High Quality QCD Axion at Gravitational Wave Observatories

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The axion solution to the strong *CP* problem is delicately sensitive to Peccei-Quinn breaking contributions that are misaligned with respect to QCD instantons. Heavy QCD axion models are appealing because they avoid this so-called quality problem. We show that generic realizations of this framework can be probed by the LIGO-Virgo-KAGRA interferometers, through the stochastic gravitational wave (GW) signal sourced by the long-lived axionic string-domain wall network and by upcoming measurements of the neutron and proton electric dipole moments. Additionally, we provide predictions for searches at future GW observatories, which will further explore the parameter space of heavy QCD axion models.

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Introduction.—A great amount of experimental effort has been aimed at discovering the QCD axion [1,2], the pseudo-Goldstone boson of a spontaneously broken axial U(1) Peccei-Quinn (PQ) symmetry [3,4] that explains the smallness of *CP* violation in strong interactions.

While attractive, the PQ mechanism is vulnerable to possible additional sources of symmetry breaking, generically misaligned with respect to the axion potential from QCD instantons. This "quality problem" (originally formulated with various perspectives in [5–11]) is alleviated in "heavy axion" models (see [6,12–14] for earlier related work), where a "heavy QCD" sector provides a larger contribution to the axion potential, aligned with that from QCD instantons.

Existing realizations of this idea rely on the QCD coupling becoming strong at high energies [6,15–19] (see also [20] for a 5D model), a separate confining gauge group, whose alignment is ensured by unification at high scales [14,21–23], or by a softly broken $\mathbb{Z}_2$ symmetry [24–26]. When the strong coupling scale Λ_H of the heavy sector is above the QCD scale $\Lambda_{\rm QCD}$, the axion mass is larger than in the standard window, and the cosmological evolution of the axion field in the early Universe is shifted to higher energy scales. Despite its appeal, it is not immediately clear what the signatures of such a scenario are since, generically, the axion can be very heavy, e.g., above the electroweak scale, while its interactions remain very weak. (Axion masses and decay constants around or below the TeV scale can be probed at colliders, see, e.g., [26–28].) Furthermore, in contrast to the standard case, a heavy QCD axion can easily decay in the early Universe and thus leaves no detectable relic dark matter today.

Nonetheless, in this Letter we show that heavy QCD axion models can be observationally probed at gravitational wave (GW) observatories (already at the currently operating LIGO-Virgo-KAGRA (LVK) [29–31] interferometers), with the exciting possibility of a correlated signature in upcoming neutron and proton electric dipole moment (nEDM, pEDM) measurements [32,33].

GWs are indeed radiated [34] by the network of axionic topological defects (domain walls, DWs, attached to strings) [35] (see also [36]), which are abundant in the early Universe if the PQ symmetry is broken after inflation. In standard QCD axion models, the network necessarily annihilates while making up only a very tiny fraction of the energy density of the Universe, and therefore, the GW signal is too weak to be detectable [37]. In contrast, the heavy QCD axion network can carry much more energy because of its larger domain wall tension. Furthermore, in generic realizations (e.g., Dine-Fischler-Srednicki-Zhitnitsky [38,39] and simple generalizations of Kim–Shifman–Vainshtein–Zakharov models [40,41]), the network can be long-lived while still avoiding the overproduction of relics, since radiated axion quanta are unstable. Annihilation of the network can be triggered by the misaligned PQ breaking effects that motivate the scenario in the first place (see [42]). These also induce a small but potentially observable shift of the QCD vacuum angle.

Our Letter points out a new source of observable gravitational waves from the dynamics of the QCD axion (see, e.g., [43–46] for previous work, unrelated to the axion quality problem and [47–50] for related scenarios with axion-like-particles).

