

## Correlations at PT-Symmetric Quantum Critical Point

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We consider a PT-symmetric Fermi gas with an exceptional point, representing the critical point between PT-symmetric and symmetry broken phases. The low energy spectrum remains linear in momentum and is identical to that of a Hermitian Fermi gas. The fermionic Green's function decays in a power law fashion for large distances, as expected from gapless excitations, although the exponent is reduced from  $-1$  due to the quantum Zeno effect. In spite of the gapless nature of the excitations, the ground state entanglement entropy saturates to a finite value, independent of the subsystem size due to the non-Hermitian correlation length intrinsic to the system. Attractive or repulsive interaction drives the system into the PT-symmetry broken regime or opens up a gap and protects PT symmetry, respectively. Our results challenge the concept of universality in non-Hermitian systems, where quantum criticality can be masked due to non-Hermiticity.

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*Introduction.*—Quantum criticality and universality play a prominent role in various branches of physics [1–3], ranging from electrons in solids through ultracold atoms to quantum plasma of quarks. The emerging scale invariance dictates the dispersion of the excitation spectrum at the quantum critical point or the collapse of the gap upon approaching criticality, which in turn determines the long distance behavior of real space correlation functions. The influence of the critical point extends over a wide temperature window, and its universality applies not only in equilibrium, but extends also to near-adiabatic processes, such as the celebrated Kibble-Zurek mechanism [4,5].

Recently, non-Hermitian systems have been extensively investigated [6–14], featuring, among many others, distinct kinds of criticality and symmetry breakings. PT-symmetric non-Hermitian Hamiltonians [15–17] possess either real or complex pairs of eigenvalues, corresponding to eigenstates preserving or breaking PT symmetry. The transition from the real to complex spectrum and the associated PT-symmetry breaking occurs at a non-Hermitian quantum critical point, which is an exceptional point (EP) [18]. Therein, not only does the spectrum become degenerate, but also two (or more) *eigenstates* coalesce, which then no longer form a complete basis.

In light of these, it would be important and interesting to explore to what extent the universality of non-Hermitian quantum critical points parallel their Hermitian counterparts. In order to shed light on this issue, we study a PT-symmetric

Fermi gas, tuned to a PT-symmetric quantum critical point. Similar systems can be realized in various experiments. We find that in spite of the gapless nature of low energy excitations, the spatial decay of the Green's function is faster at long distances than in Hermitian systems, and cannot be accounted for by the critical exponents of the PT-symmetric EP. The overlap of the Hermitian and PT-symmetric ground states resembles the fidelity near a quantum critical point. The entanglement entropy saturates to a finite value with increasing subsystem size as if a finite gap was present in the system. All these features can be explained by measurement induced quantum Zeno effect [19,20], arresting the propagation of correlations due to the underlying continuous measurement, required for non-Hermitian dynamics. Interaction effects are also subtle: repulsive or attractive interactions open up a gap and protect PT-symmetry or destroy PT-symmetry and induce a second order EP, respectively.

*Hamiltonian.*—We study a one-dimensional (direction  $x$ ), PT-symmetric non-Hermitian Hamiltonian, which reads as

$$H = \int dx i v [R^+(x) \partial_x R(x) - L^+(x) \partial_x L(x)] + \Delta R^+(x) L(x), \quad (1)$$

where  $R(x)$  and  $L(x)$  describe right and left moving fermionic fields [21,22], respectively, the system is half filled, and we assume  $\Delta > 0$  without loss of generality.

This Hamiltonian is apparently non-Hermitian due to the absence of the  $\Delta L^+(x)R(x)$  term, and also resembles a charge density wave [22,23] Hamiltonian with only half the couplings. Parity ( $P$ ) transforms  $[R(x, t), L(x, t)] \rightarrow [L(-x, t), R(-x, t)]$  while time reversal ( $T$ ) results in  $[R(x, t), L(x, t)] \rightarrow [L(x, -t), R(x, -t)]$  and takes the complex conjugate of complex numbers [24]. This Hamiltonian can be realized using an effective Lindblad description, as discussed below Eq. (11).

