Acceleration-Induced Effects in Stimulated Light-Matter Interactions

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We show that, in addition to the Unruh effect, there exist two new phenomena that are due to acceleration in the quantum theory of the light-matter interaction. The first is the phenomenon of acceleration-induced transparency which arises since acceleration impacts not only the counter-rotating terms in the light-matter interaction (the cause of the conventional Unruh effect) but also the rotating wave terms. The second new phenomenon is that the Unruh effect can be stimulated, a phenomenon that arises since not only rotating-wave terms can be stimulated (as in conventional stimulated emission) but also counter-rotating terms. The new phenomena are potentially strong enough to be experimentally observable.

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Introduction.—Gravity and acceleration are locally equivalent; so says the equivalence principle. This means that acceleration-induced quantum effects are closely related to gravity-induced quantum effects. For example, the well-known acceleration-induced Unruh effect [1–3] is closely related to the Hawking effect [4–6].

In the present Letter, we demonstrate that there are two new basic quantum effects that are caused by acceleration. These effects bridge the conceptual gap between phenomena well known in atomic physics—such as a stimulated emission and absorption—and the Unruh effect, while simultaneously generalizing all of them. Schematically, the interaction between two levels of an atom and a quantized field is described by an interaction Hamiltonian of the form $\hat{H}_{\rm int} \propto (\hat{\sigma}_- + \hat{\sigma}_+)(\hat{a} + \hat{a}^\dagger)$ where $\hat{\sigma}_+ = |e\rangle\langle g|$ raises and $\hat{\sigma}_- = \hat{\sigma}_+^\dagger$ lowers the atomic level, while $\hat{a}^\dagger$ and $\hat{a}$ create and annihilate field quanta respectively. When the atom moves inertially in free space, and its coupling with the field is weak, energy conservation leads to the condition that the energy of an absorbed or emitted photon matches the atomic energy gap. As a consequence, in the interaction Hamiltonian the so-called counter-rotating terms $\hat{\sigma}_-\hat{a} + \hat{\sigma}_+\hat{a}^\dagger$ can be neglected, resulting in the so-called Jaynes-Cummings model [7] of the form, $\hat{H}_{\rm JC} \propto \hat{\sigma}_-\hat{a}^\dagger + \hat{\sigma}_+\hat{a}$, which is paradigmatic in atomic physics and quantum optics [8,9]. If the atom is accelerated by an external agent (which of course has to invest energy), counter-rotating terms of the form $\hat{\sigma}_+\hat{a}^\dagger$ can contribute and can lead to the excitation of the accelerated atom while emitting a photon. This is the crux of the Unruh effect.

In the present Letter, we show, first, that acceleration not only activates the counter-rotating terms (the conventional Unruh effect [1,2]) but that acceleration can impact the physics of the rotating-wave terms as well. We show that this effect can be made so strong that it leads to the new phenomenon of acceleration-induced transparency. Second, we consider stimulated emission and absorption. Here, we show that, in the presence of acceleration, it is possible to stimulate not only the rotating wave terms (conventional stimulation [10]) but also the counter-rotating terms. This leads to a new phenomenon that we dub the stimulated Unruh effect. Through the stimulated Unruh effect, the probability for acceleration-induced counter-rotating transitions can be vastly enhanced. This is remarkable because, in the absence of acceleration, the effects of counter-rotating wave terms are conventionally only accessible in the ultrastrong coupling regime [11].

Simplified model of the light-matter interaction.—The basic principles underlying the two new phenomena arise as part of all quantum field theories and can be demonstrated already in the simplified model of the light-matter interaction outlined above: a two-level atom with states $\{|g\rangle, |e\rangle\}$ separated in the atom's rest frame by an energy gap Ω (in units of $\hbar = 1$) interacting with a massless scalar quantum field $\hat{\varphi}(\mathbf{x}, t)$. The free Hamiltonian reads as

$$\hat{H}_0 = \Omega\hat{\sigma}_z + \frac{1}{2} :(\partial_\mu\hat{\varphi})^2: = \Omega\hat{\sigma}_z + \int (dk)\omega_{\mathbf{k}}\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}}$$