The heavy QCD axion.—Heavy QCD axion models are characterized by an extra contribution to the axion potential that is larger than and aligned with the contribution from QCD. The zero temperature potential is

$$V_a = (\kappa_{\rm QCD}^2\Lambda_{\rm QCD}^4 + \kappa_{\rm H}^2\Lambda_{\rm H}^4)\left(1-\cos\frac{a}{f}\right), \qquad (1)$$

where f is the axion decay constant, $\Lambda_{\rm QCD,H}$ denote the strong coupling scale of QCD and the heavy sector, respectively, and $\kappa_{\rm QCD,H} \leq 1$ are prefactors that depend on details such as the fermionic spectrum. For instance, QCD gives $\kappa_{\rm QCD} \simeq (m_u/\Lambda_{\rm QCD})^{1/2}$ with m_u the up quark mass. In explicit realizations of such scenarios [6,14–18,20–26], $\kappa_H \ll 1$ can similarly arise by the presence of a light quark in the heavy sector. Having QCD subdominant, the axion mass is dictated by the heavy sector as

$$m_a \simeq 10^8\ {\rm GeV}\left(\frac{10^{12}\ {\rm GeV}}{f}\right)\left(\frac{\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)^2\kappa_{\rm H}. \qquad (2)$$

For our discussion, it is important to recall that gauge instantons generically break the original U(1) PQ symmetry to a discrete $\mathbb{Z}_{N_{\rm DW}}$ subgroup, where $N_{\rm DW}$ is a model-dependent integer number related to the axion coupling to gluons. Therefore, the periodicity $2\pi f$ induced by the potential (1) can be smaller than the fundamental axion field range $2\pi f N_{\rm DW}$ and the potential V_a can feature $N_{\rm DW}$ degenerate minima. In writing (1), we assumed that the periodicity induced by the heavy sector coincides with that of QCD instantons. This appears to be linked to the requirement of alignment between the two sectors, as is evidently the case in constructions with a $\mathbb{Z}_2$ symmetry [26] and in simple unification frameworks where standard model (SM) and heavy sector fermions descend from the same fundamental representation of a higher-rank gauge group [18,22]. This feature implies that the low-energy QCD-induced potential does not lift the degeneracy of the $N_{\rm DW}$ minima.

Generically, however, we may expect further contributions to the axion potential, misaligned with V_a. Independent of its specific origin, such a contribution can be written as

$$V_b \simeq -\mu_b^4\cos\left(\frac{N_b}{N_{\rm DW}}\frac{a}{f}-\delta\right), \qquad (3)$$

where N_b defines the subgroup $\mathbb{Z}_{N_b}$ of the PQ symmetry which is preserved by (3) and δ is a CP violating phase. In the absence of tuning, this offset is naturally $O(1)$ and $\mu_b \ll \Lambda_{\rm H}$ is required to solve the strong CP problem. The low temperature potential is $V = V_a + V_b$ and when $N_b = 1$ or is co-prime with $N_{\rm DW}$, the degeneracy of the $N_{\rm DW}$ minima is lifted. In particular, the vacuum energy difference between the global CP preserving minimum and its nearest neighbor is of the order $\Delta V \simeq \mu_b^4[1-\cos(2\pi N_b/N_{\rm DW})]$ (provided that δ is not too close to $\pi/N_{\rm DW}$). Broadly speaking, (3) can originate at a scale Λ_b, such that $\mu_b = \kappa_b^{1/2}\Lambda_b$. $\Lambda_b \gg f \gg \Lambda_{\rm H}$ can arise from uv physics via: nonperturbative effects, $\kappa_b \sim e^{-S/2}$ (see e.g., [51–53]); higher-dimensional operators when the axion is the phase of a complex scalar field (see e.g., [9]); another gauge sector with confinement scale Λ_b and a light fermion of mass m_q, $\kappa_b \sim (m_q/\Lambda_b)^{1/2}$. $\Lambda_b \ll \Lambda_{\rm H}$ can also arise from a confining gauge sector. Further details are provided in the Supplemental Material [54].