Eq. (1) has real eigenvalues, which follows from rewriting it in momentum space for a given momentum mode as

$$H_p = \begin{bmatrix} vp & \Delta \\ 0 & -vp \end{bmatrix}, \quad (2)$$

whose spectrum is purely real as  $\varepsilon_{\pm}(p) = \pm v|p|$ , independent from  $\Delta$ . It thus describes a critical system with dynamical critical exponent  $z = 1$ . Therefore, the partition function  $\text{Tr}[\exp(-H/T)]$  is formally identical to that in a Hermitian Fermi gas. In the following, we show that in spite of this, many physical properties are affected by the presence of  $\Delta$  through a *correlation length* associated to  $v/\Delta$ .

Since  $H_p$  is PT symmetric and possesses a real spectrum, it can be brought to Hermitian form after a similarity transformation [15,16,24] as

$$S^{-1}H_p S = \begin{bmatrix} vp & 0 \\ 0 & -vp \end{bmatrix}, \quad S = \begin{bmatrix} 1 & -\frac{\Delta}{2vp} \\ 0 & 1 \end{bmatrix}. \quad (3)$$

In Eq. (2), the  $p = 0$  point corresponds to an EP [11,18], when the Hamiltonian becomes defective. The  $H_p$  is tuned to the brink of PT-symmetry breaking: by replacing the 0 with  $\Gamma$ , the spectrum changes to  $\pm\sqrt{v^2 p^2 + \Gamma\Delta}$ . In Eq. (1), this amounts to adding  $\Gamma \int dx L^+(x)R(x)$  to the Hamiltonian. For  $\Gamma > 0$ , a finite gap opens, preserving PT symmetry while for  $\Gamma < 0$ , a second order EP [18] is induced as  $\pm\sqrt{v^2 p^2 - |\Gamma|\Delta}$ , and PT symmetry is broken.

Since the spectrum of Eq. (1) is real, we can construct the ground state of the system as the minimal energy configuration [25], similar to the Hermitian realm [27].

*Green's functions.*—We start analyzing the peculiar properties of Eq. (1) by evaluating the equal time fermionic Green's functions, which are the normal  $G(x) \equiv \langle R^+(x)R(0) \rangle$  and anomalous  $F(x) \equiv \langle R^+(x)L(0) \rangle$  propagators. The conventional one reads [25] as

$$G(x) = \int_0^{\infty} \frac{dp}{2\pi} \frac{\Delta^2 e^{ipx-ap}}{(2vp)^2 + \Delta^2} + \int_{-\infty}^0 \frac{dp}{2\pi} e^{ipx+ap}, \quad (4)$$

where the first term arises from the non-Hermitian coupling between the right and left movers and is absent for  $\Delta = 0$ , while the second term represents the

conventional right-moving propagator [29] and is responsible for the  $1/x$  decay of the noninteracting Hermitian Fermi gas. Here, we introduced the momentum space cutoff  $\exp(-\alpha|p|)$  for momentum integrals [21] with  $\alpha$  the short distance cutoff. In Eq. (4), the time dependence drops out completely since the spectrum is real. In our case, we find that

$$G(x) = -\frac{i}{2\pi x} \times \begin{cases} 1 & x \ll v/\Delta \\ -2(\frac{2v}{\Delta x})^2 & v/\Delta \ll x \end{cases}. \quad (5)$$

Most importantly, we observe that in spite of the gapless nature of excitations [3] with  $z = 1$  and the complete absence of  $\Delta$  in the single particle spectrum, the real space Green's function decays as  $x^{-3}$  beyond the non-Hermitian correlation length as opposed to the  $x^{-1}$  decay of free Hermitian fermions. This is interpreted as the quantum Zeno effect [12,19], where the propagation of correlations is arrested by the continuous measurement within non-Hermitian quantum mechanics; thus significantly reduced correlations are present in the system in the end. We note that within the correlation length, the conventional fermionic decay is revealed. Similar conclusions apply to the  $\langle L^+(x)L(0) \rangle$  correlator.

The decay of  $G(x)$  is analogous to that in gapped or pseudogapped systems, where correlation functions decay differently within and outside of the correlation length. However, in that case, an additional energy scale makes its presence felt already in the excitation spectrum; while this feature is completely missing in our non-Hermitian scenario, its spectrum  $\pm v|p|$  is featureless.

