Spectral Filtering Induced by Non-Hermitian Evolution with Balanced Gain and Loss: Enhancing Quantum Chaos

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The dynamical signatures of quantum chaos in an isolated system are captured by the spectral form factor, which exhibits as a function of time a dip, a ramp, and a plateau, with the ramp being governed by the correlations in the level spacing distribution. While decoherence generally suppresses these dynamical signatures, the nonlinear non-Hermitian evolution with balanced gain and loss (BGL) in an energy-dephasing scenario can enhance manifestations of quantum chaos. In the Sachdev-Ye-Kitaev model and random matrix Hamiltonians, BGL increases the span of the ramp, lowering the dip as well as the value of the plateau, providing an experimentally realizable physical mechanism for spectral filtering. The chaos enhancement due to BGL is optimal over a family of filter functions that can be engineered with fluctuating Hamiltonians.

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Non-Hermitian physics offers an exciting arena at the frontiers of physics where nonequilibrium phenomena govern. In such a scenario, the evolution no longer conserves energy and is characterized by dissipation [1]. Its relevance was soon appreciated in nuclear theory [2], chemical dynamics [3], and quantum optics [4], but its manifestations span over a wide diversity of fields such as mechanics, photonics, and active matter [1]. Condensed matter theory of many-body physics is actively being extended in this setting. An exciting frontier focuses on the interplay between the nonequilibrium dynamics of non-Hermitian systems and quantum chaos.

Statistical features of the energy spectrum of an isolated system play a crucial role in the dynamics and differentiate systems exhibiting quantum chaos from others governed by integrability, many-body localization, etc. The level spacing distribution varies in these systems from the Wigner-Dyson distribution to an exponential decay [5,6], although a clear-cut classification between chaotic and integrable systems is often more subtle [7]. Quantum chaos is associated with correlations among energy levels, as revealed by the two-point energy-level distribution [8]. In particular, correlations between energy levels can be conveniently captured by the spectral form factor (SFF) defined in terms of the Fourier transform of the energy spectrum [9–12] or its complex Fourier transform, which can be written in terms of the partition function of the system analytically continued to complex temperature [13–15]. Spectral correlations also manifest directly in the Loschmidt echo [16–18] and the quantum work statistics [19–22]. The identification of universal features in spectral statistics is generally eased by the use of spectral filters which has become ubiquitous in theoretical and numerical studies of quantum systems, chaotic or not [23–27].

At the time of writing, it remains unclear how signatures of quantum chaos are modified in open quantum systems [6,28]. To tackle this question, one can make use of random-matrix tools [29], as done conventionally for Hamiltonian systems in isolation, but to describe quantum operations instead [6,30–35]. These efforts follow the spirit of Hamiltonian quantum chaos in adopting a statistical approach to identify generators of evolution (or quantum channels) compatible with a set of symmetries. In addition, diagnostic tools to characterize open quantum chaotic systems remain to be developed. Efforts to this end can be split into two groups. The first one focuses on the spectral statistics of the generator of the evolution of an open quantum system, whether it is a Liouvillian governing the rate of change of a quantum state or a quantum channel [33,36–38]. This powerful approach leverages the elegance and universality of the Hamiltonian counterpart but seems better suited to capture the dynamics of complex systems in complex environments than to explore how Hamiltonian chaos is altered by decoherence. The second approach uses information-theoretic quantities such as the fidelity or Loschmidt echo and can provide a clear separation between the role of the environment and the spectral features of the system, singling out the correlations in the system's spectral properties that contribute directly to the quantum dynamics [39–41].

Across the quantum-to-classical transition, decoherence brings out signatures of classical chaos [42–44]. By contrast, decoherence generally suppresses dynamical manifestations of quantum chaos stemming from energy level correlations [39–41,45–48]. In this Letter, we explore the non-Hermitian evolution of chaotic quantum systems. We show that in this setting environmental decoherence can enhance dynamical signatures of quantum chaos. Specifically, we consider the nonlinear evolution of energy-diffusion processes under balanced gain and loss, which is shown to act as a spectral filter. Using the Sachdev-Ye-Kitaev model as a paradigmatic test bed, we demonstrate the amplification of quantum chaos using a fidelity-based generalization of the spectral form factor to open quantum systems, which is amenable to studies in the laboratory.

