Mechanical Squeezing via Unstable Dynamics in a Microcavity

Katja Kustura,[1,2] Carlos Gonzalez-Ballestero,[1,2] Andrés de los Ríos Sommer,[3,4] Nadine Meyer,[3,4] Romain Quidant,[3,4] and Oriol Romero-Isart[1,2]
[1]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria
[2]Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
[3]Nanophotonic Systems Laboratory, Department of Mechanical and Process Engineering, ETH Zurich, 8092 Zurich, Switzerland
[4]Quantum Center, ETH Zurich, 8083 Zurich, Switzerland

(Received 3 December 2021; accepted 10 March 2022; published 6 April 2022)

We theoretically show that strong mechanical quantum squeezing in a linear optomechanical system can be rapidly generated through the dynamical instability reached in the far red-detuned and ultrastrong coupling regime. We show that this mechanism, which harnesses unstable multimode quantum dynamics, is particularly suited to levitated optomechanics, and we argue for its feasibility for the case of a levitated nanoparticle coupled to a microcavity via coherent scattering. We predict that for submillimeter-sized cavities the particle motion, initially thermal and well above its ground state, becomes mechanically squeezed by tens of decibels on a microsecond timescale. Our results bring forth optical microcavities in the unresolved sideband regime as powerful mechanical squeezers for levitated nanoparticles, and hence as key tools for quantum-enhanced inertial and force sensing.

DOI: 10.1103/PhysRevLett.128.143601

A system of linearly coupled quantum harmonic oscillators can be dynamically unstable, even in the absence of dissipation (see Ref. [1] and references therein). Although dynamically unstable regimes are often considered undesirable [2], they can also be a resource. In this Letter, we show how to engineer and harness unstable multimode dynamics in a linear optomechanical system to induce strong mechanical squeezing. Preparing squeezed states of a mechanical oscillator—quantum states with position uncertainty smaller than its zero-point motion—is a key aim of optomechanics [3], as these states lie at the heart of many quantum-enhanced force and inertial sensing schemes [4]. This is evidenced by the many theoretical and experimental efforts aimed at developing strategies to generate mechanical squeezing, e.g., reservoir engineering based on two-tone driving [5–11], parametric squeezing [12–22], rapid frequency shifts [23–28], quantum measurement [29–36], mechanical nonlinearities [16,26,37,38], or quantum transfer of a squeezed state from a cavity mode to the mechanical oscillator [39,40].

In this Letter, we propose a novel approach based on the fast unstable quantum dynamics of a linear optomechanical system. Our protocol requires to operate in the ultrastrong coupling and far red-detuned regime, i.e., $g > \Omega/2$ and $\Delta \gg \Omega$ with Ω the mechanical frequency, Δ the laser detuning from cavity resonance, and g their optomechanical coupling rate [see Eq. (1)]. This regime is within reach for levitated optomechanics [41,42], specifically for an optically levitated nanoparticle coupled via coherent scattering [43–47] to a microcavity [48–51] in the unresolved sideband regime. Our results are particularly timely due to recent experiments demonstrating ground-state cooling and quantum control of optically levitated nanoparticles in free space [52,53]. As opposed to the first ground-state cooling experiment [44], which required a resolved-sideband cavity—and thus squeezing protocols designed for such regime [54], these recent free-space ground-state cooling experiments do not need an optical cavity. Our approach thus allows us to incorporate an independent and passive *mechanical squeezer* to these state-of-the-art experiments in the form of a properly optimized microcavity. The sole purpose of such a cavity is the generation of strong squeezing in the motion of the cooled levitated nanoparticle. This opens the door toward achieving quantum-enhanced sensing with levitated micro objects [41,42,55].

We consider a mechanical oscillator of mass m and frequency Ω coupled to an optical cavity mode of frequency ω_c that is being driven at frequency ω_t in the red-detuned regime $\Delta \equiv \omega_c - \omega_t > 0$. The linearized Hamiltonian of the system in a frame rotating at the frequency of the driving is given by

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \hbar g(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b}), \quad (1)$$

where $\hat{a}$ and $\hat{b}$ are bosonic annihilation operators of the cavity mode and the mechanical mode, respectively. In this Letter we focus solely on the regime [56]

$$\frac{4g^2}{\Delta\Omega} > 1, \quad (2)$$

which makes the system described by Eq. (1) dynamically unstable [57]. In the unstable regime defined by Eq. (2), the