Since the right and left moving fields are coupled by  $\Delta$  in Eq. (1), an anomalous Green's function is also present, similarly to density waves [23], as

$$F(x) = -\int_0^{\infty} \frac{dp}{2\pi} \frac{2\Delta v p e^{ipx-ap}}{(2vp)^2 + \Delta^2}, \quad (6)$$

exhibiting

$$F(x) = \frac{\Delta}{4\pi v} \times \begin{cases} \ln \frac{\Delta|x|e^{\gamma}}{2v} & x \ll v/\Delta \\ (\frac{2v}{\Delta x})^2 & v/\Delta \ll x \end{cases} \quad (7)$$

behavior,  $\gamma \approx 0.5772$  is Euler's constant [30], and  $F(0) = (\Delta/4\pi v) \ln(\Delta a e^{\gamma}/2v)$ . In a conventional gapped density wave, this correlation function decays exponentially as  $\exp(-|x|\Delta/v)$ . In contrast, a long distance power law decay is identified here due to gapless excitations. Since  $F(x)$  is directly proportional to  $\Delta$  and vanishes in a Hermitian Fermi gas, it can be used to reveal the presence of  $\Delta$  at short distances too. The correlator  $\langle L^+(x)R(0) \rangle$  follows similar behavior in spite of the fact that no  $L^+R$  coupling is present in the Hamiltonian. By moving away from the critical point, the Green's function decays exponentially deep in

the PT-symmetric phase with an exponent reduced by the quantum Zeno effect, while in the PT-broken regime, a power law decay  $\sim x^{-3/2}$  shows up due to the second order EP.

The ground state carries a finite current since the non-Hermitian term in Eq. (1) converts left to right movers. The particle current density [21,22]  $j$  is

$$j = v \langle R^+(x)R(x) - L^+(x)L(x) \rangle = \frac{\Delta}{4}, \quad (8)$$

independent of  $x$ . At the same time, our system realizes a charge density wave as the real space density profile  $n(x)$  oscillates as

$$n(x) - n_0 = \frac{\Delta}{2\pi v} \ln \left( \frac{\Delta \alpha e^\gamma}{2v} \right) \cos(2k_F x), \quad (9)$$

where  $n_0$  is the homogeneous particle density in the  $\Delta = 0$  system and  $k_F$  is the Fermi wave number.

*Fidelity.*—To investigate the relation between the gapless Hermitian ( $\Delta = 0$ ) and PT-symmetric ( $\Delta \neq 0$ ) ground states, their overlap, the fidelity [31,32], is evaluated to yield

$$\langle \Psi_0 | \Psi_\Delta \rangle = \frac{1}{\sqrt{\cosh(L\Delta/4v)}} = \begin{cases} e^{-\frac{L\Delta}{8v} + \frac{\ln 2}{2}} & \frac{L\Delta}{v} \gg 1 \\ 1 - \left(\frac{L\Delta}{8v}\right)^2 & \frac{L\Delta}{v} \ll 1. \end{cases} \quad (10)$$

This is analogous to the behavior expected in Hermitian systems around a Hermitian quantum critical point [32], although in the present case, both systems with  $\Delta = 0$  and  $\Delta \neq 0$  are identically critical with identical gapless spectra. This supports the narrative associated to non-Hermitian correlation length, masking the gaplessness of the spectrum in various physical quantities.

*Lattice realization.*—Our system can be realized with fermions in a one-dimensional half-filled tight binding chain with periodic boundary condition (PBC) and even number of sites as

$$H^{\text{tb}} = \sum_n \frac{J + i\delta(-1)^n}{2} (c_n^+ c_{n+1} + c_{n+1}^+ c_n) + (-1)^n g c_n^+ c_n, \quad (11)$$

where  $J$  is the uniform hopping,  $g$  is an alternating on site potential,  $\delta$  stems from a non-Hermitian alternating hopping, and  $c$ 's are fermionic annihilation operators. The non-Hermitian term arises from an effective Lindblad equation without the recycling term [33] using jump operators for bonds [34–37] as  $\sqrt{\delta}(c_n \pm c_{n+1})$  on even (+) and odd (−) bonds, and neglecting a constant shift, proportional to the total particle number. In momentum space, the ensuing Hamiltonian in the basis of  $(c_k, c_{k-\pi})$  with  $0 \leq k < \pi$  reads as

