# Boosting the Quantum State of a Cavity with Floquet Driving

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(Received 7 December 2021; revised 6 March 2022; accepted 15 April 2022; published 4 May 2022)

The striking nonlinear effects exhibited by cavity QED systems make them a powerful tool in modern condensed matter and atomic physics. A recently discovered example is the quantized pumping of energy into a cavity by a strongly coupled, periodically driven spin. We uncover a remarkable feature of these energy pumps: they coherently translate, or *boost*, a quantum state of the cavity in the Fock basis. Current optical cavity and circuit QED experiments can realize the required Hamiltonian in a rotating frame. Boosting thus enables the preparation of highly excited nonclassical cavity states in near-term experiments.

DOI: 10.1103/PhysRevLett.128.183602

Nonclassical states of cavity and circuit QED systems [1–4] serve as a resource for difficult, or even classically forbidden, tasks [5–16]. However, preparing these states is itself difficult, as it requires strong nonlinearity [2,4]. In this Letter, we present an experimentally feasible scheme for the on-demand preparation of highly excited nonclassical states, such as Fock and Schrödinger cat states. The scheme exploits topological energy pumping—the quantized pumping of energy into a cavity by a strongly coupled periodically driven spin [17–20]—which acts to coherently translate, or *boost*, a quantum state of the cavity in the Fock basis.

Energy pumping (also called frequency conversion) is well understood in the semiclassical regime, when the cavity is in a coherent state [17–19,21–23]. The spin experiences two strong periodically oscillating fields [Fig. 1(a)]—one from the external drive with phase variable $\theta_1(t) = \Omega t + \theta_{01}$, and an effective field from the cavity with phase $\theta_2(t) = \omega t + \theta_{02}$. The spin follows this magnetic field adiabatically, and in so doing winds around the Bloch sphere. If the frequency ratio $\Omega/\omega \notin \mathbb{Q}$ is irrational, and the motion of the spin covers the Bloch sphere with Chern number $C \in \mathbb{Z}$, then the spin mediates a quantized *average* number current into (or out of) the cavity:

$$[\dot{n}]_t = \frac{\Omega}{2\pi} C. \tag{1}$$

We use square brackets $[\cdot]_x$ to denote averages over the variable $x$, which in Eq. (1) is time.

The instantaneous number current $\dot{n}(t)$ is *not* quantized. It may vary substantially within the periods $2\pi/\Omega$ and $2\pi/\omega$. Thus, it is remarkable that there are special times—the *almost periods* $T_N = (2\pi/\Omega)h_N$ (where $h_N$ is an integer)—at which the number of photons pumped into the cavity is almost exactly given by $[\dot{n}]_t T_N = C h_N$, regardless of the initial phase of the drive and cavity field. At these times $\theta_1(t)$, $\theta_2(t)$, and the spin state all return close to their initial values, with a deviation decreasing like $1/h_N$.