Hamiltonian in Eq. (1) cannot be diagonalized in terms of bosonic modes, but it can be expressed in normal form [1] as

$$\hat{H} = \hbar\omega_1 \hat{c}_1^\dagger \hat{c}_1 + \frac{i\hbar r}{2}[(\hat{c}_2^\dagger)^2 - \hat{c}_2^2], \quad (3)$$

where $\hat{c}_1$ and $\hat{c}_2$ are bosonic annihilation operators of the normal modes, $\omega_1^2 \equiv (\zeta^2 + \Delta^2 + \Omega^2)/2$, $r^2 \equiv (\zeta^2 - \Delta^2 - \Omega^2)/2$, and $\zeta^4 \equiv (\Delta^2 - \Omega^2)^2 + 16\Delta\Omega g^2$. The canonical transformation between the physical modes $\{\hat{a}, \hat{b}\}$ and the normal modes $\{\hat{c}_1, \hat{c}_2\}$ is given in the Supplemental Material [58]. The Hamiltonian in Eq. (3) elucidates the dynamics in the unstable regime: the normal mode $\hat{c}_1$ is described by an uncoupled harmonic oscillator term, and the normal mode $\hat{c}_2$ by a pure squeezing term with a squeezing rate r that accounts for the unstable dynamics of the system. A key observation is that in the far-detuned regime $\Delta \gg \Omega$, the squeezed hybrid mode $\hat{c}_2$ is dominated by the contribution of the mechanical mode, namely,

$$\lim_{\Delta/\Omega \gg 1} \hat{c}_2 = -i\frac{g}{\sqrt{\Omega\Delta}}\left[(\hat{b}^\dagger + \hat{b}) + i\sqrt{\frac{\Omega}{\Delta}}(\hat{a}^\dagger - \hat{a})\right]. \quad (4)$$

This indicates that mechanical squeezing should be dynamically generated if, in addition to the instability condition defined by Eq. (2), the condition $\Delta \gg \Omega$ (far red detuning) is fulfilled. Note that both requirements can only be satisfied in the ultrastrong coupling regime $g \gg \Omega/2$ [59–62]. One can show that in the far red-detuned regime the squeezing rate in Eq. (3) is given by $r \approx 2g\sqrt{\Omega/\Delta}$. In the following we show how mechanical squeezing is generated.

Let us define the generalized mechanical quadrature $\hat{X}(\theta) \equiv (\hat{b}^\dagger e^{i\theta} + \hat{b}e^{-i\theta})/\sqrt{2}$, with $\theta \in [0, 2\pi)$. The minimal variance is obtained at a phase-space angle $2\theta_{sq} \equiv \arg\langle\hat{b}^2\rangle$ [see Fig. 1(a)] and is given by $\Delta^2 X_{sq} \equiv \langle\hat{X}^2(\theta_{sq} + \pi/2)\rangle = 1/2 + \langle\hat{b}^\dagger\hat{b}\rangle - |\langle\hat{b}^2\rangle|$. Squeezed states are defined by a variance $\Delta^2 X_{sq} < 1/2$ and their squeezing is quantified in decibels (dB) by $S \equiv -10\log_{10}(2\Delta^2 X_{sq})$. We consider the coherent dynamics generated by Eq. (1) with an initial state given by the cavity mode in vacuum (in the linearized regime) and the mechanical mode in a thermal state with mean phonon number $\bar{n}_b$ [63]. In Fig. 1(b) we show $\Delta^2 X_{sq}$ and θ_{sq} as a function of time, thereby demonstrating the generation of mechanical squeezing. One can show that the variance $\Delta^2 X_{sq}$ reaches the asymptotic value

$$\lim_{tr \gg 1} \Delta^2 X_{sq} = \frac{1}{2}\frac{\Omega}{\Delta} \ll 1. \quad (5)$$

Remarkably, the asymptotic value is independent of the mean phonon number $\bar{n}_b$. Squeezing is achieved at a phase-space angle given by $\lim_{tr\gg 1}\exp[2i\theta_{sq}] \approx -1 + \Delta\Omega/(2g^2) + i\sqrt{\Delta\Omega}/g$. Figure 1(c) shows the squeezing timescale Δ/r throughout the stability diagram, considering the exact expression for r, which shows deviation from the approximated expression $r \approx 2g\sqrt{\Omega/\Delta}$ close to the stability border. If the asymptotically squeezed state with the variance given in Eq. (5) is rotated in phase space such that the position quadrature $\hat{X}(\theta = 0)$ becomes maximally squeezed, the corresponding position fluctuations are given by $\sqrt{\hbar/(2m\Delta)}$. These fluctuations are similar to the zero-point motion associated with a hypothetical harmonic trap with frequency $\Delta \gg \Omega$. Such rotation in phase space can be done via free evolution in the harmonic trap for a time $\Omega t = \theta_{sq} + \pi/2$.