Balanced gain and loss from null-measurement conditioning.—The Markovian evolution of a quantum system in a quantum state ρ can be described by a master equation of the Lindblad form $d_t\rho = -i[H_0, \rho] + \sum_\alpha \gamma_\alpha (K_\alpha \rho K_\alpha^\dagger - \frac{1}{2}\{K_\alpha^\dagger K_\alpha, \rho\})$, in terms of the positive decay rates $\gamma_\alpha \geq 0$ and the bath operators K_α [49]. For our analysis, we rewrite this evolution as follows [50]:

$$d_t\rho = -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger) + J(\rho), \tag{1}$$

in terms of the effectively non-Hermitian Hamiltonian given by $H_{\mathrm{eff}} = H_0 - (i/2)\sum_\alpha \gamma_\alpha K_\alpha^\dagger K_\alpha$ and the jump term $J(\rho) = \sum_\alpha \gamma_\alpha K_\alpha \rho K_\alpha^\dagger$. In the absence of quantum jumps, the contribution of the latter term can be ignored, and the dynamics is exclusively governed by the non-Hermitian Hamiltonian. The trace preserving evolution for such subensemble of trajectories is given by the nonlinear Schrödinger equation for null-measurement conditioning,

$$d_t\rho = -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger) + i\mathrm{Tr}[(H_{\mathrm{eff}} - H_{\mathrm{eff}}^\dagger)\rho]\rho, \tag{2}$$

which also arises in non-Hermitian systems in scenarios characterized by balanced gain and loss (BGL) [51]. Thus, BGL dynamics can be derived as the effective evolution of an ensemble of quantum trajectories conditioned on a measurement record with no quantum jumps, e.g., in a system under continuous monitoring [50]. We note that BGL dynamics also emerges naturally in other experimental settings effectively realizing non-Hermitian Hamiltonians with broken parity-time symmetry, in which eigenvalues come in complex conjugate pairs [52,53]; see, e.g., Refs. [54–58].

Energy dephasing with and without quantum jumps.—Processes characterized by energy dephasing arise naturally in a variety of scenarios, including random quantum measurements [59,60], clock errors in timing the evolution of a quantum system [61], and fluctuations in the system Hamiltonian [62,63], such as those invoked by wave function collapse models [64–66]. The evolution of the quantum state is then exactly described by $d_t\rho = -i[H_0, \rho] - \gamma[H_0, [H_0, \rho]]$, with no restriction on γ to be in the weak-coupling limit [39,67]. This can be recast as the master equation (1) in terms of a non-Hermitian Hamiltonian $H_{\mathrm{eff}} = H_0 - i\gamma H_0^2$ and the quantum jump term $J(\rho) = 2\gamma H_0 \rho H_0$. In the absence of quantum jumps, the trace-preserving evolution is described by the nonlinear master equation (2). Given $H_0 = \sum_n E_n |n\rangle\langle n|$, for a generic initial quantum state $\rho(0) = \sum_{nm} \rho_{nm}(0)|n\rangle\langle m|$, the exact solution can be found by first solving the linear case, dropping the nonlinear term which simply accounts for the correct normalization, and subsequently including its effect. The time-dependent density matrix reads

$$\rho(t) = \frac{\sum_{nm}\rho_{nm}(0)e^{-i(E_n - E_m)t - \gamma t(E_n^2 + E_m^2)}}{\sum_n \rho_{nn}(0)e^{-2t\gamma E_n^2}} |n\rangle\langle m|. \tag{3}$$

With knowledge of the quantum state during time evolution, we turn our attention to the interplay among Hamiltonian quantum chaos, energy dephasing, and BGL. In open quantum systems, different quantities have been proposed to characterize dissipative quantum chaos using spectral properties [6,16,17,31,39,41]. An analogue of the SFF is given by the fidelity between a coherent Gibbs state

$$|\psi_\beta\rangle = \sum_n \frac{e^{-\beta E_n/2}}{\sqrt{Z_0(\beta)}}|n\rangle, \qquad Z_0(\beta) = \mathrm{Tr}[e^{-\beta H_0}], \tag{4}$$

and its time evolution [15,39–41]. For an arbitrary dynamics described by a quantum channel Λ, $\rho(t) = \Lambda[\rho(0)]$, the analogue of the SFF reads $F_t = \langle\psi_\beta|\rho(t)|\psi_\beta\rangle$. In the limit of unitary dynamics generated by a Hermitian Hamiltonian H_0, one recovers the familiar expression [13–15] $F_t = |Z_0(\beta + it)/Z_0(\beta)|^2$. The result under energy-dephasing has been explored in Refs. [39–41]. Explicit evaluation using the time-dependent density matrix under BGL (3) yields the SFF

$$F_t = \frac{|\sum_n e^{-(\beta + it)E_n - \gamma t E_n^2}|^2}{Z_0(\beta)\sum_n e^{-\beta E_n - 2t\gamma E_n^2}}. \tag{5}$$