Here, $\hat{\sigma}_z = \frac{1}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$, $\omega_{\mathbf{k}} = ck$ is the frequency of the field mode whose annihilation or creation operators are $\hat{a}_{\mathbf{k}}$, $\hat{a}_{\mathbf{k}}^\dagger$, and $(dk) = d^3k/[(2\pi)^{3/2}\omega_{\mathbf{k}}^{1/2}]$ is the Lorentz invariant integration measure and $:\cdots:$ represents normal order. Note that this simplified model captures the electromagnetic interaction between an atom and a single polarization of the electromagnetic field when the radiation of interest is of a wavelength long compared with the size of the atom [9,12–14]. (Qualitative aspects of the electromagnetic interaction arising from polarization are neglected in this model).

The interaction between the two systems, modeled after the minimal coupling between a charge distribution and the electromagnetic field, is taken to be [2]

$$\hat{H}_{\text{int}} = G\hat{\mu}\,\hat{\varphi}\,[\mathbf{x}(\tau), t(\tau)], \tag{1}$$

where G is the coupling strength, $\hat{\mu} = |e\rangle\langle g| + |g\rangle\langle e|$, and $\hat{\varphi}[\mathbf{x}(\tau), t(\tau)]$ is the field along the detector's trajectory $x^\mu(\tau) = (t(\tau), \mathbf{x}(\tau))$ in terms of its proper time τ. (This form of the interaction here matches the dipole approximated form of the electromagnetic interaction which is valid as long as the wavelength of all radiation involved is longer than the typical size of the atom [14]—this is true in all that follows.)

We are, in general, interested in the probability that an initial state $|\psi_i\rangle$ produces a state $|\psi_f\rangle$ via the interaction. Moving to the interaction picture, the interaction Hamiltonian becomes time dependent, $\hat{H}_{\text{int}}(\tau) = G\hat{\mu}(\tau)\hat{\varphi}[\mathbf{x}(\tau), t(\tau)]$, where

$$\mu(\tau) = e^{i\Omega\tau}\hat{\sigma}_+ + \text{H.c.}$$
$$\hat{\varphi}(\mathbf{x}, t) = \int (dk)(e^{-i\omega_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{x}}\hat{a}_{\mathbf{k}} + \text{H.c.}).$$

Restricting attention to the weak-coupling regime, the transition amplitude for the process $|\psi_i\rangle \to |\psi_f\rangle$ is (up to a phase),

$$\mathcal{A}_{i\to f} = \int \langle\psi_f|\hat{H}_{\text{int}}(\tau)|\psi_i\rangle d\tau = \int (dk)\mathcal{A}_{i\to f}(\mathbf{k}),$$

where

$$\mathcal{A}_{i\to f}(\mathbf{k}) = G\int d\tau\langle\psi_f|(e^{i\Omega\tau}\hat{\sigma}_+ + \text{H.c.}) \times (e^{-i\omega_{\mathbf{k}}t + i\mathbf{k}\cdot\mathbf{x}}\hat{a}_{\mathbf{k}} + \text{H.c.})|\psi_i\rangle, \tag{2}$$

is the amplitude for the transition $|\psi_i\rangle \to |\psi_f\rangle$ mediated by a field quantum of momentum $\mathbf{k}$ in proper time τ. Defining the integrals

$$I_\pm(\Omega, \mathbf{k}) = \int d\tau\, e^{i\Omega\tau \pm ik^\mu x_\mu(\tau)}, \tag{3}$$