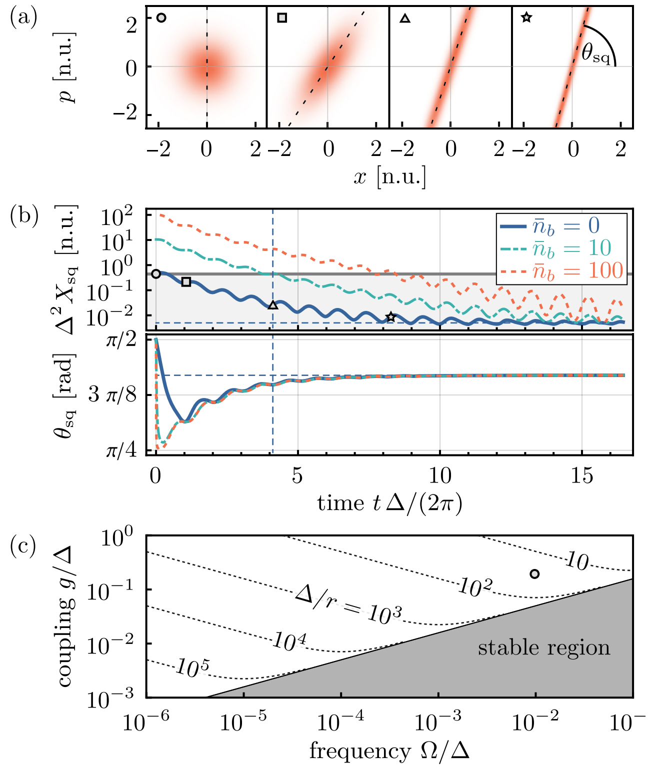

FIG. 1. Squeezing induced via dynamical instability in the absence of dissipation. (a) Wigner function of the mechanical state $W(x, p)$ at different times [denoted by points in panel (b)], for an initial state with mean phonon number $\bar{n}_b = 0$. (b) Minimal variance $\Delta^2 X_{sq}$ and the angle of squeezing θ_{sq} as a function of time, for $g/\Delta = 0.2$, $\Omega/\Delta = 0.01$, for various initial thermal states with mean phonon number $\bar{n}_b$, and for a cavity initially in the vacuum state. Dashed vertical and horizontal lines show the squeezing timescale Δ/r and the asymptotic value $\Omega/(2\Delta)$, respectively. Gray shaded area indicates squeezing. (c) Squeezing timescale Δ/r as a function of the mechanical frequency Ω and the optomechanical coupling rate g. The point denotes the configuration analyzed in panels (a) and (b).

Let us now discuss how the presence of noise and decoherence affects these results. We consider the dynamics described by the following master equation,

$$\dot{\hat{\rho}} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}] + \kappa\left(\hat{a}\,\hat{\rho}\,\hat{a}^\dagger - \frac{1}{2}\{\hat{a}^\dagger\hat{a},\hat{\rho}\}\right) - \frac{\Gamma}{2}[\hat{b}^\dagger + \hat{b},[\hat{b}^\dagger + \hat{b},\hat{\rho}]], \qquad (6)$$

where $\hat{\rho}$ is the density matrix of the cavity and the mechanical mode. The first dissipative term models cavity photon losses at a rate κ (i.e., $d\langle\hat{a}^\dagger\hat{a}\rangle/dt = -\kappa\langle\hat{a}^\dagger\hat{a}\rangle + \cdots$). The second dissipative term accounts for white mechanical displacement noise with a decoherence rate given by Γ (i.e., $d\langle\hat{b}^\dagger\hat{b}\rangle/dt = \Gamma + \cdots$). This form of mechanical dissipation models laser recoil heating, scattering of air molecules in high vacuum, and any other type of displacement noise (e.g., trap vibrations) for levitated nanoparticles [65]. In the Supplemental Material [58] we provide an analogous discussion for the standard mechanical dissipator describing the weak coupling to a thermal bath that is relevant for clamped mechanical oscillators. By writing the master equation (6) in terms of the normal modes, and neglecting rapidly rotating terms (this can be done provided that $\kappa \ll \Delta$), one can analytically obtain the asymptotic value of the minimal variance in the far-detuned regime $\Delta \gg \Omega$ (see the Supplemental Material [58] for further details). The asymptotic variance is given by

$$\lim_{t\tau\gg 1}\Delta^2 X_{\text{sq}} = \frac{\Omega}{2\Delta}\left(1 + \frac{\kappa}{4g}\sqrt{\frac{\Delta}{\Omega}} + \frac{\Gamma\Delta^2}{4g^3}\sqrt{\frac{\Delta}{\Omega}}\right). \qquad (7)$$

The second and the third term represent the noise-induced correction to the minimal variance. Equation (7) shows that in the presence of noise there is an optimal detuning Δ_{opt} for which the variance is minimized. The optimal detuning is well approximated by $\Delta_{\text{opt}} \approx g\sqrt{\kappa/(3\Gamma)}$. The condition for unstable dynamics [Eq. (2)] then reads $g/\Omega > \sqrt{\kappa/(48\Gamma)}$, and the condition for far detuning [$\Delta_{\text{opt}} \gg \Omega$] reads $g/\Omega \gg \sqrt{3\Gamma/\kappa}$. Note that typically $\kappa \gg \Gamma$, especially when considering microcavities (see below for further details). Figure 2 displays the asymptotic mechanical squeezing S at optimal detuning as a function of the rates κ/Ω and Γ/Ω for different coupling rates g/Ω. Figure 2 shows that the generation of strong squeezing is feasible in the presence of cavity losses and mechanical displacement noise. Specifically, squeezing above 3 dB is possible even at $g/\Omega = 1$, while strong squeezing ($S > 30$ dB) can be achieved deep in the ultrastrong coupling regime $g/\Omega \gtrsim 100$. Note that the resolved sideband regime, namely, $\kappa \ll \Omega$, is not a requirement to obtain strong mechanical squeezing.

