Composition of Multipartite Quantum Systems: Perspective from Timelike Paradigm

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Figuring out the physical rationale behind natural selection of quantum theory is one of the most acclaimed quests in quantum foundational research. This pursuit has inspired several axiomatic initiatives to derive a mathematical formulation of the theory by identifying the general structure of state and effect space of individual systems as well as specifying their composition rules. This generic framework can allow several consistent composition rules for a multipartite system even when state and effect cones of individual subsystems are assumed to be quantum. Nevertheless, for any bipartite system, none of these compositions allows beyond quantum spacelike correlations. In this Letter, we show that such bipartite compositions can admit stronger-than-quantum correlations in the timelike domain and, hence, indicates pragmatically distinct roles carried out by state and effect cones. We discuss consequences of such correlations in a communication task, which accordingly opens up a possibility of testing the actual composition between elementary quanta.

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Introduction.—The idea of composition plays a crucial role in fabricating our world view and accordingly guides us while constructing theories for the physical world [1]. For instance, the objects we encounter in our daily life—household items, machines, communication devices, computers, etc.—can be thought of as being composed of more elementary parts, whereas *Penrose tribar* describes an interesting composite object yet impossible to exist [2]. On the other hand, a region of spacetime with fields on it can be thought of as being composed of many smaller regions joined at their boundaries [3]. Quantum formalism also assumes a particular composition rule while describing systems consisting of more than one subsystem [4–8]. Considering the individual systems to be quantum, one can construct several mathematically consistent models to describe state and effect spaces of a multipartite system, where consistency demands the outcome probability obtained from any pair of valid state and effect to be a positive number between zero and one. Constraints arising from physical and/or information theoretic demand may further abridge the scope of possible compositions. For instance, the state space of a bipartite system satisfying *no signaling* principle and *tomographic locality* postulate lies within two extremes—*minimal* tensor product and *maximal* tensor product [9–12]. The corresponding effect spaces are specified in accordance with the "no-restriction" hypothesis [13], which demands any mathematically well-defined measurement to be physically allowed. For brevity, the resulting theories arising from these two compositions will be denoted by SEP and $\overline{\text{SEP}}$, respectively. In between these two extremes, many other compositions can be introduced, among which quantum ($\mathcal{Q}$) composition is one example.

Naturally, the question arises whether there exist input-output correlations that are specific to some particular composite structure. An interesting answer stems from the Bell nonlocal correlations [14–17] that are unavailable in SEP theory, whereas all other compositions contain such nonlocal correlations. On the other hand, a *no-go* result can be argued from the work of Barnum *et al.* [18]—any bipartite spacelike correlation obtained in $\overline{\text{SEP}}$ is also achievable in $\mathcal{Q}$. In fact, any composite model of two quantum systems satisfying the no-signaling principle cannot have a beyond quantum spacelike separated correlation within its description. It might be tempting to presume that any input-output correlation obtained in SEP should also be achievable in $\mathcal{Q}$, since the roles of state and effect cones in SEP and $\overline{\text{SEP}}$ theories just get interchanged (see Fig. 1). In this Letter, we show that this naive intuition is, in fact, not correct. There, indeed, exist timelike correlations in SEP that cannot be obtained in $\mathcal{Q}$ composition. In fact, such correlations, with different strengths, might exist in different composite models indicating empirical distinction among these compositions. In other words, mathematically consistent bipartite composition of elementary systems can have beyond quantum timelike correlations even when the elementary systems allow a quantum description. We establish the above thesis through a communication task (game) involving two parties. We first analyze the optimal qubit communication required to accomplish the task when quantum composition

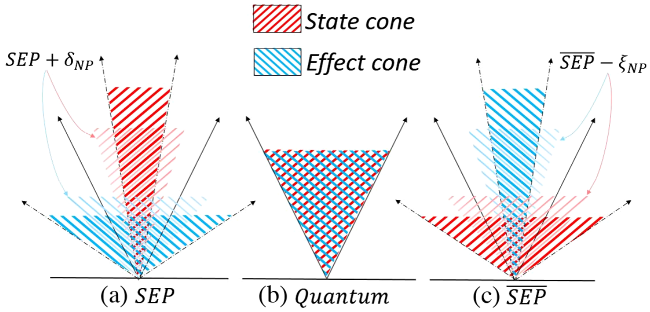

FIG. 1. Intuitive representation of the state and effect cones for different compositions of two individual quantum systems. (a) SEP theory: Only separable states are allowed, whereas effects are more exotic than the effects allowed in quantum theory. (b) Quantum theory: The state cone and effect cone exactly overlap (*self-dual*). (c) $\overline{\text{SEP}}$ theory: The effect cone is constrained in comparison to quantum theory but allows states that are more exotic than quantum states. The state cone of SEP $+ \delta_{NP}$ lies strictly in between SEP and $\mathcal{Q}$, whereas for $\overline{\text{SEP}} - \xi_{NP}$ theory it lies strictly in between $\mathcal{Q}$ and $\overline{\text{SEP}}$. Examples of such compositions are shown in (a) and (b) with light shaded cones.