the transition amplitude can be expressed as

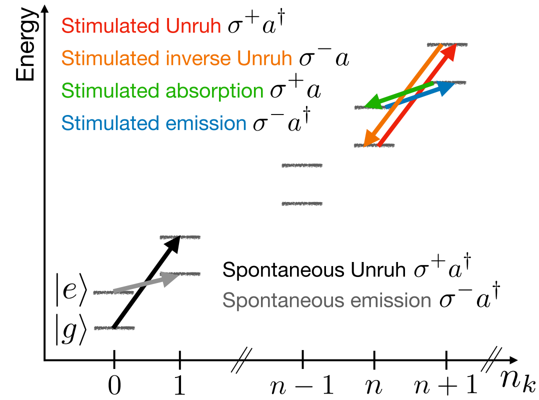

FIG. 1. Comparison between conventional first order processes in light-matter interaction that happens in inertial motion (gray, green, blue), and those that happen only in the presence of noninertial motion (black, orange, red). Shown here are the pairs of states $\{|g\rangle \otimes |n_k\rangle, |e\rangle \otimes |n_k\rangle\}$ as the field photon number n_k increases, and the transitions between them due to the various processes together with the terms that describe the processes.

$$\mathcal{A}_{i\to f}(\mathbf{k}) = G[I_-(\Omega,\mathbf{k})\langle\psi_f|\hat{\sigma}_+\hat{a}_{\mathbf{k}}|\psi_i\rangle + \text{H.c.}] + G[I_+(\Omega,\mathbf{k})\langle\psi_f|\hat{\sigma}_+\hat{a}^\dagger_{\mathbf{k}}|\psi_i\rangle + \text{H.c.}].$$

We note that in general both the rotating-wave terms (first line) and counter-rotating-wave terms (second line) contribute, weighted by I_- and I_+ respectively.

Conventional inertial absorption and emission.— Oft-studied phenomena [10] such as spontaneous emission (i.e., $|e, 0\rangle \to |g, 1_{\mathbf{k}}\rangle$), stimulated emission [i.e., $|e, n_{\mathbf{k}}\rangle \to |g, (n+1)_{\mathbf{k}}\rangle$], and absorption [i.e., $|g, (n+1)_{\mathbf{k}}\rangle \to |e, n_{\mathbf{k}}\rangle$] all arise from the rotating-wave terms when the atom is in inertial motion (see Fig. 1). [Here, $|n_{\mathbf{k}}\rangle = (n!)^{-1/2}\hat{a}^\dagger_{\mathbf{k}}|0\rangle$ is the Fock state of the field.] Indeed in inertial motion—given by the trajectory $x^\mu(\tau) = (\gamma\tau, \gamma\mathbf{v}\tau)$ where $\gamma = (1 - |\mathbf{v}|^2)^{-1/2}$—the emission and absorption amplitudes,

$$\mathcal{A}_{|e,n\rangle\to|g,n+1\rangle}(\mathbf{k}) = G\sqrt{n+1}\cdot\delta[\Omega - \gamma(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v})]$$
$$\mathcal{A}_{|g,n+1\rangle\to|e,n\rangle}(\mathbf{k}) = G\sqrt{n}\cdot\delta[\Omega - \gamma(\omega_{\mathbf{k}} - \mathbf{k}\cdot\mathbf{v})] \tag{4}$$

are nonzero only on resonance, i.e., when the excitation energy of the atom matches the (relativistically Doppler-shifted) energy of the photon. This implies that in inertial motion, emission and absorption processes are strictly due to the rotating-wave terms in the interaction. Here, the δ distribution arises from the integral I_-, which quantifies the contribution of the rotating-wave terms.

For later reference we note that the probabilities for these processes satisfy the Einstein relations,

$$|\mathcal{A}_{|e,n\rangle\to|g,n+1\rangle}|^2 = (n+1)\times|\mathcal{A}_{|e,0\rangle\to|g,1\rangle}|^2$$
$$|\mathcal{A}_{|g,n+1\rangle\to|e,n\rangle}|^2 = n\times|\mathcal{A}_{|e,0\rangle\to|g,1\rangle}|^2, \tag{5}$$

i.e., stimulated emission and absorption are respectively $(n+1)$ and n times more probable than spontaneous emission.

