Ideal Quantum Teleamplification up to a Selected Energy Cutoff Using Linear Optics

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(Received 8 October 2021; accepted 10 March 2022; published 18 April 2022)

We introduce a linear optical technique that can implement *ideal* quantum teleamplification up to the nth Fock state, where n can be any positive integer. Here teleamplification consists of both quantum teleportation and noiseless linear amplification (NLA). This simple protocol consists of a beam splitter and an $(n+1)$ splitter, with n ancillary photons and detection of n photons. For a given target fidelity, our technique improves success probability and physical resource costs by orders of magnitude over current alternative teleportation and NLA schemes. We show how this protocol can also be used as a loss-tolerant quantum relay for entanglement distribution and distillation.

DOI: 10.1103/PhysRevLett.128.160501

Introduction.—The ability to amplify an arbitrary state in a linear, or phase insensitive, manner is useful for a wide variety of quantum protocols. Unfortunately, the uncertainty principle means *deterministic* linear amplification will always introduce noise, which diminishes the output state's quantum characteristics [1,2]. However, *noiseless* linear amplification (NLA) is possible for nondeterministic amplifiers, which work with some success probability $\mathbb{P} < 1$ [3–5]. Applications of NLA include quantum secure communication [6–11], quantum repeaters [12–14], entanglement distillation [15,16], quantum sensing [17–19], and quantum error correction [20,21].

Quantum teleamplification protocols implement quantum teleportation [22] and NLA simultaneously. In this regard, Pegg *et al.* proposed a nondeterministic teleporter for low-energy states called the one-photon quantum scissor (1-QS), named for its ability to cut or truncate an arbitrary state up to its one-photon Fock state $|\psi\rangle \equiv \sum_{j=0}^{\infty} c_j|j\rangle \to \sum_{j=0}^{1} c_j|j\rangle$ [23]. Ralph and Lund later realized adjusting a beam splitter in the 1-QS modified the output state's amplitudes $|\psi\rangle \overset{\text{1-QS}}{\to} \sum_{j=0}^{1} g^j c_j|j\rangle$ [3]. Hence, for low-energy states, the 1-QS can also perform an ideal NLA operation $g^{a^\dagger a}$ up to the one-photon Fock state with $g \in (0, \infty)$ gain; this was subsequently experimentally verified [4,5]. To overcome the low-energy limitation, it was proposed to split up the input state, before applying multiple 1-QS in parallel [3,4]. However, for a finite number of 1-QS, this protocol introduces extra undesirable factors to the Fock states, distorting the output state a way from the ideal. Other NLA proposals are similarly nonideal [24–26].

Rather than multiple 1-QS in parallel, here we propose to generalize the 1-QS to the n-photons quantum scissor (n-QS), for any $n \in \mathbb{N}^+$. Previous generalizations of the QS were only for specific sizes $n \in \{1, 3, 7\}$ [27], and our fully generalized n-QS protocol contains these previous results [28]. Our n-QS protocol is a fully scalable linear optical scheme, which can perform teleamplification on an arbitrary state perfectly up to the nth Fock state. Other teleamplification proposals are restricted to specific types of input states [44]. The 2-QS case is of particular experimental interest, as it should be immediately accessible with current technology.

In this Letter, we first describe our n-QS protocol, including its operational mechanism and probability of success. We show that as an NLA it can produce amplified states with fidelities that are unreachable by previous linear-optical NLA protocols. Next, we explain how the n-QS is also useful as a high-fidelity continuous-variable teleporter, with orders of magnitude advantages over current alternatives. We then show that the n-QS can be used as a loss-tolerant relay for entanglement distillation. Finally, we discuss how our scheme is tolerant to standard resource and detector imperfections, and hence remains advantageous under practical conditions.