This expression corresponds to a single system Hamiltonian and is generally to be averaged over a Hamiltonian ensemble to reflect eigenvalue correlations, unless the system is self-averaging. The fidelity-based approach to generalize the SFF is thus naturally suited to account for non-Hermitian quantum systems, including the nonlinear evolution characterized by BGL. With these tools at hand, we proceed to explore the fate of the dynamical signatures of quantum chaos in this setting.

BGL dynamics of the Sachdev-Ye-Kitaev model.—For the sake of illustration, we consider the Sachdev-Ye-Kitaev (SYK) model which is known to be maximally chaotic. The Hamiltonian of the SYK model [68,69]

$$H_0 = \frac{1}{4(4!)} \sum_{k,l,m,n=1}^{N} J_{klmn} \chi_k \chi_l \chi_m \chi_n, \qquad (6)$$

involves N Majorana fermions satisfying $\{\chi_k, \chi_l\} = \delta_{kl}$ subject to all-to-all random quartic interactions. The coupling tensor J_{klmn} is completely antisymmetric, and independently sampled from a Gaussian distribution $J_{klmn} \in \mathcal{N}(0, [3!/(N)^3]J^2)$, where $J^2 = (1/3!)\sum_{klmn}\langle J_{klmn}^2\rangle$. We set $J = 1$ for convenience. Its experimental simulation is the subject of ongoing studies [70–74] and the features of the SFF in isolation have been characterized in depth [13]. It exhibits a decay from unit value towards a correlation hole or dip. This decay is governed by the density of states and as such, it is not universal. It stops at a characteristic dip time t_d. After the correlation hole, quantum chaos governs the evolution giving rise to a ramp as a result of the correlations between different energy levels. Such a ramp saturates to a plateau at a second characteristic time t_p, as shown in Fig. 1 for $\gamma = 0$. The occurrence of the ramp during the interval (t_d, t_p) is a clear manifestation of quantum chaos in the dynamics. Such dynamical signatures of quantum chaos are however suppressed by decoherence. Indeed, energy dephasing, which includes quantum jumps, reduces the depth of the correlation hole and delays the beginning of the ramp, while barely affecting the onset of the plateau [41].

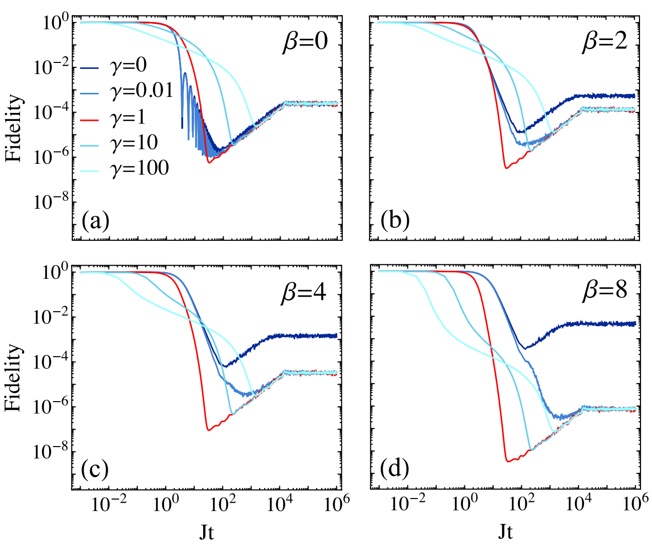

FIG. 1. Enhancement of quantum chaos under BGL in an energy-dephasing process. The time dependence of the fidelity between a coherent Gibbs state and its nonlinear time evolution is shown in the SYK model with $N = 26$, after averaging over 100 realizations of J_{klmn}. The enhancement of quantum chaos is more pronounced as the value of β is increased. Under BGL, the dip is enhanced, increasing the span of the ramp associated with fully chaotic dynamics. The value of the plateau is lowered with respect to the unitary case without a pronounced shift of its onset. For larger values of γ, the features of the SFF in isolation are gradually washed out by the nonunitary evolution. In particular, the span of the ramp is shrunk, without altering the onset of the plateau and its value.

