Damping of Pseudo-Goldstone Fields

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Approximate symmetries abound in nature. If these symmetries are also spontaneously broken, the would-be Goldstone modes acquire a small mass, or inverse correlation length, and are referred to as pseudo-Goldstones. At nonzero temperature, the effects of dissipation can be captured by hydrodynamics at sufficiently long scales compared to the local equilibrium. Here, we show that, in the limit of weak explicit breaking, locality of hydrodynamics implies that the damping of pseudo-Goldstones is completely determined by their mass and diffusive transport coefficients. We present many applications: superfluids, QCD in the chiral limit, Wigner crystal and density wave phases in the presence of an external magnetic field or not, nematic phases, and (anti)ferromagnets. For electronic density wave phases, pseudo-Goldstone damping generates a contribution to the resistivity independent of the strength of disorder, which can have a linear temperature dependence provided the associated diffusivity saturates a bound. This is reminiscent of the phenomenology of strange metal high-T_c superconductors, where charge density waves are observed across the phase diagram.

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In the simplest setting, collective excitations associated with Goldstone modes for spontaneously broken symmetries disperse linearly $\omega(q) \simeq \pm c_s q$. At finite temperature, dissipation enters to leading order in the wave vector q as a diffusive broadening of this ballistic sound mode $\omega(q) \simeq \pm c_s q - iDq^2/2$. The softness of these modes as $q \to 0$ is protected by the symmetry and its spontaneous breaking. When the symmetry is only approximate, the collective excitation is sometimes referred to as a pseudo-Goldstone mode and acquires a finite correlation length $1/q_o$. At finite temperature, it can also have a nonzero relaxation rate, so that the pole is located schematically at $\lim_{q\to 0} \omega(q) \simeq \pm c_s q_o - i\Omega/2$. The main result of this Letter is to show that these transport coefficients are not all independent in the limit of weak symmetry breaking $q_o \to 0$. We derive the following relation for the pseudo-Goldstone damping rate:

$$\Omega = q_o^2 D + O(q_o^4). \tag{1}$$

Strictly speaking, there are typically several contributions to the attenuation D and to the damping Ω, which can be obtained using various Kubo formulas; we will clarify below which ones satisfy a relation of the form (1).

In the hydrodynamic regime of thermalizing systems and away from thermal phase transitions, there are no long-range excitations other than those accounted for by global symmetries. A basic property that follows is locality of constitutive relations, i.e., expansions of operators (typically currents) in terms of conserved densities or Goldstone fields, and sources. As we will show, when the thermal correlation length is large (which happens when an approximate symmetry is spontaneously broken), the condition that sources enter locally in constitutive relations is not automatically satisfied in the Kadanoff-Martin approach to computing hydrodynamic response functions [1] and must be imposed by hand, leading to Eq. (1). The breakdown of locality is also crisply encapsulated in the noncommutativity between the $q \to 0$ and $q_o \to 0$ limits in the Kubo formulas that define Ω and D. Restoring locality with Eq. (1) restores the commutativity of limits.

Given the prevalence of Goldstones for approximate symmetries in nature and in experiments, relations of the form (1) have many applications; we survey a variety of them: superfluids, QCD in the chiral limit, Wigner crystals including in the presence of an external magnetic field, nematic phases, and (anti)ferromagnets; in so doing, we extend our result to Goldstone excitations with more complicated dispersion relations. We comment on the implications of Eq. (1) for strange metallic transport in high-temperature superconductors (HTSC). In the main

text, we give a detailed presentation for the case of a global U(1) symmetry and only sketch other cases. We provide many technical details in Supplemental Material [2].

Damping of pseudo-Goldstones at finite temperature has been studied for some time [3,4,28–37], although the relation (1) was not recognized at the time. Instead, to the best of our knowledge, Eq. (1) was first demonstrated in a gauge-gravity duality model of translation symmetry breaking [38]; see also for subsequent studies [5–10,39–45]. Equation (1) was also noted in the hydrodynamic description of soft pions in Ref. [46]. Previous efforts toward proving Eq. (1) include Refs. [8,11,12], and we comment on them in Supplemental Material [2].