Noiseless linear amplifier.—The n-QS operation on an arbitrary bosonic state $|\psi\rangle$ truncates the Fock components after n and performs NLA $g^{a^\dagger a}$ as follows:

$$|\psi\rangle \equiv \sum_{j=0}^{\infty} c_j|j\rangle \overset{n\text{-QS}}{\to} |g\psi_n\rangle = N \sum_{j=0}^{n} g^j c_j|j\rangle. \quad (1)$$

This is implemented via Fig. 1 using a beam splitter (BS) and a fixed coherent $(n+1)$ splitter called the quantum Fourier transform (QFT), with n extra resource photons and n photon detections. The amount of amplification or deamplification gain $g \in (0, \infty)$ can be freely chosen by setting the BS transmissivity to $\tau = g^2/(1+g^2)$. The n-QS

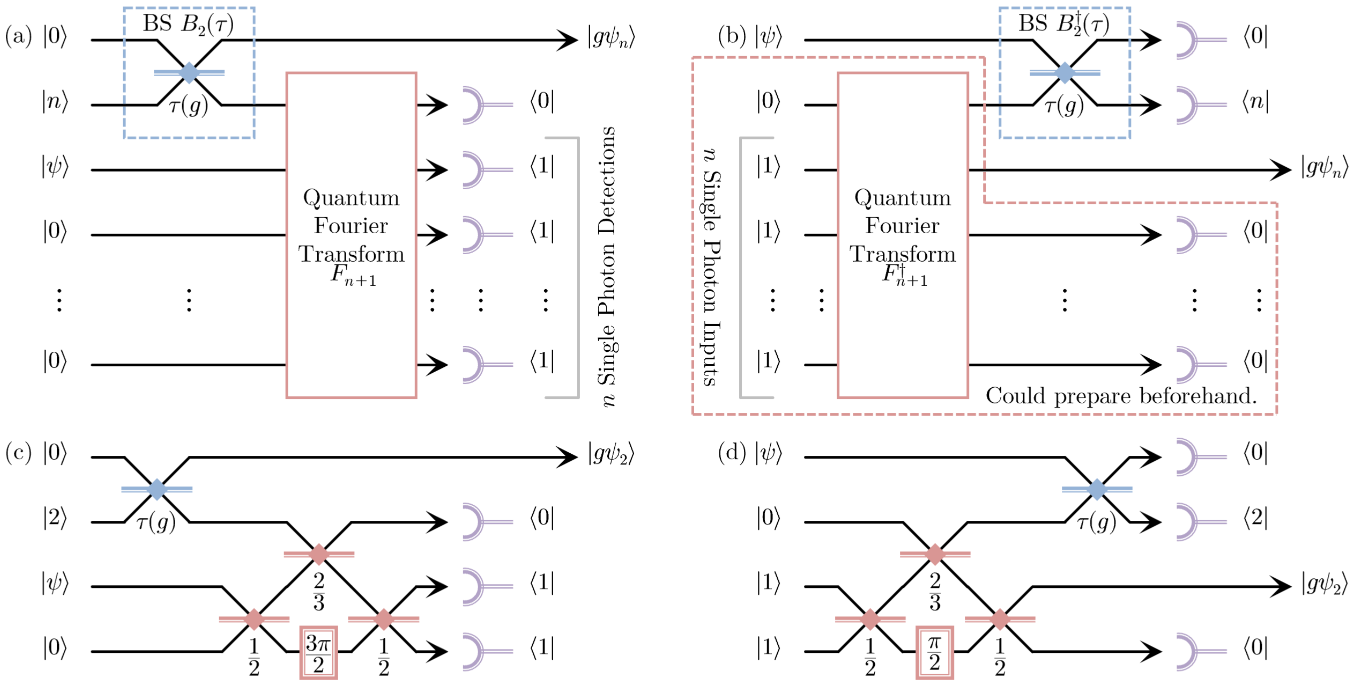

FIG. 1. Schematic of our scalable n-photons quantum scissors (n-QS) protocol, which implements noiseless linear amplification or deamplification of an arbitrary state $|\psi\rangle \rightarrow g^{a^\dagger a}|\psi_n\rangle = |g\psi_n\rangle$, up to the nth Fock state with perfect fidelity. The gain $g \in (0, \infty)$ is chosen by modifying the transmissivity $\tau = g^2/(1+g^2) \in (0,1)$ of the beam splitter. The quantum Fourier transform is a coherent $(n+1)$ splitter. This n-QS protocol requires either (a) n bunched photons (BP) or (b) n single photons (SP) as a resource. The linear optical networks for the 2-QS is shown for (c) BP or (d) SP resources.