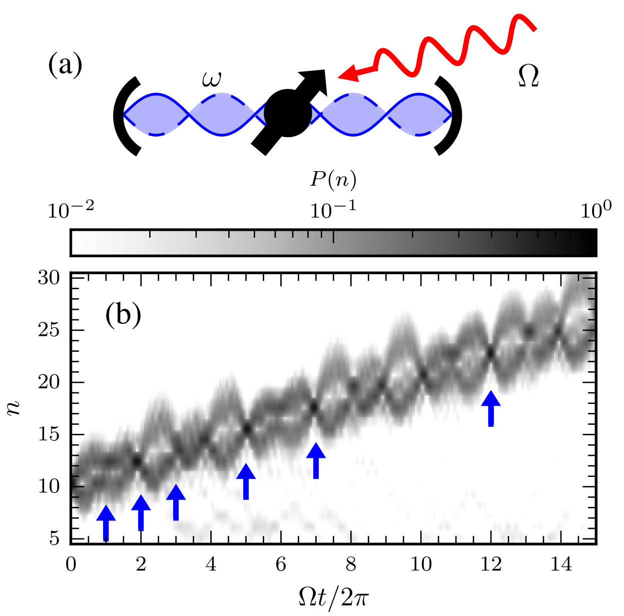


FIG. 1. (a) Model. A spin coupled to a quantum cavity with frequency $\omega$ and subject to an external periodic drive of frequency $\Omega$, such that $\Omega/\omega \notin \mathbb{Q}$. The frequencies $\hbar\omega$ and $\hbar\Omega$ are smaller than all other energy scales in the problem. (b) Cavity state boosting in a Fock state. A plot of the Fock state occupation $P(n) = \langle n|\rho_{\mathrm{cav}}(t)|n\rangle$, where $\rho_{\mathrm{cav}}(t)$ is the reduced density matrix of the cavity, shows *rephasings*, marked by blue arrows. These represent the cavity state becoming near-Fock with a larger occupation number than the initial state. Parameters in model (4) are $\Omega/\omega = (1+\sqrt{5})/2$, $\mu B_m/\hbar\omega = \mu B_d/\hbar\omega = 6$, $\mu B_0/\hbar\omega = 1.5$, and $\theta_{01} = 3\pi/2$, initial state $|\psi_0\rangle = |+\rangle_{\hat{\mathbf{x}}}|n_0\rangle$ being a product of $|+\rangle_{\hat{\mathbf{x}}}$ (the $+S$ eigenstate of $S_x$), and $|n_0\rangle$ (a Fock state) with $n_0 = 10$, and spin $S = 1/2$ (that is, a two-level system).

 

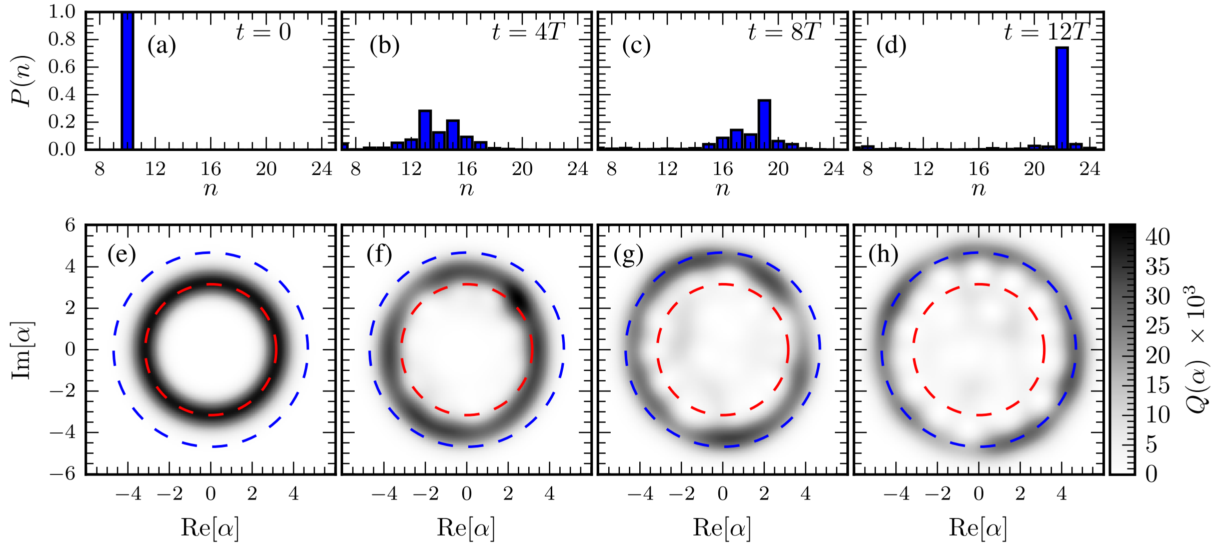


FIG. 2. (a)–(d) The photon number distribution $P(n) = \langle n|\rho_{\mathrm{cav}}(t)|n\rangle$ in Fig. 1 at multiples of the period of the classical drive $T = 2\pi/\Omega$. The distribution broadens from the initial Fock state (a), but narrows again at special times to produce a near-Fock state again (d). (e)–(h) The Husimi $Q$ function $Q(\alpha) = (1/\pi)\langle\alpha|\rho_{\mathrm{cav}}(t)|\alpha\rangle$. Initially (e) the cavity is in a Fock state, with a circularly symmetric $Q$ function. At most times (f),(g), the $Q$ function is displaced from the center of the quadrature plane, and is not circular. At special times (h) the $Q$ function is again centered and approximately circularly symmetric about the origin, but now with a larger radius. The initial radius ($n = 10$, red) and predicted final radius ($n = 22$, blue) are marked by dashed circles for reference. Parameters are as in Fig. 1.