Let us now show that the results discussed above and displayed in Fig. 2 are particularly feasible in levitated optomechanics via coherent scattering. We consider the setup shown schematically in Fig. 3(a), in which a nanoparticle is trapped in optical tweezers and placed at a node of an optical cavity. The scattering of laser photons into free

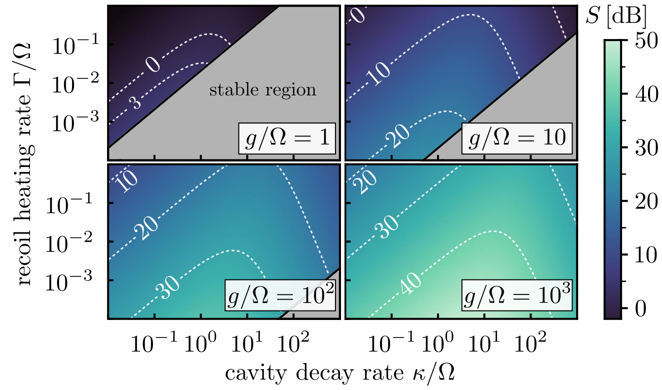

FIG. 2. Asymptotic mechanical squeezing S in the presence of dissipation, considering optimal detuning $\Delta = \Delta_{\text{opt}}$ (see main text), and an initial state with mean phonon number $\bar{n}_b = 0$.

space and into the cavity induces the coherent and dissipative dynamics that are well described by the master equation (6) with the Hamiltonian given by Eq. (1) [65]. Detailed expressions for the trapping frequency Ω, the optomechanical coupling rate g, the recoil heating rate Γ, and the cavity photon decay rate κ are given in the Supplemental Material [58,65]. Here we discuss only their dependence on the most relevant quantities, that is, tweezers power P_t, particle radius R, cavity length L_c, and cavity finesse f. In particular, $\Omega \propto P_t^{1/2}$, $g \propto P_t^{1/4}R^{3/2}L_c^{-1}$, $\Gamma \propto P_t^{1/2}R^3$, and $\kappa \propto L_c^{-1}f^{-1}$. Therefore, one obtains that $g^2/(\Omega\Delta_{\text{opt}}) \propto R^3 L_c^{-1/2} f^{1/2}$. Hence, the instability condition Eq. (2) at optimal detuning is independent of the tweezers power P_t and benefits from high-finesse cavities with small mode volumes and large (but subwavelength) nanoparticles.

The mechanical squeezing achievable in coherent scattering is shown in Fig. 3(b). We plot the asymptotic mechanical squeezing S as a function of the cavity length L_c for initial mean phonon numbers $n_b = 0$, 10, and 100, assuming a silica nanoparticle of radius $R = 100$ nm, trap frequency $\Omega/(2\pi) = 100$ kHz, and a cavity finesse $f = 10^5$ (see caption of Fig. 3 for more details). As predicted, the smaller the cavity length, the larger the generated squeezing, with S reaching values well over 10 dB for submillimeter-sized cavities [48,50,51]. The results shown in Fig. 3(b) are obtained by numerically solving the master equation (6), and they are compared with the analytical expression for the asymptotic variance Eq. (5), shown by the dashed gray line. Equation (5) is an excellent approximation away from the stable region (denoted by the shaded area) and for initial mean phonon numbers $\bar{n}_b \lesssim 100$. Figure 3(c) displays the optomechanical coupling rate g and the optimal detuning Δ_{opt} corresponding to the case analyzed in Fig. 3(b). The solid line shows the optimal detuning given by the approximation $\Delta_{\text{opt}} \approx g\sqrt{\kappa/(3\Gamma)}$, and it is nearly indistinguishable from the exact numerical