Despite its smallness, a contribution from (3) can lead to potentially observable CP violation. In particular, at low temperatures one finds

$$\Delta\theta \equiv \theta - \theta_{\rm QCD} \simeq r^4\left(\frac{N_b}{N_{\rm DW}}\right)\left(\frac{\sin\delta}{\kappa_{\rm H}^2}\right), \qquad (4)$$

where $r \equiv \mu_b/\Lambda_{\rm H}$. Current bounds from nEDM measurements [61] require $\Delta\theta \lesssim 10^{-10}$. Clearly, (4) shows that $\Lambda_{\rm H} \gg \Lambda_{\rm QCD}$ makes the PQ mechanism more robust against misaligned contributions.

In the early Universe, the mass m_a and the scale μ_b are generally temperature dependent, for instance, in the standard QCD axion case $m_a(T) \simeq m_a(T_0/T)^4$ for $T \geq T_0 \simeq 134$ MeV and $m_a(T) = m_a$ otherwise [62]. Nonetheless, our results are mostly independent of the detailed temperature dependence.

Axionic defects.—Let us now move to the cosmological evolution of topological defects, whose history begins at the PQ symmetry breaking scale $\sim N_{\rm DW} f$. Our investigation concerns scenarios where this occurs during radiation domination after inflation, which will be generic for the values of f considered in this Letter.

Axionic strings form at $T \lesssim N_{\rm DW} f$ and continuously radiate axion quanta and gravitational waves. In the absence of significant friction due to the plasma, they quickly achieve a scaling regime [63,64] (see also [65–69] for recent updates), with energy density scaling as $\rho_s = \lambda\mu H^2$, with λ a $O(1)$ parameter and $\mu \sim N_{\rm DW}^2 f^2$ the string tension.

This behavior is altered once $3H \simeq m_a(T)$. This occurs at a temperature $T_{\rm osc} \gtrsim \Lambda_{\rm H}$ (see the Supplemental Material [54]) when the axion field, with average initial value $a_i/(N_{\rm DW} f) \sim O(1)$, starts oscillating in its potential V_a and domain walls form, attached to the strings, with a tension $\sigma \simeq 8 m_a(T) f^2$. At this epoch, two possibilities arise: (i) when $N_{\rm DW} = 1$, the network of topological defects is rapidly annihilated by string-wall interactions (see, e.g., [65]); (ii) when $N_{\rm DW} > 1$, the network persists because multiple domain walls pull each string in different directions.

In both cases, the tension of the walls is larger than in the standard QCD axion case by a factor $\Lambda_{\rm H}/\Lambda_{\rm QCD} \gg 1$. For $N_{\rm DW} > 1$, in the absence of significant friction from the plasma (we show in [54] that this has a minor impact on our conclusions), the network rapidly achieves a scaling regime, with its energy density dominated by domain walls, $\rho_{\rm DW} \simeq c\sigma H$, where c is a $O(1)$ numerical prefactor (in this regime, V_a is normally already temperature independent). This scales slower than matter and radiation and thus the network is potentially dangerous for cosmology [70]. However, domain wall domination can be generically avoided in the heavy axion scenario, thanks to the misaligned potential contribution V_b. The resulting vacuum pressure causes the contraction of the false vacuum regions and the collapse of the network [36] at a temperature $T_{\rm ann}$, which can be estimated by imposing $\rho_{\rm DW} \simeq \Delta V$ and more precisely determined via numerical simulations [71]. Here we focus on the case where V_b is temperature independent below $\Lambda_{\rm H}$, as occurs generically when PQ breaking is due to physics above Λ_H; see the Supplemental Material [54] for the temperature-dependent case. To set ideas and simplify expressions, in the following we set $N_b = 1$ and $N_{\rm DW} = 6$ as example values, fix numerical prefactors according to the simulations of [71], and also fix the number of (entropy) relativistic degrees of freedom at $T_{\rm ann}$ to the SM value at high temperatures $(g_{*s,\rm ann})g_{*,\rm ann} = 106.75$ (see also [54]), although our results are only mildly affected by these precise choices. We then find

$$T_{\rm ann} \simeq \frac{10^7\ {\rm GeV}}{\sqrt{\kappa_{\rm H}}}\sqrt{\frac{10^{12}\ {\rm GeV}}{f}}\left(\frac{\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)\left(\frac{r}{0.005}\right)^2, \qquad (5)$$

showing that for $r \ll 1$ network annihilation is significant delayed.

