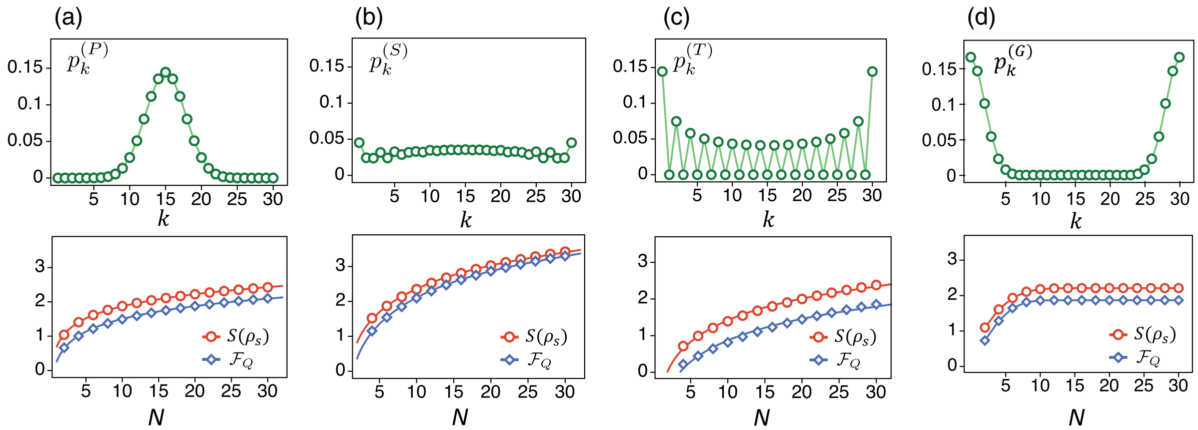

FIG. 3. Von Neumann entropy of quantum metrology based on symmetric multiqubit states. The upper panels depict the probability distribution functions $\{p_k^{(P)}, p_k^{(S)}, p_k^{(T)}, p_k^{(G)}\}$ for four types of symmetric states: product states (a), squeezed states (b), twin-Fock states (c), and GHZ-like states (d). The lower panels show the corresponding von Neumann entropy $S(\rho_s)$ (red circles), the first three ones of which are well fitted by the logarithmic functions of a form $\sim\alpha\log(N)+\beta$ (solid line) with $\alpha\approx 0.51, \beta\approx 0.7$ for (a), $\alpha\approx 0.95, \beta\approx 0.17$ for (b), and $\alpha\approx 0.84, \beta\approx -0.52$ for (c). In panel (d), the von Neumann entropy $S(\rho_s)$ saturates to a constant value for a large N. In (a)–(d), the weighted quantum Fisher information $\mathcal{F}_Q$ (blue diamonds) [Eq. (11)] shows the same scaling feature as the von Neumann entropy $S(\rho_s)$.

In the second example, we consider a spin-$\frac{1}{2}$ probe system coupled with a spin-bath environment via the Ising-type interaction. Under the assumptions of a large spin bath and weak coupling, the system undergoes a pure dephasing process. When the QFI is completely erased, the system evolves from parametrized states $|\psi_\lambda\rangle = (1/\sqrt{2})(|\uparrow\rangle + e^{-i\lambda t}|\downarrow\rangle)$ to the mixed state $\rho = (1/2)(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$ for arbitrary λ. During this process, there is no energy exchange between the probe system and the spin bath, i.e., $\Delta Q_E = 0$. The physical consequence arising from the erasure of QFI becomes entropy increase of the probe system, which equals to the bound set by the QFI, namely $S(\rho) = \log(2)t^{-2}F_Q = \log(2)$.