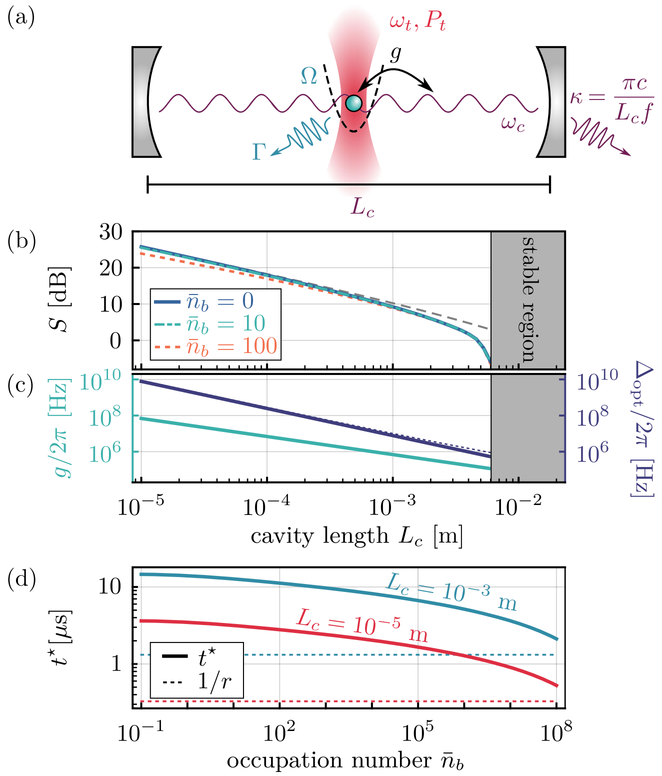

FIG. 3. Feasibility of mechanical squeezing in coherent scattering. (a) Schematic representation of the setup. A spherical nanoparticle is levitated by optical tweezers with power P_t, frequency ω_t, and waist W_t. It is placed at a node of an optical cavity with length L_c, resonance frequency ω_c, and decay rate κ. Optical tweezers provide a trapping potential with frequency Ω, an effective optomechanical coupling at a rate g, and particle recoil heating at a rate Γ. (b) Asymptotic mechanical squeezing S as a function of cavity length for a silica nanoparticle with radius $R = 100$ nm. The remaining parameters are $P_t = 29$ mW, $W_t = 0.7\ \mu$m, $\lambda_t \equiv 2\pi c/\omega_t = 1064$ nm, $\lambda_c \equiv 2\pi c/\omega_c = 1064$ nm, $f = 10^5$, and the asymmetry parameters of the tweezers $A_x = 0.9$, $A_y = 0.8$ (see the Supplemental Material [58]). (c) Optomechanical coupling rate g and optimal detuning Δ_{opt} as a function of cavity length, for the same parameters as in (b). Approximate and exact expressions for Δ_{opt} are shown by solid and dotted lines, respectively. (d) Extension time $t^\star$ as a function of the initial mean phonon number, for the same parameters as in (b).

value, shown by the dotted line. Note that the requirements $g, \Delta_{opt} \gg \Omega = 2\pi \times 100$ kHz are well satisfied.

Since squeezing occurs at a given angle θ_{sq} [see Fig. 1(a)], the spatial extension of the position probability distribution of the nanoparticle grows as a function of time. The linear dynamics of the nanoparticle in a coherent scattering setting is well described by the master equation (6) and the Hamiltonian given in Eq. (1) provided that this spatial extension is smaller than the cavity optical wavelength λ_c [65]. It is thus important to check that significant squeezing occurs before the spatial extension is too large. This is analyzed in Fig. 3(d), where we show the extension timescale $t^\star$, defined as the time such that $[\langle \hat{X}^2(\theta = 0)\hbar/(m\Omega)\rangle(t^\star)]^{1/2} = 0.1\lambda_c$, as a function of the initial mean phonon number $\bar{n}_b$ for two values of the cavity length (solid lines), and compare it to the squeezing timescale $1/r$ (dashed lines). Figure 3(d) confirms that the asymptotic value of S can be achieved faster than $t^\star$ for a mechanical mode initially in a state given by a wide range of thermal occupation numbers even well above the ground state, evidencing the broad feasibility of squeezing.

We remark that the setup considered here models recent coherent scattering experiments [43–47] that have demonstrated ground state cooling [45] and strong coupling [46,47]. We argue that the scheme we have presented is attainable in similar experiments where the cavity length is decreased, increasing the optomechanical coupling rate g at the cost of increasing the cavity decay rate κ. Such small cavities with high finesse are experimentally feasible and have been realized in Refs. [48,50,51]. In this context, motional cooling can be achieved via feedback [52,53] and the coupling to the microcavity can be switched on and off by controlling the detuning. In order to experimentally demonstrate the generation of mechanical squeezing we propose to optically measure the position of the particle during the stable harmonic dynamics generated by setting $\Delta \gg \Delta_{opt}$ at a time t_0 ($t^\star \gg t_0 \gg 1/r$), which effectively decouples the particle from the cavity. In particular, the recorded trajectories on different experimental runs can be demodulated at frequency Ω and angle $\phi \equiv -\theta_{sq} - \pi/2$ [66,67] and then ensemble averaged. As we show in the Supplemental Material [58], the variance of such a demodulated position operator is given by the squeezed variance Eq. (7) at $t = t_0$ plus a noise term of the order of Γ/Ω that is much smaller than one. This method should thus allow one to experimentally measure the squeezed variance Eq. (7).

In summary, we have shown how a dynamical multimode instability in a linear optomechanical system can be used to rapidly generate strong mechanical squeezing. The instability can be exploited by operating the optomechanical system in the far red-detuned and ultrastrong coupling regime. Our results show that it is worth exploring the different types of dynamical instabilities encountered in a multimode system [1] (e.g., three-mode dynamical instabilities might be used to generate two-mode entanglement via their separate coupling to a third mediating mode). While our results are in principle applicable to any optomechanical system, we have focused on an optically levitated nanoparticle coupled to a high-finesse optical cavity via coherent scattering. The combination of the recent achievement of free-space quantum control of nanoparticles [52,53] with a properly designed microcavity opens a direct pathway towards strong mechanical quantum squeezing induced by multimode instabilities. In this sense, our article provides a new use of optical cavities in the field of levitodynamics [42], beyond passive cooling or mediating coupling between different particles [64,68]: a microcavity is a great mechanical quantum squeezer.

We acknowledge valuable discussions with Uroš Delić. This research was supported by the European Union's Horizon 2020 research and innovation programme under Grant Agreement No. [863132] (IQLev) and by the Q-Xtreme project of the European Research Council under the European Union's Horizon 2020 research and innovation program (Grant Agreement No. 951234).