The conventional (i.e., spontaneous) Unruh effect.—The conventional Unruh effect [1,6] is the possibility that an atom in its ground state which is accelerating through the field vacuum transitions to an excited state while emitting a photon. Let us denote the atom's trajectory by $x^\mu(\tau)$. The amplitude for the conventional (i.e., spontaneous) Unruh process then reads as

$$\mathcal{A}_{|g,0\rangle\to|e,1\rangle}(\mathbf{k}) = G\int d\tau\, e^{i\Omega\tau + ik^\mu x_\mu(\tau)} = GI_+(\Omega,\mathbf{k}).$$

For inertial motion, we have that, $I_+ = \delta[\Omega + ck - \mathbf{k}\cdot\mathbf{v}]$, which does not contribute since $c > |\mathbf{v}|$ always; so in inertial motion, the above amplitude is identically zero. For noninertial trajectories, for example, for uniform acceleration of magnitude a, e.g., $x^\mu(\tau) = [\sinh(a\tau)/a, 0, 0, \cosh(a\tau)/a]$, we have that [2,6,15]

$$|\mathcal{A}_{|g,0\rangle\to|e,1\rangle}|^2 = G^2\frac{2\pi/(\Omega a)}{e^{\Omega/(k_B T_U)} - 1}; \qquad T_U = \frac{a}{2\pi k_B}, \tag{6}$$

which is a Bose-Einstein distribution at temperature T_U. An atom uniformly accelerated through the vacuum perceives an apparently thermal field.

We note a few key aspects. First, I_+ and therefore the probability of the spontaneous Unruh process can be nonzero because acceleration leads to a time-dependent and therefore chirped Doppler shift between the atom and the field modes. Mathematically, this phenomenon traces to the peculiar Fourier phenomenon of "concomitant" frequencies [16]: the Fourier transform of chirped positive frequencies necessarily also contains negative frequencies. Here, I_+ [see Eq. (3)] represents negative concomitant frequencies arising in the Fourier transform of the trajectory-dependent function $e^{ik^\mu x_\mu(\tau)}$ which represents a chirp of positive frequencies. As a consequence, even for realistically accelerated trajectories that do not involve eternal uniform acceleration, the Unruh process has a nonzero probability.

Second, the energy required to simultaneously excite the atom and create a photon comes from the accelerating agent; indeed in a more complete treatment, the excitation of the detector is accompanied by a recoil of the atom's center-of-mass degree of freedom [17–19]. Third, despite it being a robust quantum feature of accelerated bodies [20], the Unruh temperature (restoring constants) $T_U = \hbar a/(2\pi k_B c) \approx 10$ mK$(a/10^{18}$ m/s$^2)$ is, so far, too small to make its experimental study feasible. (It is worth noting that the term "Unruh effect" is sometimes reserved for the class of trajectories which possess horizons or for which the accelerated system is driven to a thermal or near-thermal state. We will consider general trajectories and will call any excitation of quantum systems due to noninertial motion an Unruh effect.)

Stimulated Unruh effect.—Given the central importance of the conventional Unruh process, we now examine whether acceleration induces further phenomena in the light-matter interaction and to what extent those may be amenable to experimental observation. Deriving inspiration from the stimulated processes that happen in inertial motion [Eq. (4)], we now consider the possibility of stimulating an Unruh-like process. That is, instead of the initial state $|g,0\rangle$, we consider the state $|g,n_\mathbf{k}\rangle$. The interaction leads to the following transformation:

$$|g,n_\mathbf{k}\rangle \to G[I_+\sqrt{n+1}|e,(n+1)_\mathbf{k}\rangle + I_-\sqrt{n}|e,(n-1)_\mathbf{k}\rangle]. \tag{7}$$