At (5) the network collapses and its energy density is transferred mostly to mildly relativistic axion quanta (see, e.g., [65,71]). In contrast to the standard QCD axion case, in the heavy axion scenario these relics can efficiently decay to SM gluons, above the QCD phase transition (PT) and to photons and/or fermions, above and below the QCD PT, depending on the specific axion model. Focusing on the decay to gluons, since $m_a \gg$ GeV in most of the parameter space of interest [72], we find that decay is efficient below the temperature

$$T_{a\to gg} \simeq 10^7\ {\rm GeV}\alpha_s\left(\frac{\sqrt{\kappa_{\rm H}}\Lambda_{\rm H}}{10^{10}\ {\rm GeV}}\right)^3\left(\frac{10^{12}\ {\rm GeV}}{f}\right)^{\frac{5}{2}} \qquad (6)$$

obtained by setting $\Gamma_{a\to gg} \simeq H$ (see [54]). This temperature can be larger than $T_{\rm ann}$ for $r \lesssim 0.001$ and/or $f \lesssim 10^{12}$ GeV. Therefore, axion relics from the network will, in general, decay immediately.

Crucially, however, the string-wall network can source a significant relic abundance of gravitational waves [73–79]. The simple quadrupole estimate for their energy density $\rho_{\rm GW}(T_{\rm ann}) \sim c^2\sigma^2/(32\pi M_p^2)$ has been confirmed by numerical simulations [79] (see also [37]). Assuming a standard radiation-dominated cosmological history after domain wall annihilation, one finds that the relic abundance of gravitational waves today is

$$\Omega_{\rm GW}h^2 \simeq 0.01(\Omega^0_{\rm rad}h^2)\tilde{\epsilon}\left(\frac{\rho_{\rm DW}}{\rho_{\rm rad}}\right)^2_{T=T_{\rm ann}}, \qquad (7)$$

where $\tilde{\epsilon} \simeq 0.1$–$1$ is a numerical efficiency factor [79] and $\rho_{\rm rad}$ and $\Omega_{\rm rad}h^2 \simeq 4\times 10^{-5}$ are the energy density and relic abundance of radiation today, respectively. The formula above shows that when the network makes up $\gtrsim O(5\%)$ fraction of the energy density of the Universe at annihilation, its gravitational wave signal is detectable by present interferometers, i.e., $\Omega_{\rm GW}h^2 \sim 10^{-9}$. This fraction at the annihilation temperature reads

$$\left.\frac{\rho_{\rm DW}}{\rho_{\rm rad}}\right|_{T=T_{\rm ann}} \simeq 0.1\kappa_H^2\left(\frac{f}{10^{11}\ {\rm GeV}}\right)^2\left(\frac{0.003}{r}\right)^4, \qquad (8)$$

for our example choice $N_b = 1$, $N_{\rm DW} = 6$.

The GW signal is peaked at a frequency corresponding to H at annihilation (see, e.g., [37]). Redshifted to today,

$$\omega_{\rm peak} \simeq \frac{5\ {\rm Hz}}{\sqrt{\kappa_{\rm H}}}\left(\frac{r}{0.005}\right)^2\left(\frac{\Lambda_H}{10^{10}\ {\rm GeV}}\right)\sqrt{\frac{10^{11}\ {\rm GeV}}{f}}. \qquad (9)$$

According to (7)–(9), the signal from a heavy axion with $f \lesssim 10^{11}$ GeV, $\Lambda_H \gtrsim 10^{10}$ GeV, and $r \gtrsim 10^{-3}$ sits right in the reach of the LVK interferometers [80,81].