In stark contrast, BGL dynamics is shown to enhance the dynamical signatures of quantum chaos. An explicit computation of the SFF for the SYK model is shown in Fig. 1 averaging over different realizations of the disorder. Results in other paradigms of chaos, such as random-matrix Hamiltonians in the Gaussian orthogonal and unitary ensembles, are detailed in Ref. [75], with an analytical expression for the latter. The effective non-Hermitian Hamiltonian accelerates the nonuniversal decay from unit value (associated with the disconnected part of the SFF), thus shifting the onset of the dip. BGL provides a physical mechanism to implement the kind of spectral filter proposed to suppress nonuniversal effects from the spectral edges in theoretical and numerical studies [23–27]. Such filters provide an analog of apodization in the time domain, suppressing fringes stemming from the sharp edges of the spectrum in the SFF. As seen from the numerator of Eq. (5), the anti-Hermitian part of the effective non-Hermitian Hamiltonian $H_{\rm eff} = H_0 - i\gamma H_0^2$ gives rise to a Gaussian spectral filter $g(E) = \exp(-\gamma t E^2)$, with a strength that increases linearly in time and width that decreases as $1/\sqrt{\gamma t}$, while the BGL dynamics enhances the signal of the fidelity by making the evolution trace preserving, giving rise to the denominator in Eq. (5). The subsequent ramp spans over a stage of the evolution which is not only longer than in the case under energy dephasing, but that also exceeds the ramp interval in the isolated case, e.g., the conventional SFF for unitary dynamics. Away from the infinite temperature limit, the ramp is prolonged up to 2 orders of magnitude over the unitary case, see Fig. 1(d).

For an isolated system ($\gamma = 0$), denoting the degeneracy of an energy level E_n by N_n, the asymptotic value of the SFF is given by $F_p = [1/Z_0(\beta)^2]\sum_n N_n e^{-2\beta E_n} \geq [Z_0(2\beta)/Z_0(\beta)^2]$, where the lower-bound holds in the absence of degeneracies ($N_n = 1\ \forall\, n$), expected in quantum chaotic systems. In the infinite-temperature limit, $F_p = 1/d$ is set by the inverse of the Hilbert space dimension and thus vanishes with increasing system size. This asymptotic value is preserved under energy dephasing, as the long-time quantum state is given by the thermal state $\rho_p = \sum_n \exp(-\beta E_n)|n\rangle\langle n|/Z_0(\beta)$. However, Fig. 1 shows that the asymptotic value varies in the BGL case. For $\gamma > 0$, the long-time limit of the fidelity reads

$$F_p \sim \frac{\sum_n N_n^2 e^{-2\beta E_n - 2\gamma t E_n^2}}{Z_0(\beta)\sum_n N_n e^{-\beta E_n - 2\gamma t E_n^2}} \geq \frac{1}{Z_0(\beta)}, \qquad (7)$$

where the inequality is saturated for systems lacking degeneracies, e.g., exhibiting quantum chaos. The discrepancy between the unitary and BGL plateau values is thus enhanced with decreasing temperature.

Even if the plateau value F_p of the SFF varies under BGL, the characteristic time t_p at which this asymptotic