operation only occurs if the correct outcomes are measured. Two different architectures are shown in Fig. 1, with (a) requiring $|n\rangle$ bunched photons (n-QSBP or BP), while (b) requiring $\otimes^n |1\rangle$ single photons (n-QSSP or SP); we will differentiate these devices by their state resources. Because of recent experimental advances, such as in boson sampling, n-QSSP may be easier to implement; for example, Ref. [45] experimentally implements the QFT up to fourth order with single-photon inputs.

The action of the BS $B_2(\tau)$ is $a_1^\dagger \rightarrow \sqrt{1-\tau}a_2^\dagger - \sqrt{\tau}a_1^\dagger$, which describes how the photons are scattered for a given transmissivity τ. Similarly, the action of an m mode linear optical network $\vec{a}^\dagger \rightarrow U_m \vec{a}^\dagger$ is captured by an $m \times m$ unitary scattering matrix U_m. The QFT optical device has the scattering matrix $(F_{n+1})_{j,k} \equiv e^{-2i\pi(j-1)(k-1)/(n+1)}/\sqrt{n+1}$. This definition justifies the interpretation of the QFT as a coherent $(n+1)$ splitter, as it scatters photons equally amongst its $n+1$ output ports with fixed phases. An arbitrary unitary U_m can always be decomposed into a network of at most $m(m-1)/2$ beam splitters and phase shifts [46,47]; however, only around half of the QFT network is needed since only the first two ports are used. As an example, we use Ref. [47] to decompose 2-QS into a four BS network, as shown in Fig. 1 for either (c) BP $|2\rangle$, or (d) SP $|1\rangle|1\rangle$ resources. The 2-QS is the smallest network whose useful teleamplification properties were previously not known.

Here we will highlight the key elements which prove Fig. 1 implements the n-QS transformation in Eq. (1) $\forall\, n \in \mathbb{N}^+$. Firstly, one can show that $B_2(g)|0\rangle|n\rangle = (1/(g^2+1)^{n/2})\sum_{j=0}^n g^j(-1)^j\sqrt{\binom{n}{j}}|j\rangle|n-j\rangle$, which already has the gain g^j coefficients, though with unwanted $(-1)^j\sqrt{\binom{n}{j}}$ factors. The red dashed box in (b) produces the two-mode output $|R_n\rangle = \otimes^{n-1}\langle 0|F_{n+1}^\dagger|0\rangle \otimes^n |1\rangle = (\sqrt{n!}/(n+1)^{n/2})\sum_{j=0}^n(-1)^j\sqrt{\binom{n}{j}^{-1}}|n-j\rangle|j\rangle$. In other words, the state $|R_n\rangle$ distorts the Fock states in an inverse manner to $B_2|0\rangle|n\rangle$. Hence, by combining one mode from each of these states, the overall action of the n-QSSP is

$$\langle 0|\langle n|B_2^\dagger|R_n\rangle = \frac{\sqrt{n!}}{(n+1)^{n/2}}\frac{1}{(g^2+1)^{n/2}}\sum_{j=0}^n g^j|j\rangle\langle j|. \qquad (2)$$

The n-QSBP is described by the same operator, since it is the conjugate transpose of this expression. The n-QS therefore applies ideal $g^{a^\dagger a}$ up to the nth Fock state

$$|g\psi_n\rangle = \frac{\sqrt{n!}}{(n+1)^{n/2}}\frac{1}{(g^2+1)^{n/2}}\sum_{j=0}^n g^j c_j|j\rangle. \qquad (3)$$

The Supplemental Material contains the full proof [28].