[1] K. Kustura, C. C. Rusconi, and O. Romero-Isart, Quadratic quantum Hamiltonians: General canonical transformation to a normal form, Phys. Rev. A **99**, 022130 (2019).

[2] X. Han, D.-Y. Wang, C.-H. Bai, W.-X. Cui, S. Zhang, and H.-F. Wang, Mechanical squeezing beyond resolved sideband and weak-coupling limits with frequency modulation, Phys. Rev. A **100**, 033812 (2019).

[3] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).

[4] C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle, Rev. Mod. Phys. **52**, 341 (1980).

[5] A. Kronwald, F. Marquardt, and A. A. Clerk, Arbitrarily large steady-state bosonic squeezing via dissipation, Phys. Rev. A **88**, 063833 (2013).

[6] H. Tan, G. Li, and P. Meystre, Dissipation-driven two-mode mechanical squeezed states in optomechanical systems, Phys. Rev. A **87**, 033829 (2013).

[7] D. Kienzler, H.-Y. Lo, B. Keitch, L. de Clercq, F. Leupold, F. Lindenfelser, M. Marinelli, V. Negnevitsky, and J. P. Home, Quantum harmonic oscillator state synthesis by reservoir engineering, Science **347**, 53 (2015).

[8] E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, Quantum squeezing of motion in a mechanical resonator, Science **349**, 952 (2015).

[9] J.-M. Pirkkalainen, E. Damskägg, M. Brandt, F. Massel, and M. A. Sillanpää, Squeezing of Quantum Noise of Motion in a Micromechanical Resonator, Phys. Rev. Lett. **115**, 243601 (2015).

[10] F. Lecocq, J. B. Clark, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Quantum Nondemolition Measurement of a Nonclassical State of a Massive Object, Phys. Rev. X **5**, 041037 (2015).

[11] C. U. Lei, A. J. Weinstein, J. Suh, E. E. Wollman, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, Quantum Nondemolition Measurement of a Quantum Squeezed State Beyond the 3 dB Limit, Phys. Rev. Lett. **117**, 100801 (2016).

[12] G. Milburn and D. Walls, Production of squeezed states in a degenerate parametric amplifier, Opt. Commun. **39**, 401 (1981).

[13] D. Rugar and P. Grütter, Mechanical Parametric Amplification and Thermomechanical Noise Squeezing, Phys. Rev. Lett. **67**, 699 (1991).

[14] M. Blencowe and M. Wybourne, Quantum squeezing of mechanical motion for micron-sized cantilevers, Physica (Amsterdam) **280B**, 555 (2000).

[15] A. Mari and J. Eisert, Gently Modulating Optomechanical Systems, Phys. Rev. Lett. **103**, 213603 (2009).

[16] A. Nunnenkamp, K. Børkje, J. G. E. Harris, and S. M. Girvin, Cooling and squeezing via quadratic optomechanical coupling, Phys. Rev. A **82**, 021806(R) (2010).

[17] J.-Q. Liao and C. K. Law, Parametric generation of quadrature squeezing of mirrors in cavity optomechanics, Phys. Rev. A **83**, 033820 (2011).

[18] A. Vinante and P. Falferi, Feedback-Enhanced Parametric Squeezing of Mechanical Motion, Phys. Rev. Lett. **111**, 207203 (2013).

[19] A. Pontin, M. Bonaldi, A. Borrielli, F. S. Cataliotti, F. Marino, G. A. Prodi, E. Serra, and F. Marin, Squeezing a Thermal Mechanical Oscillator by Stabilized Parametric Effect on the Optical Spring, Phys. Rev. Lett. **112**, 023601 (2014).

[20] A. Pontin, M. Bonaldi, A. Borrielli, L. Marconi, F. Marino, G. Pandraud, G. A. Prodi, P. M. Sarro, E. Serra, and F. Marin, Dynamical Two-Mode Squeezing of Thermal Fluctuations in a Cavity Optomechanical System, Phys. Rev. Lett. **116**, 103601 (2016).

[21] A. Chowdhury, P. Vezio, M. Bonaldi, A. Borrielli, F. Marino, B. Morana, G. A. Prodi, P. M. Sarro, E. Serra, and F. Marin, Quantum Signature of a Squeezed Mechanical Oscillator, Phys. Rev. Lett. **124**, 023601 (2020).

[22] P. Vezio, A. Chowdhury, M. Bonaldi, A. Borrielli, F. Marino, B. Morana, G. A. Prodi, P. M. Sarro, E. Serra, and F. Marin, Quantum motion of a squeezed mechanical oscillator attained via an optomechanical experiment, Phys. Rev. A **102**, 053505 (2020).

[23] J. Janszky and Y. Yushin, Squeezing via frequency jump, Opt. Commun. **59**, 151 (1986).

[24] C. F. Lo, Squeezing by tuning the oscillator frequency, J. Phys. A **23**, 1155 (1990).

[25] J. Janszky and P. Adam, Strong squeezing by repeated frequency jumps, Phys. Rev. A **46**, 6091 (1992).