In the absence of explicit breaking, Goldstones can be damped by the proliferation of vortices (and, more generally, topological defects for the spontaneously broken symmetry) which relax the winding of the phase [47,48] and contribute to Ω [3,49]. Here, we will assume that vortices do not play any significant role and, in particular, that we are far away from any melting transition, so that Ω is perturbatively activated when the symmetry is weakly explicitly broken.

Locality of constitutive relations.—In thermalizing systems, the slow excitations which govern the late time dynamics are long-wavelength modulations of conserved densities, satisfying continuity relations of the form

$$\dot{n}_a(x,t) + \nabla \cdot j_a(x,t) = 0, \tag{2}$$

where $a = 1, 2, \ldots$ labels the various densities and a dot denotes a time derivative. The current densities j_a are not themselves slow operators—their time derivatives are not suppressed by gradients—but as any operator they can be effectively expanded locally in terms of the slow densities at late times. These expansions are referred to as constitutive relations; to linear order in the densities, they read schematically (we shall ignore nonlinear hydrodynamic fluctuations in this Letter)

$$j = \alpha_0 n + \alpha_1 \nabla n + \alpha_2 \nabla^2 n + \cdots, \tag{3}$$

where α_i are *a priori* unknown coefficients. This expression should be understood as an effective operator equation: Although it is not valid microscopically, correlation functions involving operators of either side of the equation match in the hydrodynamic regime. The conservation law (2) then produces an equation of motion for the densities

$$\dot{n}_a(q,t) + M_{ab}(q) n_b(q,t) = 0, \tag{4}$$

where we have defined $n(q,t) = \int d^d x e^{iqx} n(x,t)$ and summation over repeated indices is implied. As usual, external sources are introduced by deforming the Hamiltonian

$$H_0 \to H(t) = H_0 - \int d^d x \delta\mu^a(x,t) n_a(x,t) \tag{5}$$

and enter in the conservation equations as (see [2])

$$\dot{n}_a(q,t) + M_{ab}(q)[n_b(q,t) - \chi_{bc}(q)\delta\mu_c(q,t)] = 0. \tag{6}$$

$\chi(q)$ is the matrix of static susceptibilities,

$$\chi_{ab}(x - x') \equiv -\frac{\delta^2 W}{\delta\mu^a(x)\delta\mu^b(x')}, \tag{7}$$

where $W = -T \log Z = -T \log \mathrm{Tr} e^{-\beta H}$ is the equilibrium thermal free energy and H is given by Eq. (5) but with time-independent sources $\delta\mu(x,t) = \delta\mu(x)$. The matrices χ and M satisfy various positivity and symmetry conditions due to, e.g., positivity of dissipation or Onsager relations—these are well known, and we will not comment further upon them.

We are now ready to state our constraint: Eq. (6) must be a (sufficiently) local function of the densities and sources. By local, we mean that it must satisfy an expansion in momentum q, with higher powers of q suppressed by a scale which sets the cutoff of hydrodynamics. The matrix $M_{ab}(q)$ satisfies this condition, since it originated from the local constitutive relation (3). What about $M_{ab}(q)\chi_{bc}(q)$? Static susceptibilities are local up to the thermal correlation length ξ (sometimes called inverse thermal mass). This length scale is usually smaller than the length scales of interest in hydrodynamics, except in two situations: close to thermal phase transitions and in the presence of Goldstone or pseudo-Goldstone modes. (One more exotic situation where this may arise is in systems with approximate dipole conservation [50,51].) In these situations, locality of Eq. (6) in the hydrodynamic regime is not automatic and can lead to constraints on transport parameters. We derive such a constraint in the concrete example of a conserved U(1) in the following section.

Locality of constitutive relations reflects the central assumption of hydrodynamics in thermalizing systems, namely, that all long-lived and long-range excitations are accounted for by symmetries (as densities or Goldstone modes), and these are solely responsible for nonanalyticities in thermal response functions at small wave vector q and frequency ω. Since none of these modes have been integrated out when writing expressions such as Eq. (3) [or Eq. (6) with sources], these expressions must be local.