Thus, an ensemble of spin-cavity states will *rephase* to form a boosted ensemble with a larger $n$ at the times $T_N$. This is the semiclassical mechanism underlying cavity state boosting.

Strikingly, the boosting effect persists into the quantum regime of the cavity, and also applies to nonclassical initial states. By decomposing the initial nonclassical state into a superposition of coherent states, we relate boosting in the quantum system to the corresponding semiclassical effect. An initial product state of the spin and cavity

$$|\psi(0)\rangle = |s\rangle \otimes \sum_n c_n|n\rangle \tag{2}$$

is, if the spin state is initialized correctly and the distribution of $|c_n|^2$ is sufficiently narrow, boosted to

$$|\psi(T_N)\rangle \approx |s\rangle \otimes \sum_n c_n|n + Ch_N\rangle. \tag{3}$$

Figure 1(b) shows that an initial Fock state presents the boosting phenomenon. At the almost periods, the cavity's $n$ distribution $P(n) = \langle n|\rho_{\mathrm{cav}}(t)|n\rangle$ narrows substantially [where $\rho_{\mathrm{cav}}(t)$ is the reduced density matrix of the cavity]. The cavity state has been boosted to an approximate Fock state with a larger occupation number (Fig. 2). By decoupling the spin at one of these almost periods, the boosted state can be preserved in the cavity.

More generally, highly excited nonclassical cavity states (Fock states, Schrödinger cat states, etc.) may be prepared by boosting states from lower occupations.

*Model.*—We consider a Floquet Jaynes-Cummings model with a periodically driven spin:

$$H(t) = \hbar\omega\hat{n} - \mu\vec{B}_c[\theta_1(t)]\cdot\vec{S} + \frac{\mu B_0}{2}(\hat{a}S^+ + \hat{a}^\dagger S^-). \tag{4}$$

Here, $\mu$ is the spin magnetic moment, $B_0$ is a coupling strength between the cavity and spin, $\hat{a}^{(\dagger)}$ are cavity annihilation (creation) operators, and $S^\pm$ are spin raising (lowering) operators. The spin is driven by a circularly polarized classical field with frequency $\Omega$:

$$\vec{B}_c(\theta_1) = (B_m - B_d\sin\theta_1)\hat{\mathbf{x}} + B_d\cos\theta_1\hat{\mathbf{z}}, \tag{5}$$

where the phase of the drive is $\theta_1(t) = \Omega t + \theta_{01}$. Later, we will show how this model may be achieved within a rotating frame of a typical cavity or circuit QED Hamiltonian.

*Semiclassics.*—The related semiclassical model is obtained by taking the expectation value of $H$ in a cavity coherent state $|\alpha\rangle = |\sqrt{n}e^{-i\theta_2}\rangle$, giving an effective model for the spin alone,

$$H_{\mathrm{eff}}(\theta_1,\theta_2,n) = \langle\alpha|H|\alpha\rangle - \hbar\omega n = -\mu\vec{B}_{\mathrm{eff}}\cdot\vec{S}, \tag{6}$$

where

$$\begin{aligned}\vec{B}_{\mathrm{eff}}(\theta_1,\theta_2,n) = (B_m - B_d\sin\theta_1 - B_0\sqrt{n}\cos\theta_2)\hat{\mathbf{x}} \\ - B_0\sqrt{n}\sin\theta_2\hat{\mathbf{y}} + B_d\cos\theta_1\hat{\mathbf{z}}\end{aligned} \tag{7}$$

is related to the Bernevig-Hughes-Zhang model [24,25]. For now, we assume that the motion of the cavity is unaffected by the spin, so that the phase variable arising from the cavity field $\theta_2(t) = \omega t + \theta_{02}$ rotates at a constant angular velocity. This occurs in the limit $n \to \infty$ with $B_0\sqrt{n} = O(1)$.