[26] M. Asjad, G. S. Agarwal, M. S. Kim, P. Tombesi, G. Di Giuseppe, and D. Vitali, Robust stationary mechanical squeezing in a kicked quadratic optomechanical system, Phys. Rev. A **89**, 023849 (2014).

[27] J. Alonso, F. M. Leupold, Z. U. Solèr, M. Fadel, M. Marinelli, B. C. Keitch, V. Negnevitsky, and J. P. Home, Generation of large coherent states by bang–bang control of a trapped-ion oscillator, Nat. Commun. **7**, 11243 (2016).

[28] M. Rashid, T. Tufarelli, J. Bateman, J. Vovrosh, D. Hempston, M. S. Kim, and H. Ulbricht, Experimental Realization of a Thermal Squeezed State of Levitated Optomechanics, Phys. Rev. Lett. **117**, 273601 (2016).

[29] A. A. Clerk, F. Marquardt, and K. Jacobs, Back-action evasion and squeezing of a mechanical resonator using a cavity detector, New J. Phys. **10**, 095010 (2008).

[30] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, Mechanical Squeezing via Parametric Amplification and Weak Measurement, Phys. Rev. Lett. **107**, 213603 (2011).

[31] C. Meng, G. A. Brawley, J. S. Bennett, M. R. Vanner, and W. P. Bowen, Mechanical Squeezing via Fast Continuous Measurement, Phys. Rev. Lett. **125**, 043604 (2020).

[32] R. Filip, P. Marek, and U. L. Andersen, Measurement-induced continuous-variable quantum interactions, Phys. Rev. A **71**, 042308 (2005).

[33] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, Č. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, Pulsed quantum optomechanics, Proc. Natl. Acad. Sci. U.S.A. **108**, 16182 (2011).

[34] A. Szorkovszky, G. A. Brawley, A. C. Doherty, and W. P. Bowen, Strong Thermomechanical Squeezing via Weak Measurement, Phys. Rev. Lett. **110**, 184301 (2013).

[35] M. R. Vanner, J. Hofer, G. D. Cole, and M. Aspelmeyer, Cooling-by-measurement and mechanical state tomography via pulsed optomechanics, Nat. Commun. **4**, 2295 (2013).

[36] G. Vasilakis, H. Shen, K. Jensen, M. Balabas, D. Salart, B. Chen, and E. S. Polzik, Generation of a squeezed state of an oscillator by stroboscopic back-action-evading measurement, Nat. Phys. **11**, 389 (2015).

[37] X.-Y. Lü, J.-Q. Liao, L. Tian, and F. Nori, Steady-state mechanical squeezing in an optomechanical system via Duffing nonlinearity, Phys. Rev. A **91**, 013834 (2015).

[38] K.-W. Xiao, N. Zhao, and Z.-q. Yin, Bistability and squeezing of the librational mode of an optically trapped nanoparticle, Phys. Rev. A **96**, 013837 (2017).

[39] K. Jähne, C. Genes, K. Hammerer, M. Wallquist, E. S. Polzik, and P. Zoller, Cavity-assisted squeezing of a mechanical oscillator, Phys. Rev. A **79**, 063819 (2009).

[40] G. S. Agarwal and S. Huang, Strong mechanical squeezing and its detection, Phys. Rev. A **93**, 043844 (2016).

[41] J. Millen, T. S. Monteiro, R. Pettit, and A. N. Vamivakas, Optomechanics with levitated particles, Rep. Prog. Phys. **83**, 026401 (2020).

[42] C. Gonzalez-Ballestero, M. Aspelmeyer, L. Novotny, R. Quidant, and O. Romero-Isart, Levitodynamics: Levitation and control of microscopic objects in vacuum, Science **374**, eabg3027 (2021).

[43] D. Windey, C. Gonzalez-Ballestero, P. Maurer, L. Novotny, O. Romero-Isart, and R. Reimann, Cavity-Based 3D Cooling of a Levitated Nanoparticle via Coherent Scattering, Phys. Rev. Lett. **122**, 123601 (2019).

[44] U. Delić, M. Reisenbauer, D. Grass, N. Kiesel, V. Vuletić, and M. Aspelmeyer, Cavity Cooling of a Levitated Nanosphere by Coherent Scattering, Phys. Rev. Lett. **122**, 123602 (2019).

[45] U. Delić, M. Reisenbauer, K. Dare, D. Grass, V. Vuletić, N. Kiesel, and M. Aspelmeyer, Cooling of a levitated nanoparticle to the motional quantum ground state, Science **367**, 892 (2020).

[46] A. de los Ríos Sommer, N. Meyer, and R. Quidant, Strong optomechanical coupling at room temperature by coherent scattering, Nat. Commun. **12**, 276 (2021).

[47] A. Ranfagni, P. Vezio, M. Calamai, A. Chowdhury, F. Marino, and F. Marin, Vectorial polaritons in the quantum motion of a levitated nanosphere, Nat. Phys. **17**, 1120 (2021).

[48] D. Hunger, T. Steinmetz, Y. Colombe, C. Deutsch, T. W. Hänsch, and J. Reichel, A fiber Fabry–Perot cavity with high finesse, New J. Phys. **12**, 065038 (2010).