Here, the transformed state is unnormalized for brevity. The first term ($\propto I_+$) corresponds to a stimulated Unruh process, which arises from counter-rotating terms in the interaction, is absent in inertial motion, and does not depend on the stimulating photon being resonant with the atom. The second term ($\propto I_-$) corresponds to conventional (resonant) absorption, which is due to rotating wave terms in the interaction, and therefore relies on atom-photon resonance. The stimulated Unruh process stands in the same relation to the conventional (spontaneous) Unruh process in an accelerating frame, as conventional stimulated emission is to spontaneous emission in an inertial frame. Importantly, the probability of the stimulated Unruh processes is enhanced by a factor of $n+1$ compared with the spontaneous version

$$|\mathcal{A}_{|g,n\rangle\to|e,n+1\rangle}|^2 = (n+1)\times|\mathcal{A}_{|g,0\rangle\to|e,1\rangle}|^2, \tag{8}$$

where $|\mathcal{A}_{|g,0\rangle\to|e,1\rangle}|^2$ is the probability for the spontaneous Unruh process [which is, e.g., for the special case of uniform acceleration, given in Eq. (6)]. This equation elicits the well-known Einstein relation [Eq. (5)] for inertial stimulated emission.

The scaling with photon number immediately suggests that the experimental obstruction to observing the spontaneous Unruh effect can be alleviated by stimulation. However two aspects need to be addressed. First, in order to take advantage of the n-scaling, realistic experiments would need to use a large mean photon number, whereas large-n Fock states are challenging to prepare. Second, the state transformation in Eq. (7) produces an undesirable resonant absorption component in addition to the stimulated Unruh process. (Processes of higher order in the interaction Hamiltonian are suppressed, since they roughly scale as some power of the first order process, which, even with stimulation, has a less-than-unity probability in the weak-coupling regime.) In the following we show that both issues can be solved.

First we show that stimulation with the desired scaling can also be achieved with readily available states rather than with Fock states. To see this, we represent general field states in terms of the (over-)complete coherent state basis [21,22] $|\alpha_{\mathbf{k}}\rangle = e^{\alpha\hat{a}^\dagger_{\mathbf{k}} - \alpha^*\hat{a}_{\mathbf{k}}}|0\rangle$. The transition amplitude for the stimulated Unruh process where the field is in these basis states is (up to a phase factor),

$$\mathcal{A}_{|g,\alpha\rangle\to|e,\beta\rangle}(\mathbf{k}) = \frac{Ge^{-|\alpha-\beta|^2/2}}{(2\pi)^{3/2}\sqrt{\omega_{\mathbf{k}}}}(\alpha I_- + \beta^* I_+).$$

Here $|\alpha\rangle$, $|\beta\rangle$ are nonorthogonal field coherent states: $|\langle\alpha\beta\rangle|^2 = e^{-|\alpha-\beta|^2}$). The probability that the atom gets excited, irrespective of the final field state in mode $\mathbf{k}$, is

$$\begin{aligned}\mathcal{P}_\alpha(\mathbf{k}) &= \int \frac{d^2\beta}{\pi}|\mathcal{A}_{|g,\alpha\rangle\to|g,\beta\rangle}(\mathbf{k})|^2 \\ &= \frac{G^2}{(2\pi)^2\omega_{\mathbf{k}}}(|\alpha|^2|I_+ + I_-|^2 + |I_+|^2). \end{aligned} \tag{9}$$

In the special case $\alpha = 0$, i.e., if the initial field state is the vacuum, this expression reduces to $\mathcal{P}_0(\mathbf{k}) \propto |GI_+|^2$, i.e., we recover the conventional, i.e., spontaneous Unruh effect for a general trajectory.

For an arbitrary initial field state $\hat{\rho}_i = \int P(\alpha)d^2\alpha/\pi$, where $P(\alpha)$ is a generalized probability distribution in the coherent state basis [21,22], the probability that the atom is excited on an arbitrary trajectory can be shown to be $\mathcal{P}(\mathbf{k}) = \int P(\alpha)\mathcal{P}_\alpha(\mathbf{k})d^2\alpha/\pi$, or explicitly,

$$\mathcal{P}(\mathbf{k}) = \frac{G^2}{(2\pi)^3\omega_{\mathbf{k}}}(\langle\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}\rangle|I_+ + I_-|^2 + |I_+|^2). \tag{10}$$

Here we have used the relation between integrals of P and expectations of normal-ordered operators [22]. The implication is that in general—irrespective of the input field state—the probability of the stimulated Unruh process grows with the average photon number.