is assumed. In fact, a necessary and sufficient condition for perfect accomplishment of the task is derived in the generalized probabilistic theory (GPT) framework. We then show that for perfect success of the game the required number of qubits to be communicated is strictly less than quantum if SEP composition is considered. Thus, SEP composition makes certain communication complexity problems trivial in comparison to quantum composition. It is worth mentioning that, starting with an information-theoretic axiom "*communication complexity is nontrivial,*" researchers have identified "unphysical" consequences of beyond quantum spacelike correlations [19–24] (see [25] for a review on communication complexity). Our study establishes that the same axiom can be efficiently utilized to isolate beyond quantum correlations in the timelike domain. Our proposed communication task provides an empirically testable criterion toward natural selection of the bipartite composite structure among different possible compositions lying in between SEP and $\overline{\text{SEP}}$.

Preliminaries.—First, we briefly recall the framework of GPT, since some of our results will be proved in this generalized framework. Although the framework had originated much earlier [26–28], the advent of quantum information theory brought renewed interest in this framework [13,29–32]. For an overview of this framework, we also refer to Refs. [33–37]. A GPT is specified by a list of system types and the composition rules specifying a combination of several systems. In the prepare and measure scenario, a system ($\mathcal{S}$) is described by the tuple (Ω, E). Here, Ω represents the collection of normalized states, generally a compact-convex set embedded in some positive cone V_+ (collection of unnormalized states) of some real vector space V. On the other hand, E denotes the collection of effects, where an effect $e \in E$ corresponds to a linear functional $e: \Omega \mapsto [0,1]$, with $e(\omega)$ denoting the success probability of filtering the effect e on the state $\omega \in \Omega$ when some measurement $M := \{e_i | e_i \in E \; \forall \; i, \sum_i e_i = u$, and $u(\omega) = 1 \; \forall \; \omega \in \Omega\}$ is performed. The unnormalized effects form a cone $V_+^\star$ which is dual to the state cone V_+. For instance, the state cone of a quantum system associated with Hilbert space $\mathcal{H}$ consists of the set of positive operators $\mathcal{T}_+(\mathcal{H}) \subset \mathcal{T}(\mathcal{H})$ acting on $\mathcal{H}$, where $\mathcal{T}(\mathcal{H})$ denotes the set of all Hermitian operators on $\mathcal{H}$. A normalized state ρ is an element of $\mathcal{T}_+(\mathcal{H})$ with trace one, and their collection is the convex-compact set of density operators $\mathcal{D}(\mathcal{H})$. A generic quantum measurement corresponds to positive operator valued measure $M := \{\pi_i | \pi_i \in \mathcal{T}_+(\mathcal{H}), \sum_i \pi_i = \mathbf{1}_\mathcal{H}\}$, with $\mathbf{1}_\mathcal{H}$ being the identity operator on $\mathcal{H}$.

A set of states $\{\omega_i\} \subset \Omega$ are called perfectly distinguishable if there exists a measurement $M \equiv \{e_i | \sum_i e_i = u\}$ such that $e_i(\omega_j) = \delta_{ij}$. Given a system $\mathcal{S} \equiv (\Omega, E)$, the maximal cardinality of the set of states that can be perfectly distinguished is known as the operational dimension of the system. On the other hand, the maximal cardinality of the set of states that can be perfectly distinguished pairwise is known as the information dimension of the system [38]. Note that, in the case of operational dimension, only one measurement is allowed to distinguished the states, whereas for information dimension different measurements for distinguishing different pairs are allowed.

Given two systems $\mathcal{S}_A \equiv (\Omega_A, E_A)$ and $\mathcal{S}_B \equiv (\Omega_B, E_B)$, the state space of the composite system $\mathcal{S}_{AB} \equiv (\Omega_{AB}, E_{AB})$ is embedded in the tensor product space $V_A \otimes V_B$, although the choice of Ω_{AB} is not unique. However, the *no-signaling* principle and *tomographic locality* [9] postulate narrow down the choices within two extremes—the minimal tensor product space and maximal tensor product space, resulting in theories we will call SEP and $\overline{\text{SEP}}$, respectively. For two systems, each described by quantum theory individually, the state cone in SEP theory is given by

$$(V_{AB}^{\text{SEP}})_+ := \left\{ \sum_i \pi_i^A \otimes \pi_i^B | \; \forall \; i, \pi_i^A \in \mathcal{T}_+(\mathcal{H}_A), \right.$$
$$\left. \text{and } \pi_i^B \in \mathcal{T}_+(\mathcal{H}_B) \right\}.$$