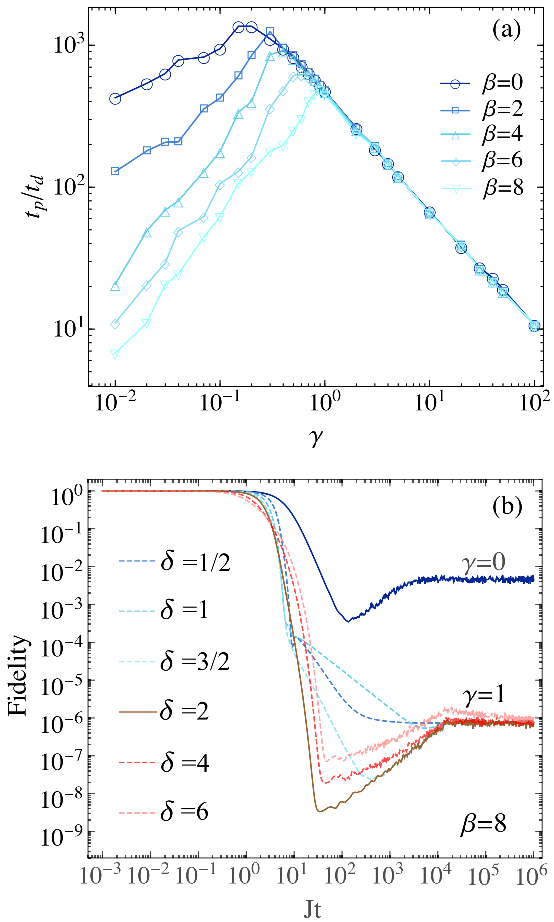

FIG. 2. (a) Quantifying the enhancement of quantum chaos under BGL in the SYK model. The ratio between the dip and the plateau times is shown as a function of the dephasing rate γ for different values of the inverse temperature β. (b) The Gaussian filter $\delta = 2$ that emerges under energy diffusion with BGL is shown to be optimal over the family of filter functions $g(E) = \exp(-\gamma t|E|^{\delta})$ in maximizing the duration of the ramp. Data for $N = 26$ is averaged over 100 realizations of J_{klmn}.

value is reached is only weakly affected by BGL with respect to the unitary case and the enhancement of the ramp can be traced to the shortening of the dip time t_d induced by the BGL spectral filter. As the ramp is governed by the eigenvalue correlations stemming from quantum chaos, their enhancement can be quantified by the ratio t_p/t_d as a function of the dephasing rate γ that enters the Gaussian filter term in Eq. (5); see Fig. 2(a). There is a critical value of γ above which BGL minimizes the ramp as decoherence mechanisms generally do. However, for values of γ below the critical one, the duration of the ramp is enhanced with increasing dephasing. The enhancement is more pronounced for larger values of β for which it exceeds 2 orders of magnitude.

Given an energy spectrum $\{E_n\}$, obtained by theoretical or experimental means, one may wonder whether other filter functions provide an advantage over the Gaussian filter in enhancing signatures of quantum chaos. A filter function $g(E) \geq 0$ yields the modified SFF

$$F_t = \frac{|\sum_n e^{-(\beta+it)E_n} g(E_n)|^2}{Z_0(\beta)\sum_n e^{-\beta E_n} g(E_n)^2}. \qquad (8)$$

We consider the family of filter functions $g(E) = \exp(-\gamma t|E|^{\delta})$, which includes the Gaussian case for $\delta = 2$. Those with $\delta \geq 2$ can be engineered by generalized energy dephasing processes conditioned on BGL as discussed in the Supplemental Material [75]. For completeness, we also consider $0 < \delta < 2$. Figure 2(b) shows that the Gaussian filter is optimal in the sense that it maximizes the duration of the ramp with respect to the family of higher-order Gaussian filter functions $\exp(-\gamma t|E|^{\delta})$ with $\delta \geq 0$. More general filters are discussed in Ref. [75].

Quantum simulation of energy dephasing under BGL.— The features presented here are not exclusive to the SYK model and we have reproduced them in other quantum chaotic systems, e.g., random-matrix Hamiltonians, as shown in Ref. [75]. However, the phenomenology described thus far is specific to energy dephasing processes governed by BGL. Other open quantum systems unrelated to energy dephasing do not exhibit an enhancement of the dynamical signatures of quantum chaos in the presence of BGL. Indeed, when the quantum jump operators K_α do not commute with the system Hamiltonian H_0, the SFF generally loses the signatures of quantum chaos, with or without quantum jumps. Said differently, a general Markovian dehasing evolution (e.g., of the kind considered in Ref. [76]) suppresses completely the dip and ramp in the SFF as shown in Ref. [75]. Thus, energy dephasing stands out as the *only* kind of time evolution that can be used to enhance dynamical manifestations of chaos in the laboratory, when conditioned to BGL. From an experimental point of view, energy dephasing is amenable to quantum simulation by coarse graining in time the evolution of an isolated system [39–41]. The SFF under BGL can be expressed as

$$F_t = \frac{|\int_{-\infty}^{\infty} \mathrm{d}s K(t,s) Z_0(\beta + is)|^2}{Z_0(\beta)\int_{-\infty}^{\infty} \mathrm{d}s\mathrm{d}s' K(t,s)K(t,s') Z_0[\beta + i(s-s')]}, \qquad (9)$$