Implication for pseudo-Goldstone modes.—The simplest setting involving a pseudo-Goldstone field is a system with a single approximate continuous symmetry [U(1) or $\mathbb{R}$] which is spontaneously broken. This could describe, e.g., the ordered phase of the XY model in the presence of a small symmetry-breaking deformation. At finite temperature, the system will thermalize and hydrodynamics will emerge—the hydrodynamic modes are the charge density n

associated with the symmetry, its conjugate the Goldstone phase ϕ, and energy and momentum densities. Energy and momentum densities will not play an important role here, and we shall ignore them (or assume they decouple) for simplicity.

The hydrodynamics of this system fits in the general framework established in the previous section, with n and ϕ playing the role of densities. ($\nabla\phi$ is, in fact, the density of a higher-form symmetry associated with conservation of winding, with the Josephson relation corresponding to the constitutive relation for the higher-form current $\dot{\phi}$ [52,53].) Consider first the situation where the symmetry is exact, and the hydrodynamic theory is well known [13]. The matrix of susceptibilities (7) is obtained by coupling the theory to static sources $H_0 \to H$:

$$H = H_0 - \int d^d x \delta\mu(x) n(x,t) + \delta s_\phi(x)\phi(x,t). \quad (8)$$

Since the thermal correlation length diverges due to the presence of the Goldstone, it is convenient not to integrate out the Goldstone phase ϕ [54], so that instead of working with $W[\delta\mu, \delta s_\phi]$ we will consider the free energy F defined as

$$e^{-\beta W} = \int D\phi e^{-\beta F[\delta\mu, \delta s_\phi, \phi]}. \quad (9)$$

Only gradients of ϕ can appear in F, since ϕ shifts under the symmetry. Let us now assume the symmetry is weakly broken. This will introduce a new length scale $1/q_o$ in the system, the thermal correlation length of ϕ, which is parametrically larger than the cutoff length of hydrodynamics—a hydrodynamic description of the system should exist that is valid across this new scale. A lower-gradient, symmetry-breaking term is allowed in $F = \int d^d x f$:

$$f = \frac{f_s}{2}[(\nabla\phi)^2 + q_o^2\phi^2] - \delta s_\phi \phi - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots, \quad (10)$$

with χ_{nn} the charge susceptibility and f_s the superfluid stiffness. By integrating out ϕ and using Eq. (9), we obtain the susceptibility matrix

$$\chi(q) \simeq \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{f_s(q^2+q_o^2)} \end{pmatrix}. \quad (11)$$

Let us also review how the constitutive relations and conservation laws change [3,49]; see also [45]. To leading order in gradients, the most general way the conservation law can be weakly broken is

$$\dot{n} + \nabla\cdot j = -\Gamma n + f_s q_o^2 \phi + \cdots, \quad (12)$$

where we introduced a charge relaxation rate Γ. The last term is fixed by the symmetry-breaking term in the free energy (10). (We fix the sign of the Goldstone field with the convention $[\int d^d x n(x), \phi(x')] = i$.) In the absence of sources, the constitutive relation for the current and the Josephson relation are (a different choice of hydrodynamic frame would lead to analogous constraints; see [2] for further details)

$$j \simeq f_s \nabla\phi - D_n \nabla n, \qquad \dot{\phi} \simeq -\Omega\phi - \frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \phi. \quad (13)$$

In the absence of explicit symmetry breaking, the coefficients of the leading terms are fixed in terms of the coefficients appearing in the free energy (10) with $q_o = 0$, while two transport coefficients D_n and D_ϕ appear at subleading order in gradients. The M matrix defined by Eq. (6) is given by

$$M(q) \simeq \begin{pmatrix} \Gamma + D_n q^2 & -f_s(q_o^2 + q^2) \\ \frac{1}{\chi_{nn}} & \Omega + D_\phi q^2 \end{pmatrix} \quad (14)$$

and is local. However, the other matrix appearing in the hydrodynamic equation of motion in the presence of sources (6),

$$M(q)\cdot\chi(q) \simeq \begin{pmatrix} \chi_{nn}(\Gamma + D_n q^2) & -1 \\ 1 & \frac{\Omega + D_\phi q^2}{f_s(q_o^2+q^2)} \end{pmatrix}, \quad (15)$$

is generically not—the last term has an expansion as $q \to 0$ with higher powers of q suppressed by q_o instead of the hydrodynamic cutoff, which is parametrically larger. Locality is restored only if the transport parameters satisfy the relation

$$\Omega \simeq q_o^2 D_\phi, \quad (16)$$

to leading order in q_o. This relation was recently checked in two holographic models [10,45].