$$H_k^{\text{tb}} = \begin{bmatrix} J \cos(k) & g + \delta \sin(k) \\ g - \delta \sin(k) & -J \cos(k) \end{bmatrix} \quad (12)$$

with dispersion  $E_\pm(k) = \pm \sqrt{(J^2 + \delta^2) \cos^2(k) + g^2 - \delta^2}$ . Expanding this around the Fermi wave number  $\pi/2$  for half filling, we get Eq. (1) for  $g = \delta$  as  $v = J$  and  $\Delta = 2g$ . As long as  $g \geq \delta$ , the system is PT symmetric, and the spectrum is real. However, for  $g < \delta$ , there is always a region close to  $k = \pi/2$  with imaginary pairs of eigenvalues; thus, the PT symmetry is broken. From this point on, we consider the  $g = \delta$  case, when  $H^{\text{tb}}$  is tuned to the boundary of PT symmetry and represents a lattice realization of Eq. (1). For more details of  $H^{\text{tb}}$ , see Ref. [25].

First, we evaluate numerically the fermionic Green's function,  $G^{\text{tb}}(m) = \langle c_{m+n}^+ c_n \rangle$ . Due to the commensurate Fermi wave number  $\pi/2$ , this measures directly  $G(x)$  or  $F(x)$  for odd or even  $n$ , because they are multiplied by the  $\sin(k_F m)$  and  $\cos(k_F m)$  factors in the full fermionic Green's function, respectively. In particular, for  $g = \delta = 0$ , we recover the conventional expression  $G^{\text{tb}}(m) = \sin(\pi n/2)/n\pi$ . For finite  $g = \delta$ , the system indeed exhibits a charge density wave pattern with gapless, linearly dispersing excitations. In Fig. 1, we numerically evaluate the tight binding Green's function in the thermodynamic limit, which agrees with  $G(x)$  and  $F(x)$ , as advertised above.

*Entanglement.*—With the knowledge of the single particle Green's function, we address the entanglement properties of our system. In particular, we evaluate the von-Neumann entanglement entropy  $S(m)$  between the subsystem of size  $m$  and the rest of the chain using Refs. [38–41]. We have also checked on small systems up to 26 sites that by numerically exact diagonalizing the many-body problem of Eq. (11) and brute force

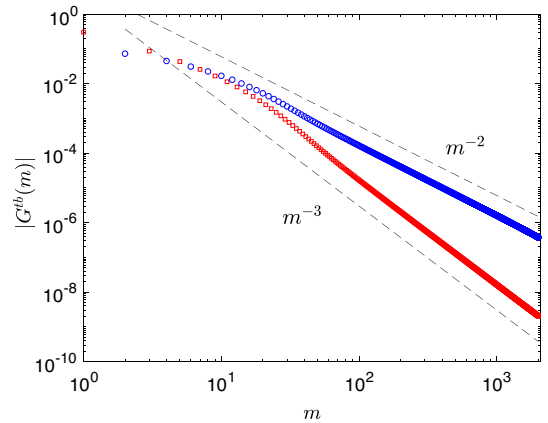


FIG. 1. The absolute value of the lattice Green's function is plotted for even (blue) and odd (red) spatial separation and  $g = \delta = 0.2J$ . The reduction caused by the quantum Zeno effect is clearly visible in the long distance asymptotics, which agrees with the continuum limit calculation. The black dashed lines show the  $m^{-2}$  and  $m^{-3}$  power law decays.