The n-QS has a success probability of $\mathbb{P} = \langle g\psi_n | g\psi_n \rangle$, which can be improved depending on whether we are considering the BP or SP configuration. For n-QSBP, it is not required that the vacuum state $\langle 0|$ be detected at the first output port of F_{n+1}, since the QFT is highly symmetric. If $\langle 0|$ was instead detected in the (m_0+1)th output port $\otimes^{m_0} \langle 1| \otimes \langle 0| \otimes^{n-m_0} \langle 1|$, $m_0 \in \{0,\ldots,n\}$, the output state will be $|g\psi_n\rangle$ with an extra phase shift that can be corrected by $C_1(m_0) = e^{(2i\pi m_0/(n+1))a^\dagger a}$ [28]. Utilizing all $n+1$ heralding events enhances the success probability by $\mathbb{P}_{\mathrm{BP}} = (n+1)\mathbb{P}$. For n-QSSP, the $|R_n\rangle$ resource state from the red dashed box in Fig. 1(b) could be prepared and stored beforehand; assuming $|R_n\rangle$ is deterministically available increases the success probability $\mathbb{P}_{\mathrm{SP}}$ to at least $((n+1)^n/(n+1)!)\mathbb{P}$ [28]. Note that $\mathbb{P}_{\mathrm{SP}} < \mathbb{P}_{\mathrm{BP}}$ for $n \in \{1,2\}$, $\mathbb{P}_{\mathrm{SP}} = \mathbb{P}_{\mathrm{BP}}$ for $n=3$, and $\mathbb{P}_{\mathrm{SP}} > \mathbb{P}_{\mathrm{BP}}$ for $n>3$ [28]. Since there is no difference between the output states of these configurations, we will use $\mathbb{P}_{\mathrm{XP}} = \max(\mathbb{P}_{\mathrm{SP}}, \mathbb{P}_{\mathrm{BP}})$ depending on the size n under consideration.

We will now contrast our protocol with Xiang *et al.* (X10) linear optical NLA protocol [4]. An n sized X10 network has n copies of 1-QS in parallel between two n splitters, and hence requires approximately the same amount of physical resources as an n-QS. One advantage of the simplified n-QS structure is that setting a particular gain requires changing just one BS, while n-X10 requires changing n BS concurrently. The output state from n-X10 has both the cutoff and distorted coefficients

$$|\psi\rangle \overset{n\text{-X10}}{\rightarrow} |g\phi_n\rangle = N' \sum_{j=0}^{n} \frac{1}{(n-j)!n^j} g^j c_j |j\rangle, \tag{4}$$

hence the NLA is not ideal in general [3]. The fidelity F can quantify how far away these output states are from the ideal NLA output state $g^{a^\dagger a}|\psi\rangle = |g\psi\rangle$ [28]. It is clear that an $n_{\max}$-QS can amplify any arbitrary state with a $n_{\max}$ upper energy limit with perfect fidelity. This feat cannot be replicated by any finite sized n-X10, or by any previous linear-optical NLA protocol [24,25].

In Fig. 2 we consider amplifying a coherent state and a single-mode squeezed vacuum (SMSV) state. Our n-QS has superior fidelity scaling, and hence for a required target fidelity needs much less resources with better success probability than the n-X10. For example, Figs. 2(a) and 2(b) show amplifying the coherent state by $g \approx 3$ with 99.9% fidelity requires only an 4-QS with 10^{-5} success probability, as opposed to a much larger 24-X10 with 10^{-24} success probability. Figures 2(c) and 2(d) emphasize the flexibility of our n-QS protocol, in that we can choose the best n size for a given input; since SMSV states contain only even photon numbers, it is best to use even sized n-QS (odd sizes will give the same fidelity as one size down). These graphs also show the n-QS has fidelity advantages even with amplifying SMSV states near maximum squeezing given by $g_{\max}^2 s = 1$ (here $g_{\max} \approx 1.9$).