The spin model (6) has been shown to exhibit energy pumping in the adiabatic limit, where $\hbar\Omega$ and $\hbar\omega$ are much less than all other energy scales in the problem [17]. Energy pumping proceeds with $C = \pm 1$ if the spin is initially aligned with the field, $\Omega/\omega \notin \mathbb{Q}$ is irrational, and $(|B_m| - |B_d|)^2 < B_0^2 n < (|B_m| + |B_d|)^2$ [19].

In this regime, the spin follows the effective field, $\langle \vec{S} \rangle = S\hat{B}_{\text{eff}} + O(\Omega)$. Importantly, the spin state only depends on the instantaneous values of $\theta_1$, $\theta_2$, and $n$. Explicitly calculating the instantaneous rate of change of $n$ using $\hbar\dot{n} = -\langle \partial_{\theta_2} H_{\text{eff}} \rangle$ gives [21]

$$\hbar\dot{n}(\theta_1, \theta_2, n) = \mu S \partial_{\theta_2} |\vec{B}_{\text{eff}}| + \hbar\Omega F + O(\Omega^2), \tag{8}$$

where

$$F = S\hat{B}_{\text{eff}} \cdot (\partial_{\theta_1} \hat{B}_{\text{eff}} \times \partial_{\theta_2} \hat{B}_{\text{eff}}), \tag{9}$$

is the Berry curvature of the spin state aligned to the field $\vec{B}_{\text{eff}}$ [26].

We neglect the effect of the changing cavity population $n$ on the spin dynamics, and so fix $n = n_0$ on the right hand side of Eq. (8). This is justified if the right hand side of Eq. (8) changes slowly with $n$. Then the change in cavity population

$$\Delta n(t, \vec{\theta}_0, n_0) = \int_0^t \dot{n}(\vec{\theta}_s, n_0) ds \tag{10}$$

is computed as the integral of a quasiperiodic function over the trajectory $\vec{\theta}_t = [\theta_1(t), \theta_2(t)]$ on the torus. As $\Omega/\omega$ is irrational, this trajectory densely fills the torus as $t \to \infty$, and the integral [Eq. (10)] approximates the uniform integral of $\dot{n}$ over the torus. At the *almost periods* $T_N$, the trajectory comes close to its initial position ($\vec{\theta}_{T_N} \approx \vec{\theta}_0$), and Eq. (10) approximates the uniform integral especially well:

$$\begin{aligned}\Delta n(T_N, \vec{\theta}_0, n_0) &= \frac{T_N}{(2\pi)^2} \int \dot{n}(\vec{\theta}, n_0) d^2\theta + O(T_N^{-1}) \\ &= \frac{\Omega T_N}{2\pi} C + O(T_N^{-1}).\end{aligned} \tag{11}$$

These almost periods may be computed from the continued fraction expansion of $\Omega/\omega$ [27,28].

Crucially, Eq. (11) implies that $\Delta n(T_N)$ is only $O(T_N^{-1})$ different between trajectories with different initial conditions $\vec{\theta}_0$. An ensemble of spins initiated in coherent cavity