We note that the atom can get excited through three different processes [see Eq. (10)]: (1) the spontaneous Unruh effect (last term $\propto |I_+|^2$)—which is challenging to observe, (2) the new phenomenon of the stimulated Unruh effect ($\propto \langle\hat{a}^\dagger\hat{a}\rangle|I_+|^2$)—whose probability can be dramatically larger, or (3) the conventional (resonant) absorption ($\propto \langle\hat{a}^\dagger\hat{a}\rangle|I_-|^2$). In fact, as Eq. (10) shows, stimulation amplifies resonant and nonresonant terms equally: the actual probability is $\propto \langle\hat{a}^\dagger\hat{a}\rangle|I_+ + I_-|^2$. Excitation due to the new stimulated Unruh effect appears, therefore, to be "contaminated" by conventional resonant absorption. We will now continue our search for new acceleration-induced effects in the light-matter interaction. We will thereby find a new effect that happens to provide means to suppress I_- relative to I_+, i.e., that will allow one to remove the contamination of the stimulated Unruh effect by conventional absorption.

Indeed, on one hand, it is clear that it is possible to make I_- smaller than I_+ simply by choosing the stimulating field mode to be far detuned from the atomic resonance. This is because absorption is a resonant process whereas the Unruh process is nonresonant. In this case, assuming a finite linewidth γ for the resonant transition, and a detuning Δ from resonance, the probability of exciting the resonance decreases as [9] $[1 + (\Delta/\gamma)^2]^{-1} \approx (\gamma/\Delta)^{-2}$. On the other hand, we now demonstrate a new effect that allows not only for an attenuation of conventional absorption but, in principle, for its complete suppression, i.e., $I_- = 0$.

Acceleration-induced transparency.—As we now show, acceleration impacts not only the counter-rotating but also the rotating-wave terms; in fact, by choosing suitably accelerated trajectories, one can, in principle, completely suppress resonant absorption, $I_- = 0$, while also keeping $I_+ \neq 0$. This phenomenon may be called "acceleration-induced transparency." (Acceleration-induced transparency is unrelated to the so-called electromagnetically induced transparency [23] which is caused by destructive interference of excitation amplitudes in a three-level atom induced by resonant excitation with coherent field states).

To demonstrate acceleration-induced transparency, let us assume that an atomic gap Ω and the stimulating mode's wave vector $\mathbf{k}$ are chosen. Our task is to show that there are trajectories for which $I_- = 0$ and $I_+ \neq 0$. To this end, we consider for any trajectory $x_\mu(\tau)$ its "phase function" $\alpha(\tau) = k^\mu x_\mu(\tau)$, so that $I_\pm(\Omega, \mathbf{k}) = \int d\tau\, e^{i\Omega\tau \pm i\alpha(\tau)}$. The phase functions of physical trajectories obey: $\dot{\alpha}(\tau) > 0\ \forall\ \tau$. This is because $\dot{\alpha}(\tau)$ is scalar and, in an instantaneous rest frame, it reads as $\dot{\alpha}(\tau) = k^\mu\dot{x}_\mu(\tau) = k^0\dot{x}_0(\tau) > 0$ since $k_0, \dot{x}_0 > 0$. Conversely, and more importantly, to any phase function $\alpha(\tau)$ obeying $\dot{\alpha}(\tau) > 0\ \forall\ \tau$, there exists a corresponding physical trajectory. We prove this by construction: choose a coordinate system such that $k = (k_0, 0, 0, k_0)$; then the trajectory

$$\dot{x}_\mu(\tau) = \left[\frac{1}{2}\left(\frac{k_0}{\dot{\alpha}(\tau)} + \frac{\dot{\alpha}(\tau)}{k_0}\right), 0, 0, \frac{1}{2}\left(\frac{k_0}{\dot{\alpha}(\tau)} - \frac{\dot{\alpha}(\tau)}{k_0}\right)\right] \tag{11}$$

is timelike with τ its proper time (i.e., $\dot{x}^\mu\dot{x}_\mu = 1$). It is straightforward to verify that this trajectory obeys $k^\mu\dot{x}_\mu(\tau) = \dot{\alpha}(\tau)$, i.e., that it produces the desired phase function $\alpha(\tau)$ up to an irrelevant integration constant that translates the trajectory. Notice that the trajectory is inertial if $\dot{\alpha}$ is constant, and accelerating otherwise.