Restoring locality through Eq. (16) implies that the order of limits $q \to 0$ and $q_o \to 0$ commutes in the Kubo formulas defining D_ϕ and Ω:

$$D_\phi = f_s \lim_{\omega\to 0}\lim_{q\to 0}\lim_{\Gamma\to 0}\lim_{q_o\to 0}\frac{1}{\omega}\mathrm{Im}G^R_{\partial_t\phi\partial_t\phi}(\omega, q), \quad (17)$$

$$\frac{\Omega}{q_o^2} = f_s \lim_{\omega\to 0}\lim_{\Gamma\to 0}\lim_{q_o\to 0}\lim_{q\to 0}\frac{1}{\omega}\mathrm{Im}G^R_{\partial_t\phi\partial_t\phi}(\omega, q). \quad (18)$$

The limits $\Gamma \to 0$ and $q_o \to 0$ can be taken in any order.

In Supplemental Material [2], we rederive Eq. (16) using the Schwinger-Keldysh formalism for effective theories of hydrodynamics [14–17]. This is advantageous as locality is built in from the start when constructing the effective action. There are only two independent symmetry-breaking terms in the action, with coefficients ω_o and Γ, and Eq. (16) follows automatically.

With Eq. (16) in hand and turning external sources $\delta\mu$ and s_ϕ back on, we observe that the Josephson relation can be rewritten as

$$\dot{\phi} \simeq \delta\mu - \frac{n}{\chi_{nn}} + \frac{D_\phi}{f_s}(s_\phi - h_\phi), \tag{19}$$

where $h_\phi \equiv \delta f/\delta\phi = f_s q_o^2 \phi - f_s \nabla^2 \phi$ is the field conjugate to ϕ. In static equilibrium, $\langle h_\phi \rangle = s_\phi$. In practice, writing out the dissipative terms in constitutive relations in terms of the conjugate fields directly leads to local equations of motion. In Refs. [54,55], it was emphasized how symmetry and consistency with static equilibrium with external sources constrains constitutive relations.

QCD.—At temperatures below the chiral phase transition, the hydrodynamics of QCD includes pions as long-lived degrees of freedom; see, e.g., [32,33,46,56]. The spontaneously broken SU(2) symmetry is only approximate, due to the quark masses; this case, thus, falls into the class of systems considered in this Letter. At linear order in fields, the Josephson relation for pions is identical to the Abelian one studied above. Imposing that the hydrodynamic equations of motion be local in the presence of sources, one thus finds the relation (16) between the pion thermal mass, diffusivity, and relaxation rate.

This relation was, in fact, noticed recently in the context of QCD in Ref. [46], where it was shown to follow from positivity of entropy production. This argument, however, does not straightforwardly apply to Goldstones for Abelian symmetries, because additional terms can be added to the entropy current to guarantee positivity of entropy production without imposing the relation (16). (However, as shown in Ref. [18], coupling the fields to external sources fixes this ambiguity, and the entropy production argument applies.) Instead, the locality argument presented here applies to all of these situations.

Goldstones for translation.—The free energy density for an isotropic Wigner crystal in mechanical equilibrium in two spatial dimensions is [13]

$$f = \frac{B-G}{2}(\nabla^l u_l)^2 + G(\nabla_{(i} u_{j)})^2 + \frac{1}{2} G q_o^2 u^2, \tag{20}$$

where the u_i are the displacements, B and G are the bulk and shear elastic moduli, respectively, and indices run over the spatial dimensions. The term proportional to q_o^2 is assumed to be small and explicitly breaks translation symmetry.