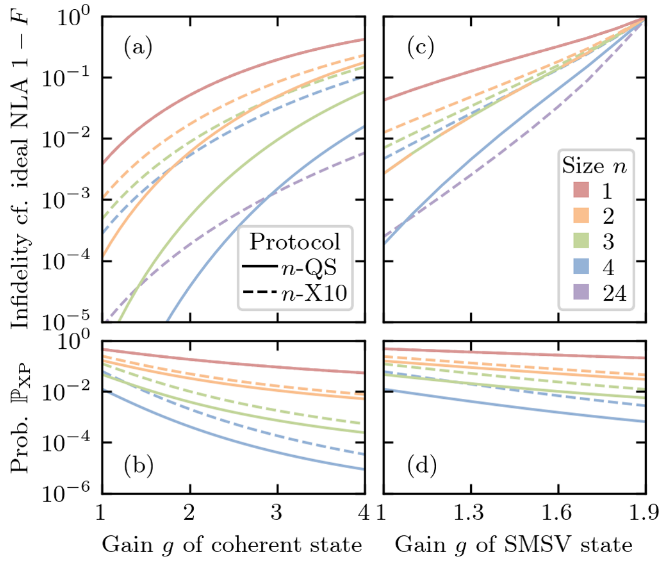

FIG. 2. A comparison of our n-QS NLA protocol, as per Fig. 1, against the n-X10 NLA protocol [4]. The left plots considers a coherent state input with $\alpha = 0.3$ amplitude, and the right plots considers a single-mode squeezed vacuum (SMSV) state input with $s \approx 0.29$ squeezing (such that these states have the same average photon number). (a),(c) Infidelity $1-F$ relative to a perfect NLA output state. (b),(d) Probability of success $\mathbb{P}_{\mathrm{XP}}$.

Quantum teleporter.—Quantum teleportation is a key primitive in quantum protocols [48–50], since it allows for the transfer and manipulation of quantum information through a shared entangled state; this is possible in both discrete variable [22] and continuous variable (CV) [51] regimes. Andersen and Ralph (AR13) proposed a CV teleportation scheme [29], which could in principle reach high fidelities with lower energy requirements than standard CV teleportation [51]. However, in a similar manner as X10, a finite sized AR13 protocol distorts the output state. We will demonstrate our n-QS with $g=1$, as in Fig. 3(a), is a better protocol for high-fidelity teleportation. We restrict ourselves to linear-optical systems, hence both n-AR13 and n-QS are nondeterministic, and require a comparable amount of physical resources.

We consider teleporting coherent and SMSV states with various amplitudes in Fig. 4; we chose higher valued energy states to show the advantage of our scheme for larger n. It is clear our n-QS scales with many orders of magnitude better fidelity in comparison to n-AR13, while the probability of success scales comparatively. For example, teleporting an SMSV using a 4-QS results in superior fidelity and success probability, while requiring less resources compared to a 10-AR13.

The AR13 paper illustrated the effectiveness of their protocol by analyzing the teleportation of a coherent state superposition $|\alpha\rangle + |-\alpha\rangle$ with $\alpha = 2$. The authors note that to achieve just over 99% fidelity, the standard

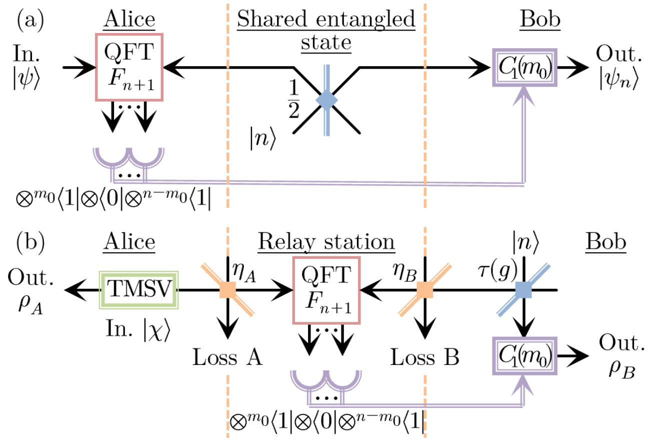

FIG. 3. Our scalable n-QS structure can be applied to many situations besides NLA, with significant improvements over existing protocols. We investigate applications for (a) quantum teleportation and (b) entanglement distillation as a loss-tolerant quantum relay. Shown are the BP variants.

teleportation approach requires 500 average photons (30 dB of squeezing) [51], while n-AR13 requires an $n = 100$ photon entangled state [29]. To reach the same fidelity, our n-QS protocol requires just $n = 10$ photons.