Our remaining task is to find examples of phase functions $\alpha(\tau)$ obeying $\dot{\alpha}(\tau) > 0\ \forall\ \tau$ for which $I_- = 0$ and $I_+ \neq 0$. Through Eq. (11) we then obtain corresponding trajectories for which the Unruh effect can be arbitrarily strongly stimulated while conventional absorption vanishes. Consider a phase function satisfying

$$\dot{\alpha}(\tau) := k^0 \begin{cases} s_0 & \tau < 0 \\ s_0 + \frac{s_1 - s_0}{T_1}\tau & \tau \in [0, T_1) \\ s_1 + \frac{s_2 - s_1}{T_2 - T_1}(\tau - T_1) & \tau \in [T_1, T_2) \\ s_2 & \tau \geq T_2, \end{cases} \tag{12}$$

where the constants $\{s_i, T_i\}$ can be chosen arbitrarily except that we require $\dot{\alpha} > 0$ and $0 < T_1 < T_2$.

The corresponding trajectories are relatively simple. They are initially inertial, then possess two phases of acceleration, followed by inertial motion. In the nonrelativistic regime, the trajectories further simplify, since then the accelerations are uniform.

The question is whether among these trajectories there are ones which exhibit acceleration-induced transparency for some arbitrarily fixed gap Ω. To produce explicit examples, we held the parameters s_0, s_2, and T_1 fixed and varied the parameters s_1 and T_2. In the (s_1, T_2) plane, we plotted the curves for which $\mathrm{Re}[I_-] = 0$ or $\mathrm{Im}[I_-] = 0$. These curves intersect (see Supplemental Material [24]), which shows that there are parameter values for which the corresponding trajectory possesses acceleration-induced transparency at the chosen gap Ω. This means that it is possible, in principle, to make the stimulated Unruh process dominate arbitrarily strongly over all conventional, i.e., resonant, processes, at this order in perturbation theory and for a given gap Ω.

In Fig. 2, we plot $|I_\pm(\Omega)|$ for such a trajectory. The plot shows that the resonant effects described by I_- tend to dominate over the nonresonant effects described by I_+, except for the arbitrarily strong acceleration-induced suppression of I_- at the chosen value of Ω. Notice that this plot in effect also shows the k_0 dependence. Mathematically, we have here shown a phenomenon of Fourier theory (related to that of concomitant frequencies [16]): by suitably chirping positive frequencies, a single positive frequency can always be suppressed in the overall Fourier spectrum. Physically, this means that the amplitude for resonant absorption, being a coherent superposition of contributions from all parts of the trajectory, can be made to vanish for suitable trajectories. The corresponding trajectories can be experimentally realized, for example, by accelerating a charged Unruh-DeWitt detector using external electromagnetic fields (see also the Supplemental Material [24]). In addition to the effects considered above where the accelerating atom starts in the ground state, it is possible to consider the case where the atom starts in the excited state. Variants of the stimulated Unruh effect can then be considered with this initial condition for the atom's internal state. Such processes are time-reversed versions of the ones