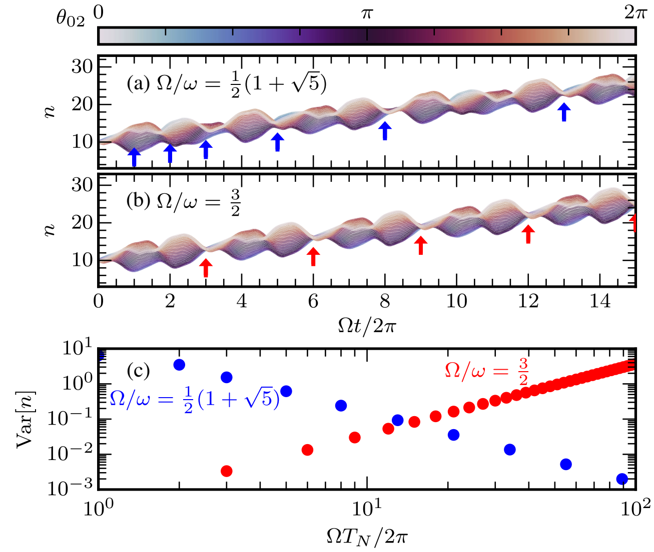


FIG. 3. Semiclassical rephasings. The prediction for the Fock occupation number $n(t)$ (10) for an ensemble of initial phases $\vec{\theta}_0$ and a (a) quasiperiodic and (b) periodic drive. Both show rephasings at their almost periods and periods respectively. (c) Inspecting the variance of $n(t)$ between $N_\theta = 32$ different values of $\theta_{02}$ shows that the rephasings improve in quality with increasing $T_N$ for quasiperiodic drives, but decay linearly for periodic drives.

states with different $\theta_{02}$ will each pump the same number of photons into the cavity at the almost periods, with a correction which decays as larger almost periods are considered (Fig. 3). We say the ensemble *rephases*.

In contrast, if $\Omega/\omega = p/q \in \mathbb{Q}$ are rationally related [17,29], then trajectories do not densely fill the torus, and the long-time averages $[\dot{n}]_t$ depend on $\vec{\theta}_0$, so that rephasings at subsequent periods $T_N = N(2\pi/\Omega)p$ decay in quality linearly with $T_N$.

*Quantum.*—The rephasing of the classical ensemble of states initiated with different $\theta_{02}$ can be used to explain cavity state boosting in the full quantum model (4). An arbitrary initial state $|\psi(0)\rangle$ of the spin and cavity can be decomposed into a superposition of coherent states $|\alpha\rangle = |\sqrt{n}e^{-i\theta_2}\rangle$ and spin states $|m\rangle_{\hat{B}_{\text{eff}}}$ ($m \in \{-S, \dots, S\}$) quantized along the axis $\hat{B}_{\text{eff}}$ defined by $n$ and $\theta_2$. For the simplest case of a spin-$\frac{1}{2}$, we have

$$|\psi(0)\rangle = \int d^2\alpha [c_+(\alpha)|+\rangle_{\hat{B}_{\text{eff}}} + c_-(\alpha)|-\rangle_{\hat{B}_{\text{eff}}}]|\alpha\rangle, \tag{12}$$

where $d^2\alpha$ is a normalized measure on the coherent states [30]. When $c_- \approx 0$, the initial state is approximately a superposition of states where the spin is aligned with an effective field $\vec{B}_{\text{eff}}$. The dynamics of each component of this superposition can then be described semiclassically. The requirement $c_- \approx 0$ is typically unrestrictive, and for the