Momentum is relaxed:

$$\dot{\pi}^i + \nabla_j \tau^{ji} = -\Gamma \pi^i - G q_o^2 u^i, \tag{21}$$

while the Josephson equation is

$$\dot{u}_i = \frac{\pi_i}{\chi_{\pi\pi}} - \Omega_{ij} u^j + D_\parallel \nabla_i \nabla^j u_j + D_\perp \epsilon_{ij} \nabla^j \nabla \times u. \tag{22}$$

(We neglect charge and heat fluctuations. We give a more complete analysis in Supplemental Material [2]. Both the electric and heat currents receive new dissipative terms when $q_o \neq 0$.) Locality of the $M \cdot \chi$ matrix following from Eqs. (20)–(22) constrains both the damping and the diffusivities (see [2])

$$\frac{D_\parallel}{B+G} = \frac{D_\perp}{G} \equiv \frac{D}{G}, \qquad \Omega_{ij} = q_o^2 D \delta_{ij}, \tag{23}$$

as reported in holographic models of broken translations [5,38,41].

Strange metallic transport.—There is mounting experimental evidence that dynamical charge fluctuations play an important role across the phase diagram of cuprate HTSC [57], which could play an important role in strange metallic transport [34,38,58].

Restoring charge and heat fluctuations and assuming approximate invariance under Galilean boosts, the resistivity is (see [2])

$$\rho_{\mathrm{dc}} = \frac{m^\star}{ne^2}\left(\Gamma + \frac{\omega_o^2}{\Omega}\right) = \frac{m^\star}{ne^2}\left(\Gamma + \frac{c_s^2}{D}\right), \tag{24}$$

with n the density, e the unit charge, $m^\star$ the effective mass, $c_s = \sqrt{G/(m^\star n)}$ the Goldstone sound velocity, and D defined as in Eq. (23). Equation (23) produces a finite resistivity, which is rooted in the relaxation of the pseudo-Goldstone in the bath of thermal excitations. We expect this result to extend to weakly pinned electronic charge density wave systems where the Fermi surface is fully gapped and some electrons remain uncondensed, providing a bath of gapless excitations and giving rise to Eq. (23). By contrast, if the Fermi surface is fully gapped, the damping (23) will be exponentially suppressed, recovering earlier hydrodynamic descriptions of pinned charge density waves [29].

This result straightforwardly extends to a unidirectional charge density wave. Since Γ is typically $O(g^2)$ in the strength $g \ll 1$ of explicit breaking, while c_s^2/D is $O(g^0)$, the resistivity is large in the "hard" direction of the spontaneous modulation and small in the transverse "easy" direction, in general agreement with transport experiments in 2DES [59] and in HTSC [60].

Combining Eq. (24) with arguments [61] that diffusivities are bounded from below by the Planckian timescale [62] in strongly correlated materials

$$D \simeq \frac{\hbar}{k_B T} v^2 \simeq \frac{\hbar}{k_B T} c_s^2 \tag{25}$$

(where, for simplicity, we took the characteristic velocity $v = c_s$) leads to

$$\rho_{\rm dc} \simeq \frac{m^\star}{ne^2}\left(\Gamma + \frac{k_B T}{\hbar}\right). \tag{26}$$

This gives a natural mechanism for the ubiquitous T-linear resistivity in these materials. Moreover, the slope of the linear term is independent from the strength of explicit breaking, as observed in experiments where samples are disordered by ion irradiation [63]. In contrast, Γ originates from more conventional scattering mechanisms (umklapp and disorder), which allows one to account for experimental reports of competing scattering mechanisms with distinct temperature dependencies [64,65].

Wigner crystals in a magnetic field.—When placed in a magnetic field, the longitudinal and transverse hydrodynamic modes of a 2D Wigner crystal couple. The magneto-phonon sector is described by the effective Lagrangian [4,66]

$$\mathcal{L} \simeq \frac{1}{2}\omega_c \epsilon^{ij}\varphi_i \dot{\varphi}_j - \frac{1}{2}\varphi_i[\omega_o^2 \delta^{ij} - \mu^{ijab}\nabla_a \nabla_b]\varphi_j. \tag{27}$$

The presence of a term with a single time derivative reflects the breaking of time reversal due to a background magnetic field and leads to the canonical commutation relation between the Goldstones

$$[\varphi_i(x), \varphi_j(y)] = -\frac{i\epsilon_{ij}}{\omega_c}\delta(x-y), \tag{28}$$

which are no longer independent degrees of freedom.