The GW spectrum away from the peak frequency [37] decreases as ω^3 for $\omega < \omega_{\rm peak}$, whereas for $\omega > \omega_{\rm peak}$ it behaves as $\sim\omega^{-1}$, until a cutoff frequency corresponding to the domain wall width. However, further numerical simulations are required to understand the precise behavior of the spectrum around the peak frequency.

Predictions.—Although the $N_{\rm DW} = 1$ case does not leave observable GW signals (see the Supplemental Material [54]) due to the quick decay of the network, the situation is radically different for $N_{\rm DW} > 1$, where network annihilation is delayed. To simplify the presentation, we fix $N_b = 1$, $N_{\rm DW} = 6$, $\kappa_{\rm H} = 1$, and $g_{*,\rm ann} = g_{*s,\rm ann} = 106.75$ (see [54] for the case $\kappa_{\rm H} \ll 1$), and $\delta = 0.3$ and present results varying Λ_H, $r \equiv \mu_b/\Lambda_{\rm H}$, and f. According to (4), r can then be traded for $\Delta\theta$.

We first present results for large values of Λ_H, which maximally reduce the sensitivity to misaligned contributions.

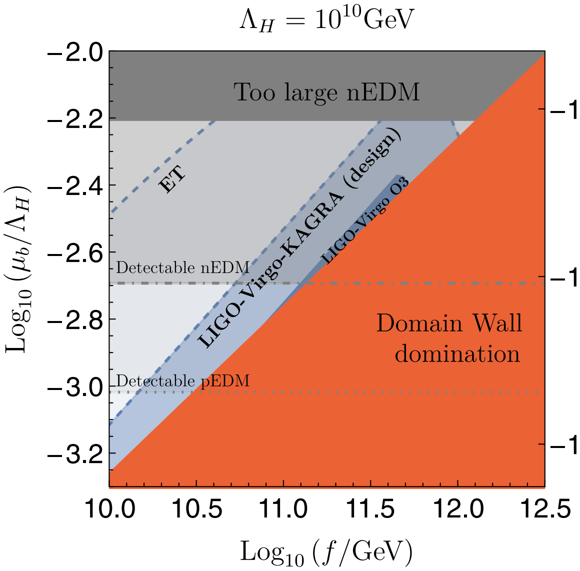

FIG. 1. Regions of parameter space that can be probed by GW and/or nEDM experiments, for $\Lambda_H = 10^{10}$ GeV and $\kappa_H = 1$. Constraints are also shown, as dark-shaded regions, from domain wall domination (lower right corner), nEDM [61] (upper part), LIGO-Virgo O3 run [82] (dark-blue shaded). Dashed contours bound regions probed by LVK at design sensitivity and ET (sensitivity curves taken from [83]). The gray-shaded region will be also probed by neutron [84] (dot-dashed line) and pEDM [85] measurements.