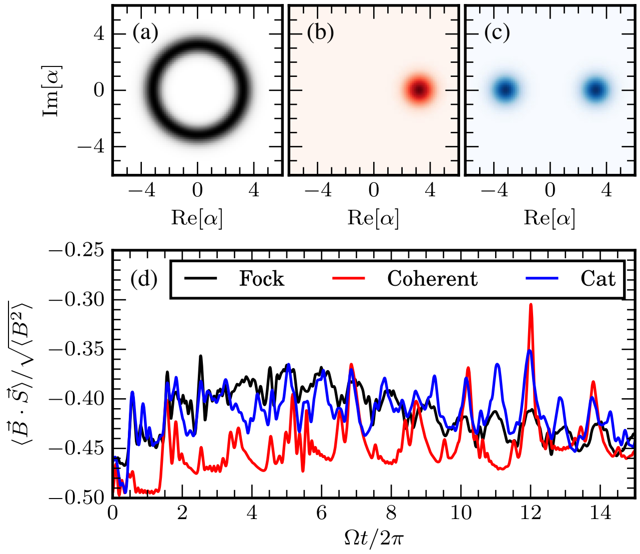


FIG. 4. Alignment of spin and field. (a)–(c) Cavity $Q$ functions for different initial states, $|+\rangle_{\hat{\mathbf{x}}}|\psi_0\rangle$, with (a) $|\psi_0\rangle = |n=10\rangle$ a Fock state, (b) $|\psi_0\rangle = |\alpha = \sqrt{10}\rangle$ a coherent state, and (c) $|\psi_0\rangle \propto |\alpha = \sqrt{10}\rangle + |\alpha = -\sqrt{10}\rangle$ a Schrödinger cat state. (d) The expectation value $M = \langle \vec{B}\cdot\vec{S}\rangle/\sqrt{\langle \vec{B}^2\rangle}$ quantifies how closely aligned the spin is to an effective cavity field in a basis of coherent states. We see that $M$ remains close to its extremal value of $-S$. Parameters are as in Fig. 1.

model (4) an initial product state $|\psi(0)\rangle = |+\rangle_{\hat{\mathbf{x}}}|\psi_0\rangle$ is sufficient.

In each component of the superposition [Eq. (12)], the dynamics of the spin is described by the semiclassical description leading to Eq. (11)—the spin remains aligned with the effective field as it evolves under the cavity dynamics (Fig. 4). Thus, at the almost periods the spin will return to its initial state in each component of the superposition, while the cavity coherent state returns to the same angular position $\theta_2(T_N) \approx \theta_{02}$ but with a larger $n(T_N) \approx n_0 + T_N[\dot{n}]_t$.

Furthermore, the quantum mechanical phase accumulated by each component may be expressed within the semiclassical approximation as the integral of the energy. In the $c_+$ components of Eq. (12), this is

$$\phi(t, \vec{\theta}_0, n_0) = \frac{1}{\hbar}\int_0^t (\hbar\omega n_0 - \mu S|\vec{B}_{\mathrm{eff}}(\vec{\theta}_s, n_0)|)ds. \quad (13)$$

The phase $\phi$ is also an integral of a quasiperiodic function, just as $\Delta n$ in Eq. (10). Thus, $\phi(T_N, \vec{\theta}_0, n_0)$ rephases at the almost periods $T_N$, becoming almost $\vec{\theta}_0$ independent. This extends our observations about rephasings in a classical ensemble to rephasings in the full quantum superposition.

The result of this rephasing is the boosting phenomenon: at the almost periods $T_N$, the quantum state of the cavity rephases to form a state which has been boosted in the Fock basis, as described in Eq. (3) (up to a global phase).

We have neglected several effects in the above arguments. We enumerate these approximations in the Supplemental Material [28], and demonstrate that there is a regime of parameters and initial states in which the boosting phenomenon occurs as claimed.

*Experimental considerations.*—Cavity boosting requires a periodic classical drive, which is routine in essentially all experimental architectures. In Eq. (4), it also requires that $\hbar\Omega$ and $\hbar\omega$ be the smallest energy scales in the problem, which, naively, necessitates ultrastrong coupling [31–34]. However, this hierarchy can be achieved in a rotating frame starting from a strong coupling Hamiltonian in the lab frame.

A typical lab frame cavity QED Hamiltonian takes the form [1–4]

$$\begin{aligned} H_{\mathrm{lab}}/\hbar = \omega_{\mathrm{cav}}\hat{n} + [\omega_q + f(t)]S_z + g(\hat{a}+\hat{a}^\dagger)S_x \\ + 2V(t)\cos(\omega_q t)S_x, \end{aligned} \quad (14)$$

where $\omega_{\mathrm{cav}}$ is the lab frame cavity frequency, and $\omega_q$ is the mean level splitting of the spin. The splitting of the spin is modulated slowly by $f(t)$, while the $x$ field on the spin is amplitude modulated by $2V(t)$ at the resonant carrier frequency $\omega_q$.