Loss-tolerant quantum relay.—Here we consider distilling entanglement through a loss channel with $\eta \in [0, 1]$ total transmissivity. The n-QS can function as a quantum relay by distributing the QFT measurement component over the channel, as shown in Fig. 3(b), such that $\eta_A \eta_B = \eta$. The distributed 1-QS has previously been shown to be uniquely loss tolerant, in that it can overcome the repeaterless Pirandola-Laurenza-Ottaviani-Banchi bound [52] without quantum memories [30]; the only other known scheme that can also do this feat is the twin-field quantum key distribution protocol and its variants [53–56]. We confirm that the complete set of distributed n-QS are also loss tolerant with improved usage rates. In other words, instead of having the entire NLA at Bob's side ($\eta_A = \eta, \eta_B = 1$), by placing the QFT measurement half way ($\eta_A = \sqrt{\eta}, \eta_B = \sqrt{\eta}$), we improve the success probability scaling from η^n to $\eta^{n/2}$ [28]. Note here we consider distilling a two-mode squeezed vacuum or EPR state with $\chi = 0.25$ squeezing.

The logarithmic negativity (LN) is an entanglement monotone [57,58], and an upper bound for distillable entanglement [59]. The LN is shown by the solid lines in Fig. 5(a), which increases with larger n sizes. Maximum LN occurs with gain approximately $g_{\max}\chi\sqrt{\eta_A/\eta_B} \approx 1$ (here $g_{\max} \approx 4$), which corresponds to an output state that is uniformly distributed in the Fock basis [28]. The dashed lines in these graphs only consider the second moment covariance correlations, which are more relevant for Gaussian and CV protocols [60].

The entanglement of formation (EOF) is an entanglement metric [61], whose properties for multimode Gaussian states are known [31,62–65]. Figure 5(b) is the Gaussian EOF, which closely resembles the covariance-based LN, as expected. The gray horizontal lines are pure loss channels

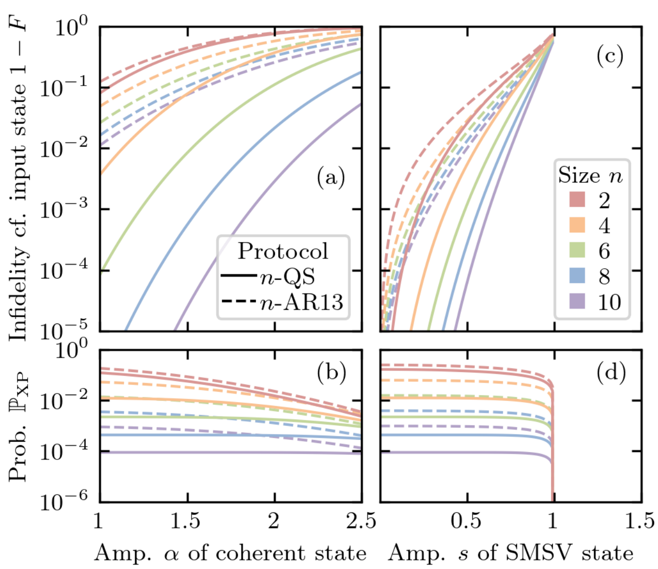

FIG. 4. A comparison of our n-QS teleportation protocol, as per Fig. 3(a), against the n-AR13 high-fidelity teleportation protocol [29]. We consider teleporting an α amplitude coherent state on the left and an s squeezed SMSV state on the right. (a),(c) Infidelity $1 - F$ of the teleported output state relative to the input state. (b), (d) Protocol's probability of success $\mathbb{P}_{XP}$.