calculating the subsystem entanglement agrees with the Green's function based approach. We find that for a small subsystem size  $m$ , the entanglement entropy follows a  $S(m) = \frac{1}{3}\ln(m)$  growth [42,43]. This indicates that the central charge [31,44–46] of the non-Hermitian system with EP in Eq. (1) remains 1. Upon further increasing the subsystem size such that  $m \gg J/g$ , the entanglement entropy saturates to a constant value, shown in Fig. 2, similarly to what happens in one-dimensional gapped systems. However, our Hamiltonian is gapless, but the presence of non-Hermitian coherence length stops the logarithmic entanglement growth, similarly to how it affects the decay of the Green's function through the quantum Zeno effect [19]. The scaling of the saturation value of the entanglement entropy for a large subsystem agrees surprisingly with Ref. [42] as  $S(m \gg J/g) = \frac{1}{3}\ln(J/g) + \frac{2}{3}$  after identifying the correlation length with the non-Hermitian coupling. This behavior is highly unusual and challenges the interpretation of criticality and universality in non-Hermitian systems: in spite of the typical gapless excitation spectrum in one dimension, the entanglement entropy still saturates as if the system was gapped. This “contradiction” is cured upon realizing that in spite of the gapless spectrum, the asymptotic decay of the Green's function gets suppressed by the quantum Zeno effect, which results in an effective gapped type subsystem entanglement entropy.

We mention that in related nonunitary conformal field theories and critical systems [47–50], the conventional  $S(m) \sim \ln(m)$  behavior was found. The difference between these and our findings follows from the fact that either a distinct definition of the reduced density matrix is used [49] or in the models of Refs. [47,48,50] the right ground states

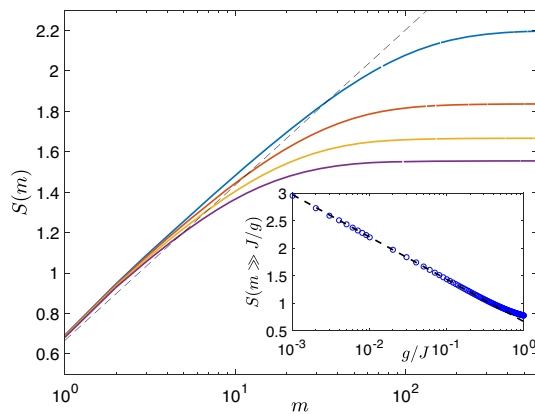


FIG. 2. The von-Neumann entanglement entropy is plotted for  $g = \delta = 0.01J, 0.03J, 0.05J$ , and  $0.07J$  from top to bottom. The black dashed line denotes a  $\frac{1}{3}\ln(m) + \frac{2}{3}$  curve. The inset shows the saturation value of the entanglement entropy for a large subsystem size as a function of  $g = \delta$ . The black dashed line denotes  $\frac{1}{3}\ln(J/g) + \frac{2}{3}$ .

of both  $H$  and  $H^+$  are equal to each other, which is not the case [25] for Eq. (2).

*Interaction effects.*—Since low dimensional systems are susceptible to various instabilities, we address here the role of electron-electron interactions on Eqs. (1) and (11). On the lattice, this amounts to considering the nearest-neighbor interaction as  $H_{\text{int}} = V \sum_m c_m^+ c_m c_{m+1}^+ c_{m+1}$ . This interaction preserves PT symmetry. In similar situations [21], one typically performs a renormalization group calculation to address the relevance of interaction processes. In our case, however, a perturbative Hartree-Fock calculation already suffices as

$$H_{\text{int}}^{\text{HF}} = V \sum_m 2 \langle c_{m+1}^+ c_{m+1} \rangle c_m^+ c_m - \langle c_{m+1}^+ c_m \rangle c_m^+ c_{m+1} - \langle c_m^+ c_{m+1} \rangle c_{m+1}^+ c_m + \text{const}, \quad (13)$$

where the expectation values are taken with respect to the noninteracting ground state. The first Hartree term on the rhs renormalizes  $g$  by  $2Vg/(\pi J) \ln(2J/g)$ , while the last two Fock terms mostly induce a shift to the Fermi wave vector, which turns out to be irrelevant at weak interactions, and the first term in Eq. (13) governs the weak coupling physics. For repulsive interactions, the renormalized  $g$  is enhanced; thus a clean gap opens up in the spectrum and protects PT symmetry. On the attractive side,  $g$  gets renormalized to smaller values; hence  $\delta$  becomes larger than the renormalized  $g$ , the PT symmetry gets broken, and part of the excitation spectrum becomes imaginary.