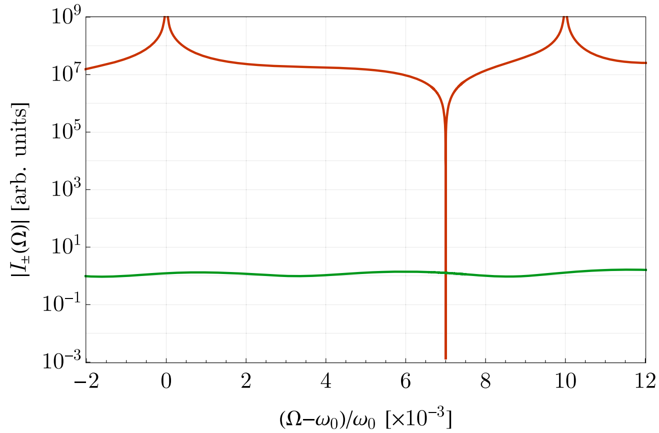

FIG. 2. The two curves display $|I_\pm(\Omega)|$ for an example of a trajectory of the form given in Eq. (12). The red curve is the resonant contribution $|I_-|$ which shows the strength of conventional absorption while the green curve is the nonresonant contribution $|I_+|$ which shows the strength of Unruh-type counter-rotating effects. Importantly, we see that the resonant contribution dips below that of the latter (here $|I_-/I_+| \approx 10^{-3}$), so that at that frequency, the probability of resonant absorption is $\sim 10^{-6}$ of the stimulated Unruh process. The frequency at which the resonant contribution dips is designed to be the detector gap frequency. The two peaks in I_- correspond to absorption at the Doppler shifted frequencies $\omega_0 = k^0 s_0$, $\omega_2 = k^0 s_2$ that are due to the initial and final inertial velocities of the trajectory.

already studied above. The amplitudes for such processes are obtained by replacing $I_\pm \to I_\mp^*$ in the equations above.

Conclusions.—We have shown that, beyond the Unruh effect, there are two new phenomena by which acceleration impacts the light-matter interaction: On one hand, we showed that acceleration not only activates the counter-rotating terms (the Unruh effect) but it also impacts the rotating-wave terms, leading to the new phenomenon of acceleration-induced transparency. On the other hand, we also showed that, in the presence of acceleration, stimulation can be used not only on the rotating-wave terms (conventional stimulation) but also on the counter-rotating terms, leading to the new phenomenon of a stimulated Unruh-like effect.

Further, we showed that by using suitably designed trajectories and by simultaneously stimulating, these two new phenomena can be combined to highly enhance acceleration-induced counter-rotating transitions while simultaneously arbitrarily strongly suppressing the conventional rotating-wave effects. While the Unruh effect is as yet unobservable, the new findings could bring acceleration-induced activations of the counter-rotating terms into the range of observability, even in the weak-coupling regime.

For example, the stimulated Unruh effect can be detected via the recoil of the emitted or absorbed photon on an accelerated low-mass two-level system [19]. A single electron, whose spin state degeneracy is lifted by a magnetic field, and whose motion can be continuously monitored, is such a transducer. Although accelerations of $\sim 10^{20}$ m/s^2 are possible for a trapped electron, corresponding to an Unruh temperature $T_U \sim 1$ K, the challenge so far has been the exceedingly small rate of the spontaneous Unruh process [25] $\Gamma_0 \approx 10^{-18}$/s. We have shown that, in a scalar model of the electromagnetic interaction, the rate scales to [Eq. (8)] $\Gamma_n = (n+1)\Gamma_0$ when stimulated with n photons on average. Thus, the accelerated motion of a single electron trapped in a Penning trap (a combination of a static axial magnetic field and a quadrupolar electric field) [26], which is colocated inside a microwave cavity loaded with photons, can serve as a potential experimental platform for detecting the stimulated Unruh effect. Indeed, state-of-the-art microwave cavities resonating at $\omega \sim 2\pi \times 1$ GHz with modest quality factors [27,28] of $Q \sim 10^5$ can store on average $n = (4Q/\omega)(P/\hbar\omega) \approx 10^{15}(P/1\ \mathrm{mW})$ photons. That is, the stimulated counter-rotating processes that we have described here can be made 15 orders of magnitude more likely than the spontaneous Unruh process, and therefore resolvable in a few hours of observation.

Finally, similar to how the equivalence principle relates the Unruh effect to the Hawking effect, the equivalence principle now suggests the existence of the gravity-induced analogs of the two new phenomena that we found here, such as, possibly, phenomena of gravity-induced transparency and the stimulation of Hawking radiation. Work in this direction is in progress.

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