in terms of the kernel $K(t,s) = (1/\sqrt{4\pi\gamma t})e^{-[(t-s)^2/4\gamma t]}$. Knowledge of the analytically continued partition function $Z_0(\beta + is)$ thus suffices to determine the SFF under BGL. A variety of experimental techniques have been demonstrated to measure the partition function in the complex plane, given its manifold applications that range from the study of Lee-Yang zeroes in critical systems [77–80] to the full counting statistics of many-body observables [81] and positive operator-valued measures such as work in quantum thermodynamics [82,83]. A ubiquitous approach relies on single-qubit interferometry that utilizes a two-level system or qubit as a probe [84–88]. Experimental demonstrations include, e.g., NMR systems [89] and ultracold gases [90]. An alternative experimental approach can be conceived by engineering energy dephasing using noise as a resource [67,91] and measuring the overlap between the initial coherent Gibbs state and its time-evolution either via learning quantum algorithms [92] or interferometry [93].

In summary, we have considered the nonlinear non-Hermitian evolution of a quantum chaotic system under balanced gain and loss. Using a fidelity-based generalization of the spectral form factor we have shown that the interplay between energy dephasing and BGL enhances the dynamical signatures of quantum chaos by providing an experimentally realizable physical mechanism for spectral filtering, i.e., the optimal filter function of Gaussian type. Spectral filtering has become a ubiquitous tool in theoretical and numerical studies of many-body systems, chaotic or not. As a result, our findings motivate the use of BGL as a generic practical tool to probe the spectral features in complex quantum systems. In addition, our results advance the understanding of dissipative quantum chaos and could be explored in quantum simulators by making use of established experimental techniques such as noise engineering and single-qubit interferometry.

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[1] Y. Ashida, Z. Gong, and M. Ueda, Adv. Phys. **69**, 249 (2020).
[2] G. Gamow, Z. Phys. **51**, 204 (1928).
[3] R. Levine, *Quantum Mechanics of Molecular Rate Processes* (Dover Publications, New York, 2011).
[4] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).
[5] T. Guhr, A. Müller-Groeling, and H. A. Weidenmüller, Phys. Rep. **299**, 189 (1998).
[6] F. Haake, *Quantum Signatures of Chaos* (Springer, Berlin, 2010).
[7] F. Borgonovi, F. Izrailev, L. Santos, and V. Zelevinsky, Phys. Rep. **626**, 1 (2016).
[8] O. Bohigas, M. J. Giannoni, and C. Schmit, Phys. Rev. Lett. **52**, 1 (1984).
[9] L. Leviandier, M. Lombardi, R. Jost, and J. P. Pique, Phys. Rev. Lett. **56**, 2449 (1986).
[10] J. Wilkie and P. Brumer, Phys. Rev. Lett. **67**, 1185 (1991).
[11] Y. Alhassid and N. Whelan, Phys. Rev. Lett. **70**, 572 (1993).
[12] J.-Z. Ma, J. Phys. Soc. Jpn. **64**, 4059 (1995).
[13] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, J. High Energy Phys. 05 (2017) 118.
[14] E. Dyer and G. Gur-Ari, J. High Energy Phys. 08 (2017) 075.
[15] A. del Campo, J. Molina-Vilaplana, and J. Sonner, Phys. Rev. D **95**, 126008 (2017).
[16] T. Gorin, T. Prosen, T. H. Seligman, and M. Žnidarič, Phys. Rep. **435**, 33 (2006).
[17] P. Jacquod and C. Petitjean, Adv. Phys. **58**, 67 (2009).
[18] B. Yan, L. Cincio, and W. H. Zurek, Phys. Rev. Lett. **124**, 160603 (2020).
[19] A. Chenu, I. L. Egusquiza, J. Molina-Vilaplana, and A. del Campo, Sci. Rep. **8**, 12634 (2018).
[20] I. García-Mata, A. J. Roncaglia, and D. A. Wisniacki, Phys. Rev. E **95**, 050102(R) (2017).
[21] E. G. Arrais, D. A. Wisniacki, L. C. Céleri, N. G. de Almeida, A. J. Roncaglia, and F. Toscano, Phys. Rev. E **98**, 012106 (2018).
[22] A. Chenu, J. Molina-Vilaplana, and A. del Campo, Quantum **3**, 127 (2019).
[23] M. R. Wall and D. Neuhauser, J. Chem. Phys. **102**, 8011 (1995).
[24] V. A. Mandelshtam and H. S. Taylor, J. Chem. Phys. **106**, 5085 (1997).
[25] H. Gharibyan, M. Hanada, S. H. Shenker, and M. Tezuka, J. High Energy Phys. 07 (2018) 124.
[26] M. Schiulaz, E. J. Torres-Herrera, and L. F. Santos, Phys. Rev. B **99**, 174313 (2019).
[27] J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, Phys. Rev. E **102**, 062144 (2020).
[28] D. Braun, *Dissipative Quantum Chaos and Decoherence*, Springer Tracts in Modern Physics (Springer, Berlin Heidelberg, 2003).
[29] M. L. Mehta, *Random Matrices*, 3rd ed. (Elsevier, San Diego, 2004).
[30] S. Denisov, T. Laptyeva, W. Tarnowski, D. Chruściński, and K. Życzkowski, Phys. Rev. Lett. **123**, 140403 (2019).
[31] T. Can, J. Phys. A **52**, 485302 (2019).
[32] T. Can, V. Oganesyan, D. Orgad, and S. Gopalakrishnan, Phys. Rev. Lett. **123**, 234103 (2019).
[33] L. Sá, P. Ribeiro, and T. Prosen, Phys. Rev. X **10**, 021019 (2020).
[34] L. Sá, P. Ribeiro, T. Can, and T. Prosen, Phys. Rev. B **102**, 134310 (2020).
[35] K. Wang, F. Piazza, and D. J. Luitz, Phys. Rev. Lett. **124**, 100604 (2020).
[36] F. Haake, F. Izrailev, N. Lehmann, D. Saher, and H.-J. Sommers, Z. Phys. B **88**, 359 (1992).
[37] G. Akemann, M. Kieburg, A. Mielke, and T. Prosen, Phys. Rev. Lett. **123**, 254101 (2019).
[38] J. Li, T. Prosen, and A. Chan, Phys. Rev. Lett. **127**, 170602 (2021).
[39] Z. Xu, L. P. García-Pintos, A. Chenu, and A. del Campo, Phys. Rev. Lett. **122**, 014103 (2019).
[40] A. del Campo and T. Takayanagi, J. High Energy Phys. 02 (2020) 170.
[41] Z. Xu, A. Chenu, T. Prosen, and A. del Campo, Phys. Rev. B **103**, 064309 (2021).