[49] L. Magrini, R. A. Norte, R. Riedinger, I. Marinković, D. Grass, U. Delić, S. Gröblacher, S. Hong, and M. Aspelmeyer, Near-field coupling of a levitated nanoparticle to a photonic crystal cavity, Optica **5**, 1597 (2018).

[50] G. Wachter, S. Kuhn, S. Minniberger, C. Salter, P. Asenbaum, J. Millen, M. Schneider, J. Schalko, U. Schmid, A. Felgner, D. Hüser, M. Arndt, and M. Trupke, Silicon microcavity arrays with open access and a finesse of half a million, Light Sci. Appl. **8**, 37 (2019).

[51] J. Fait, S. Putz, G. Wachter, J. Schalko, U. Schmid, M. Arndt, and M. Trupke, High finesse microcavities in the optical telecom O-band, Appl. Phys. Lett. **119**, 221112 (2021).

[52] L. Magrini, P. Rosenzweig, C. Bach, A. Deutschmann-Olek, S. G. Hofer, S. Hong, N. Kiesel, A. Kugi, and M. Aspelmeyer, Real-time optimal quantum control of mechanical motion at room temperature, Nature (London) **595**, 373 (2021).

[53] F. Tebbenjohanns, M. L. Mattana, M. Rossi, M. Frimmer, and L. Novotny, Quantum control of a nanoparticle optically levitated in cryogenic free space, Nature (London) **595**, 378 (2021).

[54] O. Černotík and R. Filip, Strong mechanical squeezing for a levitated particle by coherent scattering, Phys. Rev. Research **2**, 013052 (2020).

[55] D. C. Moore and A. A. Geraci, Searching for new physics using optically levitated sensors, Quantum Sci. Technol. **6**, 014008 (2021).

[56] C. Genes, D. Vitali, P. Tombesi, S. Gigan, and M. Aspelmeyer, Ground-state cooling of a micromechanical oscillator: Comparing cold damping and cavity-assisted cooling schemes, Phys. Rev. A **77**, 033804 (2008).

[57] G. A. Peterson, S. Kotler, F. Lecocq, K. Cicak, X. Y. Jin, R. W. Simmonds, J. Aumentado, and J. D. Teufel, Ultrastrong Parametric Coupling between a Superconducting Cavity and a Mechanical Resonator, Phys. Rev. Lett. **123**, 247701 (2019).

[58] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.143601 for the canonical transformation between the physical modes and the normal modes, further details about the derivation of the asymptotic minimal variance Eq. (7), a discussion of squeezing in the presence of a standard mechanical dissipator describing weak coupling to a thermal bath, expressions for all coherent and dissipative rates in a coherent scattering setup, and a detailed analysis of the detection scheme.

[59] A. A. Anappara, S. De Liberato, A. Tredicucci, C. Ciuti, G. Biasiol, L. Sorba, and F. Beltram, Signatures of the ultrastrong light-matter coupling regime, Phys. Rev. B **79**, 201303(R) (2009).

[60] A. Bayer, M. Pozimski, S. Schambeck, D. Schuh, R. Huber, D. Bougeard, and C. Lange, Terahertz light–matter interaction beyond unity coupling strength, Nano Lett. **17**, 6340 (2017).

[61] A. Frisk Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori, Ultrastrong coupling between light and matter, Nat. Rev. Phys. **1**, 19 (2019).

[62] F. Fogliano, B. Besga, A. Reigue, P. Heringlake, L. Mercier de Lépinay, C. Vaneph, J. Reichel, B. Pigeau, and O. Arcizet, Mapping the Cavity Optomechanical Interaction

with Subwavelength-Sized Ultrasensitive Nanomechanical Force Sensors, Phys. Rev. X **11**, 021009 (2021).

[63] We remark that the conservative dynamics generated by Eq. (1) and discussed in Fig. 1 can be understood in terms of the approximated exponentiation discussed in [64].

[64] I. Brandão, D. Tandeitnik, and T. Guerreiro, Coherent scattering-mediated correlations between levitated nanospheres, Quantum Sci. Technol. **6**, 045013 (2021).

[65] C. Gonzalez-Ballestero, P. Maurer, D. Windey, L. Novotny, R. Reimann, and O. Romero-Isart, Theory for cavity cooling of levitated nanoparticles via coherent scattering: Master equation approach, Phys. Rev. A **100**, 013805 (2019).

[66] A. C. Doherty, A. Szorkovszky, G. I. Harris, and W. P. Bowen, The quantum trajectory approach to quantum feedback control of an oscillator revisited, Phil. Trans. R. Soc. A **370**, 5338 (2012).

[67] M. Rossi, D. Mason, J. Chen, and A. Schliesser, Observing and Verifying the Quantum Trajectory of a Mechanical Resonator, Phys. Rev. Lett. **123**, 163601 (2019).

[68] H. Rudolph, K. Hornberger, and B. A. Stickler, Entangling levitated nanoparticles by coherent scattering, Phys. Rev. A **101**, 011804(R) (2020).