This conclusion is corroborated by studying  $H^{\text{tb}} + H_{\text{int}}$  numerically using exact diagonalization on a system with 26 lattice sites and 13 particles and PBC. We study the spectrum around the ground state energy  $E_0$  for weak interactions, where  $E_0$  represents the eigenenergy with the lowest real part. This turns out to be unique and purely *real* in this case. The many-body spectrum for the six lowest lying states above the ground state are plotted in Fig. 3. There is a finite gap in the noninteracting,  $V = 0$  limit due to finite level spacing; therefore the PT-symmetry breaking is shifted to finite negative  $V$ . This together with the finite noninteracting gap vanishes with increasing system size.

*Relation to sine-Gordon model.*—One-dimensional quantum systems are often studied via bosonization [21]. By applying it to Eq. (1), we get

$$H = \int \frac{dx}{2\pi} v \{ [\pi \Pi(x)]^2 + [\partial_x \phi(x)]^2 \} + \frac{\Delta}{2\pi\alpha} e^{-i2\phi(x)}, \quad (14)$$

where  $\Pi$  and  $\phi$  are dual fields satisfying  $[\phi(x_1), \Pi(x_2)] = i\delta(x_1 - x_2)$  at equal times. This is the non-Hermitian “half” of the conventional sine-Gordon model, and both the imaginary sine and real cosine potentials have equal strengths [24,36]. Due to this, the absolute value of the potential term is constant and is unable to localize the  $\phi$  field. This is manifested in the linear energy-momentum

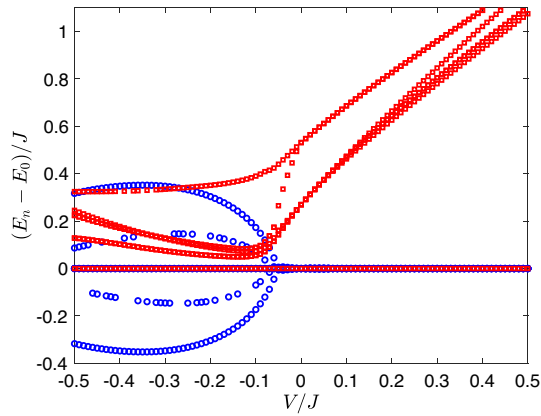


FIG. 3. The many-body spectrum  $E_n$  of  $H^{\text{tb}} + H_{\text{int}}$ , consisting of the six lowest lying states above the *real* ground state energy  $E_0$  is shown for 26 lattice sites with  $g = \delta = 0.5J$ . The real and imaginary part of the eigenenergies, measured from the ground state energy, are plotted in red squares and blue circles, respectively.

relationship from the original, fermionic treatment. The fate of adding  $e^{i2\phi(x)}$  potential to Eq. (14) is discussed in Ref. [25]. We speculate that a mapping of this bosonic Hamiltonian back into the fermionic one could play an important role when analyzing strongly correlated non-Hermitian systems [37] at their respective Luther-Emery point [51].

*Experimental possibilities.*—Eqs. (1) and (11) can be realized using a waveguide lattice [52], simulating an effective non-Hermitian Hamiltonian experimentally. From Refs. [53–55], our setup can be created with ion trap physics, simulating our system in a minimal setting consisting of two atomic levels and a motional degree of freedom. Ultracold bosonic atoms loaded into an optical lattice as in Refs. [36,37] with controlled losses represent another viable route. In this case, the density matrix of the system evolves according to a Lindblad equation. By continuously measuring the environment [12,33,56] and postselecting the data to ensure that the state of the environment remains unchanged, the effective Hamiltonian becomes non-Hermitian and of the form of Eq. (1). Additionally, one can use single photon interferometry to realize our momentum space Hamiltonians [57].

*Summary.*—We have studied the properties of a PT-symmetric quantum critical point. We find that it is “less universal” than its Hermitian counterpart: the equal time Green’s function decays faster for longer distances than expected from the linear energy-momentum relationship. This occurs due to the quantum Zeno effect, which slows down the propagation of excitations due to continuous measurement. As a result, the spatial entanglement entropy saturates to a finite value with increasing subsystem size, and the saturation value is determined by the non-Hermitian correlation length, in spite of the gapless nature of excitations. Repulsive electron-electron interaction opens up a gap

by protecting PT symmetry while attractive interaction breaks PT symmetry and induces a second order exceptional point.

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