[42] F. M. Cucchietti, D. A. R. Dalvit, J. P. Paz, and W. H. Zurek, Phys. Rev. Lett. **91**, 210403 (2003).
[43] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[44] S. Habib, K. Jacobs, and K. Shizume, Phys. Rev. Lett. **96**, 010403 (2006).
[45] J. Rammensee, J. D. Urbina, and K. Richter, Phys. Rev. Lett. **121**, 124101 (2018).
[46] A. Touil and S. Deffner, PRX Quantum **2**, 010306 (2021).
[47] P. Zanardi and N. Anand, Phys. Rev. A **103**, 062214 (2021).
[48] N. Anand, G. Styliaris, M. Kumari, and P. Zanardi, Phys. Rev. Research **3**, 023214 (2021).
[49] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).
[50] H. Carmichael, *Statistical Methods in Quantum Optics 2: Non-Classical Fields*, Theoretical and Mathematical Physics (Springer Berlin Heidelberg, 2009).
[51] D. C. Brody and E.-M. Graefe, Phys. Rev. Lett. **109**, 230405 (2012).
[52] C. M. Bender and S. Boettcher, Phys. Rev. Lett. **80**, 5243 (1998).
[53] C. M. Bender, Rep. Prog. Phys. **70**, 947 (2007).
[54] J. Schindler, A. Li, M. C. Zheng, F. M. Ellis, and T. Kottos, Phys. Rev. A **84**, 040101(R) (2011).
[55] B. Peng, Ş. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. **10**, 394 (2014).
[56] B. Peng, Ş. K. Özdemir, S. Rotter, H. Yilmaz, M. Liertzer, F. Monifi, C. M. Bender, F. Nori, and L. Yang, Science **346**, 328 (2014).
[57] L. Chang, X. Jiang, S. Hua, C. Yang, J. Wen, L. Jiang, G. Li, G. Wang, and M. Xiao, Nat. Photonics **8**, 524 (2014).
[58] L. Feng, Z. J. Wong, R.-M. Ma, Y. Wang, and X. Zhang, Science **346**, 972 (2014).
[59] N. Gisin, Phys. Rev. Lett. **52**, 1657 (1984).
[60] J. K. Korbicz, E. A. Aguilar, P. Ćwikliński, and P. Horodecki, Phys. Rev. A **96**, 032124 (2017).
[61] I. L. Egusquiza, L. J. Garay, and J. M. Raya, Phys. Rev. A **59**, 3236 (1999).
[62] G. J. Milburn, Phys. Rev. A **44**, 5401 (1991).
[63] S. L. Adler, Phys. Rev. D **67**, 025007 (2003).
[64] I. C. Percival, Proc. R. Soc. A **447**, 189 (1994).
[65] A. Bassi and G. Ghirardi, Phys. Rep. **379**, 257 (2003).
[66] A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, Rev. Mod. Phys. **85**, 471 (2013).
[67] A. Chenu, M. Beau, J. Cao, and A. del Campo, Phys. Rev. Lett. **118**, 140403 (2017).
[68] S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
[69] A. Kitaev, A simple model of quantum holography, in *Proceedings at KITP* (unpublished).
[70] I. Danshita, M. Hanada, and M. Tezuka, Prog. Theor. Exp. Phys. **2017**, 083I01 (2017).