In the absence of pinning ($\omega_o = 0$), φ has to appear with a gradient in the potential because of invariance under translations $\varphi^i \to \varphi^i + c^i$. The most general stiffness tensor consistent with isotropy and PT symmetry is $\mu_{ijab}q^a q^b = Bq_i q_j + Gq^2\delta_{ij}$, where the stiffnesses must satisfy $B, G > 0$ for the potential to be positive definite. The φ_i static susceptibility matrix takes the same form as in the case without a magnetic field.

Locality enforces the following dissipative Josephson relation:

$$\dot{\varphi}_i \simeq -\left(\frac{\epsilon_{ij}}{\omega_c} + D_\varphi \delta_{ij}\right)(\delta_{jk}\omega_o^2 - \mu^{jkab}\nabla_a \nabla_b)\varphi_k. \tag{29}$$

Both the longitudinal $\Omega = D_\varphi \omega_o^2$ and Hall $\Omega^H = \omega_o^2/\omega_c$ relaxation rates obey a relation analogous to Eq. (1) and are completely determined by ω_o and parameters of the unpinned theory. Where applicable, our results are consistent with Refs. [7–9]. The type-II nature of the Goldstones is manifest in their dispersion relation ($q_y = 0$) without pinning:

$$\omega \simeq \pm q_x^2\sqrt{\frac{G(B+G)}{\omega_c^2} - \frac{B^2 D_\varphi^2}{4}} - \frac{i}{2}(B+2G)D_\varphi q_x^2, \tag{30}$$

or with pinning:

$$\omega \simeq \pm\frac{\omega_o^2}{\omega_c} - iD_\varphi \omega_o^2 \pm \frac{B+2G}{2\omega_c}q_x^2 - \frac{i}{2}(B+2G)D_\varphi q_x^2. \tag{31}$$

Nematic phases.—Consider a phase in a translation invariant isotropic system where rotations are spontaneously broken. Examples include nematic or hexatic liquid crystals [67,68] and quantum Hall systems [69] (spontaneous breaking of a discrete rotation symmetry also arises in the cuprates [70]). Spontaneous breaking of isotropy produces a Goldstone field that shifts under rotations $\theta \to \theta + c$. Since the generator of rotations is given by $J = \int d^2x \epsilon_{ij} x^i \pi^j$, where $T_{0i} = \pi_i$ is the momentum density, this realization of the symmetry is implemented by the commutation relation

$$[\pi_i(x), \theta(y)] = \frac{1}{2} i\epsilon_{ij}\partial_j \delta^2(x-y) + \cdots, \tag{32}$$

where $\cdots$ denote terms that are linear in θ. The hydrodynamic equations are well known [13,68]. When rotation symmetry is only approximate, θ acquires a small gap q_o^2 as in the examples in previous sections, which can be measured in the susceptibility

$$\lim_{\omega\to 0} G^R_{\theta\theta}(\omega, q) = \chi_{\theta\theta}(q) \simeq \frac{1}{f_\theta}\frac{1}{q^2 + q_o^2}, \tag{33}$$

where f_θ is the stiffness which appear in the free energy as in Eq. (10). The Josephson relation and stress tensor constitutive relation are

$$\dot{\theta} \simeq -q_o^2 D_\theta \theta + \frac{1}{2}\frac{1}{\chi_{\pi\pi}}\nabla \times \pi + D_\theta \nabla^2 \theta, \tag{34a}$$

$$\tau_{ij} \simeq P\delta_{ij} + \frac{f_\theta}{2}\epsilon^{ij}(\nabla^2 - q_o^2)\theta + \pi_{ij} \tag{34b}$$