Application to multiqubit quantum metrology.—As a unique advantage, quantum metrology based on multiqubit entangled states can beat the standard quantum limit (SQL) of measurement precision [20]. We consider symmetric multiqubit states, which include a wide range of entangled states that can surpass the SQL such as squeezed spin states, twin-Fock states and GHZ states [22]. Using the basis of Dicke states $\{|D_N^k\rangle\}_{k=0}^N$ (N is the system size), the symmetric multiqubit states can be expanded as follows

$$|\psi\rangle = \sum_{k=0}^{N} c_k |D_N^k\rangle, \tag{10}$$

where $\{p_k = |c_k|^2\}_{k=0}^N$ is the corresponding probability distribution function [22].

We choose the interrogation Hamiltonian as $H_\lambda = \lambda\sum_j \sigma_j^z/2$, and consider four types of symmetric multiqubit states, the probability distribution functions of which are shown in Figs. 3(a)–3(d). In this scenario, the average heat dissipation is lower bounded as $\Delta Q_E \geq k_B T S(\rho_s)$ [see Eq. (7) and (8)], thus we compare the von Neumann entropy $S(\rho_s)$ for these different symmetric multiqubit states. It can be seen that the scaling of the von Neumann entropy with the system size can be fitted by functions of the form $S(\rho_s) = \alpha\log(N)+\beta$ for product states (a), squeezed states (b), and twin-Fock states (c). Figure 3(d) shows a different type of symmetric multiqubit states with a probability distribution function $p_k^{(G)} \sim \exp[-k^2/2\nu^2] + \exp[(N-k)^2/2\nu^2]$ (referred to as GHZ-like states below). Particularly, we arrive at the GHZ state in the limit of $\nu\to 0$. It is remarkable that the von Neumann entropy $S(\rho_s)$ for GHZ-like states saturates to a constant value in the large N limit, which further implies that the physical limit on energy consumption of quantum metrology would not increase with the number of qubits involved. Our results demonstrate that, although squeezed spin states, twin-Fock states, and GHZ-like states can all beat the SQL of measurement precision [41], their energy efficiencies in quantum metrology are radically different. The GHZ-like states represent a class of resource states for quantum metrology that can offer superior advantages not only in measurement precision but also in energy consumption.

Such an interesting phenomenon of energy consumption in multiqubit quantum metrology results from the fact that the von Neumann entropy $S(\rho_s)$ is related to a weighted summation of the QFI, $\mathcal{F}_Q$, contributed by all level pairs, namely,

$$S(\rho_s) \geq \mathcal{F}_Q = \frac{1}{t^2}\sum_{a<b}\frac{F_Q^{ab}}{2(a-b)^2}\log\left(\frac{2}{p_a+p_b}\right), \tag{11}$$

where F_Q^{ab} represents the QFI contributed by the level pair (a,b), the direct summation of which equals to the total

QFI F_Q [41]. It can be seen from Fig. 3 that $\mathcal{F}_Q$ provides a well-behaved bound for $S(\rho_s)$, and reveals its scaling feature. Therefore, by focusing on the weighted QFI $\mathcal{F}_Q$, we are able to analyze energy consumption and engineer suitable quantum resource states in order to reduce heat dissipation while sustaining high measurement precision.

Summary and outlook.—The work establishes a fundamental link between the concepts of entropy, QFI and heat dissipation, and clearly states the physical limit on energy consumption of quantum metrology to achieve a certain level of measurement precision. These results are generally relevant only in the asymptotic and local regimes of quantum estimation theory. It would be interesting to extend them into more realistic scenarios with prior information beyond the quantum CRB [46–53]. The revealed thermodynamic principle for quantum metrology provides a new perspective to explore quantum advantage, apart from the considerable focusing on the measurement precision, offered by quantum resources. We show that multiqubit states that can achieve similar performances in measurement precision may demonstrate very different energy efficiencies. Besides, the QFI, equivalent to quantum metric, can characterize the property of a given band defined over the Brillouin zone [54–57]. Therefore, the present connection between quantum Fisher information and thermodynamic quantities may offer a new way to explore the topological properties of energy bands from a thermodynamic perspective.

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