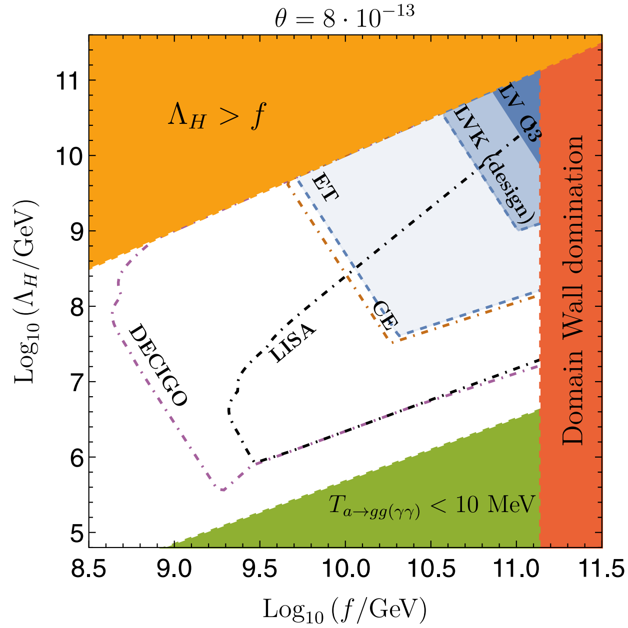

FIG. 2. Regions of parameter space detectable at GW observatories, fixing $\theta = 8 \times 10^{-13}$ according to upcoming nEDM measurements [84] and $\kappa_H = 1$. Same description and color code as in Fig. 1, with the addition of CE, *LISA*, and *DECIGO*'s sensitivity curves (dot-dashed lines, taken from [83]) and two constraints (dark-shaded regions) corresponding to axion decays below 10 MeV (lower right corner) and to $\Lambda_H > f$ (upper left corner).

We consider V_b to be temperature independent at $T \lesssim \Lambda_H$ and Λ_H to have a QCD-like temperature behavior (see also [54]).

Fixing $\Lambda_H = 10^{10}$ GeV as a representative example, we show values of r and f that can be probed by gravitational wave observatories (dashed contours), together with constraints (solid contours), in Fig. 1. In the lower right half, the string-wall network dominates before annihilation. While this region might not be completely ruled out, annihilation of the network in this case would require a dedicated study. In the upper part of the parameter space, the PQ solution is spoiled, i.e., $\Delta\theta \gtrsim 10^{-10}$ [61]. In the dark-blue-shaded region, the GW signal is incompatible with the latest 2σ upper bound from LIGO-Virgo (LV) [82], and this corresponds to the region close to DW domination. The dashed blue contours bound regions where the GW signal is detectable at the design sensitivity of LVK and Einstein Telescope (ET), respectively. The change of slope in the GW regions arises because of an intermediate phase of matter domination driven by the axions produced by the string-wall annihilation. This occurs at small decay temperatures (6), corresponding to the right half of the figure.

Very interestingly, we find that a significant fraction of these GW-observable regions also predicts a detectable nEDM (and/or pEDM) in the near future [32,84,85], i.e., $\Delta\theta \gtrsim 10^{-12}$ ($\Delta\theta \gtrsim 10^{-14}$) (above the dot-dashed and dotted gray contours, respectively). Motivated by the exciting possibility of a combined heavy axion discovery via nEDM experiments and GW observatories, we fix $\Delta\theta = 8 \times 10^{-13}$ and broaden our analysis to different values of Λ_H in Fig. 2. We find that any $\Lambda_H \gtrsim 10^6$ GeV leads to a GW signal in the foreseen reach of future ground [design LVK, ET, Cosmic Explorer (CE)] and space-based interferometers [Laser Interferometer Space Antenna (*LISA*) [86,87], Decihertz Interferometer Gravitational Wave Observatory (*DECIGO*) [88,89]]. The lower right corner in the figure is strongly constrained by the slow decay of axion quanta and a phase of matter domination which spoils big bang nucleosynthesis. As in Fig. 1, the change of slope in the GW contours is due to an intermediate phase of matter domination, in the lower right part of the figure.

Values of Λ_H smaller than 10^6 GeV can also lead to viable cosmologies, observable GWs, and detectable nEDM and/or pEDM, if the potential V_b is temperature dependent below Λ_H, see [54].

Finally, let us mention that, for $\Lambda_H \lesssim 10^{10}$ GeV, LVK are expected to probe the high frequency tail of the GW signal ($\sim\omega^{-1}$), ET can investigate the peak, and *LISA* can probe the low frequency tail ($\sim\omega^3$). Full GW spectra for some representative choices of parameters are shown in Fig. 3.