[71] L. García-Álvarez, I. L. Egusquiza, L. Lamata, A. del Campo, J. Sonner, and E. Solano, Phys. Rev. Lett. **119**, 040501 (2017).
[72] D. I. Pikulin and M. Franz, Phys. Rev. X **7**, 031006 (2017).
[73] R. Babbush, D. W. Berry, and H. Neven, Phys. Rev. A **99**, 040301(R) (2019).
[74] Z. Luo, Y.-Z. You, J. Li, C.-M. Jian, D. Lu, C. Xu, B. Zeng, and R. Laflamme, npj Quantum Inf. **5**, 53 (2019).
[75] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.190402 for details regarding the nonlinear master equation, the SFF with general filters, the BGL dynamics in the SYK and random-matrix Hamiltonians, and the role of decoherence under BGL.
[76] A. R. Kolovsky and D. L. Shepelyansky, Ann. Phys. (Berlin) **531**, 1900231 (2019).
[77] C. N. Yang and T. D. Lee, Phys. Rev. **87**, 404 (1952).
[78] T. D. Lee and C. N. Yang, Phys. Rev. **87**, 410 (1952).
[79] B.-B. Wei and R.-B. Liu, Phys. Rev. Lett. **109**, 185701 (2012).
[80] B.-B. Wei, S.-W. Chen, H.-C. Po, and R.-B. Liu, Sci. Rep. **4**, 5202 (2014).
[81] Z. Xu and A. del Campo, Phys. Rev. Lett. **122**, 160602 (2019).
[82] R. Dorner, S. R. Clark, L. Heaney, R. Fazio, J. Goold, and V. Vedral, Phys. Rev. Lett. **110**, 230601 (2013).
[83] L. Mazzola, G. De Chiara, and M. Paternostro, Phys. Rev. Lett. **110**, 230602 (2013).
[84] A. Recati, P. O. Fedichev, W. Zwerger, J. von Delft, and P. Zoller, Phys. Rev. Lett. **94**, 040404 (2005).
[85] J. Goold, T. Fogarty, N. Lo Gullo, M. Paternostro, and T. Busch, Phys. Rev. A **84**, 063632 (2011).
[86] M. Knap, A. Shashi, Y. Nishida, A. Imambekov, D. A. Abanin, and E. Demler, Phys. Rev. X **2**, 041020 (2012).
[87] D. Hangleiter, M. T. Mitchison, T. H. Johnson, M. Bruderer, M. B. Plenio, and D. Jaksch, Phys. Rev. A **91**, 013611 (2015).
[88] T. J. Elliott and T. H. Johnson, Phys. Rev. A **93**, 043612 (2016).
[89] X. Peng, H. Zhou, B.-B. Wei, J. Cui, J. Du, and R.-B. Liu, Phys. Rev. Lett. **114**, 010601 (2015).
[90] A. J. Roncaglia, F. Cerisola, and J. P. Paz, Phys. Rev. Lett. **113**, 250601 (2014).
[91] A. Smith, Y. Lu, S. An, X. Zhang, J.-N. Zhang, Z. Gong, H. T. Quan, C. Jarzynski, and K. Kim, New J. Phys. **20**, 013008 (2018).
[92] L. Cincio, Y. Subaşı, A. T. Sornborger, and P. J. Coles, New J. Phys. **20**, 113022 (2018).
[93] G. Ness, M. R. Lam, W. Alt, D. Meschede, Y. Sagi, and A. Alberti, Sci. Adv. **7**, eabj9119 (2021).