with $\pi_{ij} = -D_\eta(\partial_i \pi_j + \partial_j \pi_i - \delta_{ij}\nabla\cdot\pi) - D_\zeta \nabla\cdot\pi\delta_{ij}$. The damping term $\Omega = q_o^2 D_\theta$ in Eq. (34a) follows from locality and obeys a relation analogous to Eq. (1). The q_o^2 term in Eq. (34b) follows from consistency with Eq. (32). Differently from previous examples, there is no analog of a relaxation rate Γ for the angular momentum density: Assuming that only rotation symmetry is broken but translation symmetry is preserved, the stress tensor is still conserved. One could, of course, lift this assumption and also break translations. Since $\Omega = q_o^2 D_\theta$, there is only one parameter introduced by the weak breaking of isotropy: q_o.

At the linear level, the Goldstone mixes with only the transverse momentum density $\nabla \times \pi$ and the longitudinal sector is unchanged from regular hydrodynamics; we will, therefore, focus on the transverse sector $\{\pi_\perp, \theta\}$. Solving the continuity relations, one finds when $q_o = 0$ that this pair of modes disperses as

$$\omega_{\pm} \simeq -\frac{i}{2}q^2\left(D_\eta + D_\theta \pm \sqrt{(D_\eta - D_\theta)^2 - \frac{f_\theta}{\chi_{\pi\pi}}}\right). \quad (35)$$

When $(D_\eta - D_\theta)^2 \geq f_\theta/\chi_{\pi\pi}$, these form two diffusive modes $\omega \sim -iq^2$, while in the opposite case $\omega_{\pm} \sim \pm q^2 - iq^2$.

When $q_o \neq 0$, transverse momentum diffuses:

$$\omega_- \simeq -iq^2\left(D_\eta + \frac{f_\theta}{4\chi_{\pi\pi}D_\theta}\right), \quad (36)$$

while the Goldstone relaxes:

$$\omega_+ \simeq -iD_\theta q_o^2 - iq^2\left(D_\theta - \frac{f_\theta}{4\chi_{\pi\pi}D_\theta}\right). \quad (37)$$

(Anti)ferromagnets.—Ferromagnets spontaneously break SU(2) spin symmetry—which can be viewed as an internal symmetry in the nonrelativistic limit (see [71] for a recent formulation of relativistic spin hydrodynamics)—down to U(1), with a finite magnetization $\langle n_a \rangle = M_0\delta_a^3$. Antiferromagnets have the same symmetry-breaking pattern, but $M_0 = 0$; their hydrodynamics is structurally similar to that of pions discussed above, and the results there apply with minor modifications. In this section, we therefore focus on ferromagnets and briefly mention the constraints obtained from locality, leaving a more detailed study for future work.

In the absence of explicit breaking of SU(2) symmetry, magnons disperse as [72]

$$\omega = \pm\frac{f_s}{M_0}q^2 - i\gamma q^4 + \cdots. \quad (38)$$

In practice, the SU(2) symmetry is always approximate and broken by spin-orbit effects. This leads to several possible explicit symmetry-breaking scenarios: For example, there may or may not be an unbroken U(1), which may or may not be aligned with the magnetization. For the purposes of illustration, we focus on the situation where the entire SU(2) is weakly explicitly broken. This will generate a finite magnon correlation length $1/q_o$, allow for a relaxation term in the continuity relation as in Eq. (12), and allow for new terms in constitutive relations (see [2] for details), leading to a dispersion relation of the form

$$\omega = \pm\frac{f_s}{M_0}(q_o^2 + q^2) - i(\Gamma + D_o q^2 + \gamma q^4) + \cdots. \quad (39)$$

Explicit breaking of the symmetry introduced three new parameters: Γ, D_o, and q_o. However, imposing that the hydrodynamic equations of motion be local shows that only two are independent:

$$\Gamma = q_o^2(D_o - \gamma q_o^2), \quad (40)$$

which is the analog to Eq. (1) in this case.

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Note added.—Recently, Ref. [18] appeared with a derivation of Eq. (1) using positivity of entropy production. They also report that extra coefficients arise when coupling to external sources. These do not affect our main result (1). We give more details in Supplemental Material [2].

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