A caveat is in order before our conclusions: the parameter space shown in Figs. 1 and 2 is further constrained if

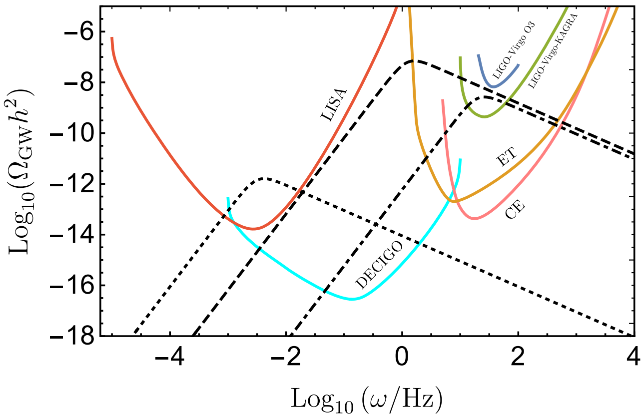

FIG. 3. Representative GW spectra (dashed, dotted, dot-dashed lines) for $\kappa_H = 1$ and $N_b = 1$, $N_{DW} = 6$, $\delta = 0.3$. Dashed: $\Lambda_H = 10^{10}$ GeV, $f \simeq 3 \times 10^{11}$ GeV and $\Delta\theta \simeq 9 \times 10^{-11}$. Dotted: $\Lambda_H = 10^7$ GeV, $f = 10^{10}$ GeV, and $\Delta\theta \simeq 2 \times 10^{-12}$. Dot-dashed: $\Lambda_H = 10^{11}$ GeV, $f = 1.6 \times 10^{11}$ GeV, and $\Delta\theta \simeq 1.2 \times 10^{-11}$. Sensitivity curves are taken from [82,83] for LIGO-Virgo O3. See also the Supplemental Material [54].

V_b arises from dimension-five (and to a less relevant extent from dimension-six) operators with large coefficients. While we discuss this quantitatively in the Supplemental Material [54], we note here that a large region of parameter space remains unaffected if such operators originate from nonperturbative effects (as can be expected if they are due to gravity, see, e.g., [7,52,90]).

Conclusions.—Heavy QCD axion models can feature a long-lived network of topological defects. The main finding of this Letter is that these models predict (i) a stochastic gravitational wave signal measurable by the design LVK interferometers in a large region of parameter space (further broadened by ET and CE, with the possibility of correlated signals also at *LISA* and *DECIGO*) and (ii) a nEDM and/or pEDM measurable in the near future, when (a) the new heavy QCD scale is large, i.e., $\Lambda_H \gtrsim 10^{10}$ GeV, thus making the PQ mechanism more robust, and (b) misaligned PQ breaking terms that motivate these models in the first place are not strongly suppressed.

Furthermore, we showed that combined GW (at *LISA* and *DECIGO*) and nEDM and pEDM signatures also arise for $10^6 \lesssim \Lambda_H \lesssim 10^{10}$ GeV.

Our results do not strongly depend on the specific heavy QCD axion model, as long as its potential has approximately degenerate minima.

We necessarily left several interesting points for future work. First, in order to precisely characterize the GW signal, numerical simulations of axionic string-wall networks beyond the current literature, possibly including friction and plasma effects, are crucial [71].

Second, we left unspecified the particle content and properties of the heavy QCD and of the misaligned PQ breaking sectors. However, these sectors may contain dark matter candidates (see, e.g., [91]) or light states that can contribute to the number of extra relativistic degrees of freedom, $\Delta N_{\rm eff}$ [92,93].

Furthermore, it is interesting to understand whether the collapse of the network of topological defects may also lead to a significant fraction of primordial black holes, in a scaled-up version of the mechanism proposed in [94].

Finally, a more complete exploration of the parameter space of heavy axion models may lead to further interesting signatures. For instance, GWs of very low frequency may arise for misaligned sectors lighter than QCD and may provide a new explanation for recent NANOGrav observations [95]. An investigation of all these aspects is ongoing [96].

Our work is relevant for the ongoing effort to probe well-motivated regions in the parameter space of the PQ mechanism. Guided by the theoretical pursuit of "higher quality" models, we suggest that gravitational wave interferometers and nEDM/pEDM experiments may be the right laboratories to discover the heavy QCD axion.

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