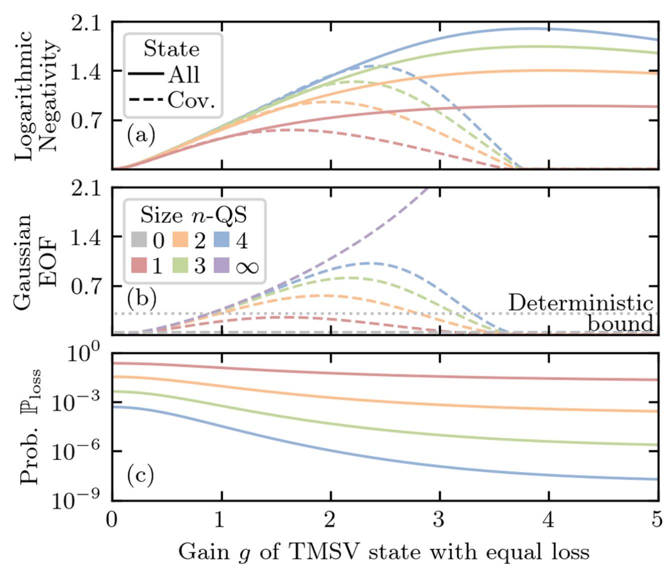

FIG. 5. The amount of entanglement which can be recovered, using an equally $\eta_A = \eta_B = \sqrt{\eta}$ distributed n-QS as a quantum relay, as per Fig. 3(b). We consider a $\chi = 0.25$ amplitude two-mode squeezed vacuum (TMSV) state into a lossy channel with $\eta = 0.05$ total transmission. The entanglement is measured using (a) log negativity and (b) Gaussian entanglement of formation. The solid line considers all correlations (i.e., the entire state), while the dashed lines only consider second moments (i.e., the covariance matrix). (c) Protocol's probability of success $\mathbb{P}_{\text{loss}}$.

with no QS, where the dashed line has the same initial squeezing $\chi = 0.25$, and the dotted line is the deterministic bound with infinite squeezing $\chi = 1$; it is clear this bound can be beaten by small sized n-QS. Increasing loss does not significantly change the maximum amount of distillable entanglement, which is another experimental appealing feature of this loss tolerant protocol [28].

Experimental imperfections.—Finally, we examined the effect of noisy, inefficient photon detectors and sources. We showed that our n-QSBP protocol is tolerant to experimental imperfections, in the same sense as the already experimentally verified 1-QS [4]. In other words, an imperfect n-QSBP as an amplifier, teleporter, or relay results in relative improvements with increased n, in a similar fashion as the ideal graphs in this Letter. Unfortunately, the n-QSSP is not tolerant to experimental imperfections. This is because of how the entanglement resource is prepared, and a different preparation scheme could help to improve an imperfect n-QSSP. See the Supplemental Material for more details [28].

Conclusion.—We introduced the generalized $n \in \mathbb{N}^+$ quantum scissors, which can perform perfect fidelity tele-amplification up to the nth Fock state. We proved that this operation can be implemented using two simple scalable linear-optical networks, with either n single or n bunched ancillary photons. As a consequence, our n-QS protocol is shown to have substantial advantages over existing NLA and teleportation schemes, in terms of fidelity scaling, success probability, and physical resources. Finally, we showed that a distributed n-QS quantum relay is loss tolerant with fast rates, and hence is useful as a building block for quantum repeater networks.

A. P. L. acknowledges support from BMBF (QPIC-1, PhoQuant), and the Einstein Research Unit on Quantum Devices. This research was supported by the Australian Research Council (ARC) under the Centre of Excellence for Quantum Computation and Communication Technology (Project No. CE170100012).

Note added.—The authors recently became aware of a new related work which investigated noiseless quantum tele-amplifiers from a different angle [66], based on the continuous-variable teleportation protocol [51].

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