Making a rotating frame transformation $|\psi\rangle \to U|\psi\rangle$ with $U(t) = \exp[i\omega_q t(\hat{n} + S_z)]$ and dropping terms which oscillate rapidly with frequency $2\omega_q$ produces a Hamiltonian

$$\begin{aligned} H_{\mathrm{rot}}/\hbar = (\omega_{\mathrm{cav}} - \omega_q)\hat{n} + f(t)S_z \\ + \frac{g}{2}(\hat{a}S^+ + \hat{a}^\dagger S^-) + V(t)S_x, \end{aligned} \quad (15)$$

at leading order in $\omega_q^{-1}$. Making the identifications

$$\begin{aligned} \omega_{\mathrm{cav}} - \omega_q &= \omega, \\ \hbar f(t) &= -\mu B_d\cos(\Omega t), \\ \hbar g &= \mu B_0, \\ \hbar V(t) &= -\mu[B_m - B_d\sin(\Omega t)] \end{aligned} \quad (16)$$

reproduces Eq. (4) in the rotating frame. As the transformation $U$ rigidly rotates the phase space of the cavity, boosting in the rotating frame implies boosting in the lab frame. We verify this in the Supplemental Material [28].

Boosting requires a hierarchy of scales

$$\omega_{\mathrm{cav}} - \omega_q, \Omega \ll f, g, V \ll \omega_q. \quad (17)$$

This hierarchy is achievable in a variety of microwave-frequency superconducting architectures, where naturally high coupling strengths, on the order of 100 MHz, and lifetimes in excess of 100 $\mu$s provide an ample window for the required slow drive timescales $\omega_{\mathrm{cav}} - \omega_q$ and $\Omega$ [3,4].

It is also possible to satisfy this hierarchy in optical cavity QED, although the achievable separation of scales between dissipation rates and light-matter couplings is typically smaller [1,2].

*Discussion*—Cavity state boosting allows the preparation of nonclassical states of a quantum cavity with larger occupation number $n$ than may otherwise be possible. The potential to realize boosting in optical cavities is particularly intriguing, as the deterministic generation of even single photons is challenging in this regime.

Boosting is topological, in the sense that it occurs even if the instantaneous Hamiltonian is continuously deformed, provided the drive frequency $\Omega$ remains incommensurate to the cavity frequency. Changing the parameters of the Hamiltonian may alter the positions of the almost periods, but will not change the fact that they occur.

There is a close analogy between rephasings and Bloch oscillations. Electronic wave packets in an electric field show center-of-mass oscillations, and coherently expand and contract [35]. If the packet also has a nonzero Hall velocity, then at Bloch periods it has the same shape, but is translated perpendicular to the electric field—that is, it has been boosted. This analogy can be made precise through the construction of synthetic dimensions, and the frequency lattice [36–41].

If photon losses in the cavity, or dephasing of the qubit, are significant, boosting degrades in quality. As the rate of photon loss from the cavity increases with increasing $n$, the cavity populations achievable with boosting (and all methods) are limited by the cavity quality factor. Quality factors larger than $10^6$ have been reported in many architectures [42–44].

Boosting offers a qualitatively distinct method of preparing highly nonclassical cavity states—for instance, Fock states—compared with current methods [45–47]. Presently, preparing Fock states requires detailed and precise control of the coupled spin [45–47]. In contrast, boosting has an immensely simpler drive protocol for the spin—a sine wave in Eq. (4). Related protocols may also be used to prepare many-body scar states in other systems [48].

Boosting also provides a way of preparing Schrödinger cat states for use in bosonic encoded qubits [9–16]. Remarkably, the drive protocol to boost a cat state is the same as for a Fock state. Indeed, boosting does not require any knowledge of the current state of the cavity.

The authors would like to thank E. Boyers, M. Kolodrubetz, C. Laumann, B. Lev, I. Martin, A. Polkovnikov, A. M. Rey, and A. Sushkov for useful discussions. D. L. and A. C. were supported by NSF Grant No. DMR-1752759, and AFOSR Grant No. FA9550-20-1-0235. P. C.'s work at MIT was supported by the NSF STC "Center for Integrated Quantum Materials" under Cooperative Agreement No. DMR-1231319. A. K. was supported by AFOSR Grant No. FA95502110129 and NSF Grant No. PHY2047732. This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958. Numerical calculations were performed using QuSpin [49,50].

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