The effect cone E_{AB}^{SEP} is dual to Ω_{AB}^{SEP} and is given by

$$(V_{AB}^{\text{SEP}})_+^\star := \{Y \in \mathcal{T}(\mathcal{H}_A \otimes \mathcal{H}_B) | \text{Tr}(XY) \geq 0 \; \forall \; X \in (V_{AB}^{\text{SEP}})_+\}.$$

SEP theory allows only separable states, whereas the state cone $(V_{AB}^{\mathcal{Q}})_+ := \{Y \in \mathcal{T}_+(\otimes \mathcal{H}_B)\}$ of quantum theory contains both product and entangled states. On the other hand, effects in SEP theory can be more exotic than quantum entangled effects. For instance, entanglement witnesses [39] can be a valid effect in this theory, as they yield positive probability on any separable state. Thus, we have the following set inclusion relations:

$$(V_{AB}^{\mathrm{SEP}})_+ \subset (V_{AB}^{Q})_+ = (V_{AB}^{Q})_+^{\star} \subset (V_{AB}^{\mathrm{SEP}})_+^{\star}.$$

Equality between $(V_{AB}^{Q})_+$ and $(V_{AB}^{Q})_+^{\star}$ for quantum composition is known as self-duality, which also holds true for a class of elementary GPT models [40–42]. The $\overline{\mathrm{SEP}}$ theory is obtained by interchanging the role of state cone and effect cone of SEP theory (see Fig. 1) and, hence, allows more exotic states than quantum states, e.g., the *positive on pure tensors* states [43]. In between SEP and $\overline{\mathrm{SEP}}$, one can consider many other compositions, either by adding a subset of nonseparable states δ_{NP} with the state cone of SEP or by excluding a subset of nonseparable states ξ_{NP} from the state cone of $\overline{\mathrm{SEP}}$. The resulting theories we will denote as $\mathrm{SEP} + \delta_{NP}$ and $\overline{\mathrm{SEP}} - \xi_{NP}$. Excluding SEP, many of these theories are at par with quantum theory in the sense of containing nonlocal correlations. The result of Ref. [18] implies that one cannot obtain any spacelike correlation in any of these models that is not available in quantum theory. More precisely, any input-output spacelike correlation obtained in any of the above compositions between two qudits is, indeed, achievable in $\mathbb{C}^d \otimes_Q \mathbb{C}^d$. In the following, we will show that such a thesis is not true anymore if we consider correlations in the timelike domain. Formally, spacelike correlations represent joint input-output conditional probability distributions arising from local measurements performed on spatially separated composite systems where no communication is allowed between the subsystems. On the other hand, the timelike scenario allows communication from one subsystem to the other (see Fig. 2). While the study of Bell nonlocality has

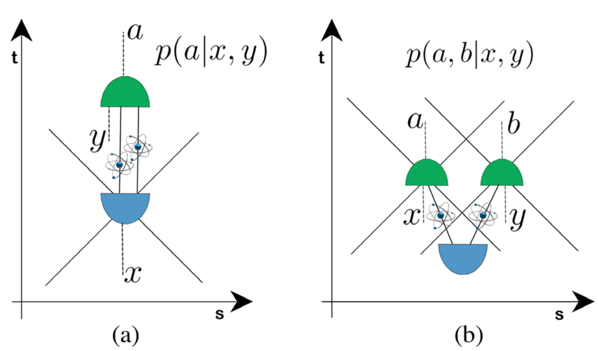

FIG. 2. Timelike and spacelike correlations. (a) The preparation (blue) and measurement (green) devices receive inputs x and y, respectively, and finally an outcome a is obtained. The blue device prepares two elementary systems which are individually described as quantum bits, but the global description of the preparation is unknown. This setting can generate stronger-than-quantum correlation as shown in Theorem 1. In (b), correlations generated by spacelike separated measurement devices acting on two qubits always imply a quantum description of the joint preparation [18].

motivated a vast literature in the former scenario [17], the latter avenue has been comparatively less explored. In the following, we introduce a game that helps to grasp the strength of correlations in the timelike domain.

Pairwise distinguishability game $\mathcal{P}_D^{[n]}$.—The game involves two players (say) Alice and Bob and a referee. In each run of the game, the referee provides a classical message η to Alice, randomly chosen from some set of messages $\mathcal{N}$, where $|\mathcal{N}| \coloneqq n$, and asks Bob a question $\mathbb{Q}(\eta, \eta')$—whether the message given to Alice in that run is $\eta \in \mathcal{N}$ or $\eta' \in \mathcal{N}$, where $\eta' \neq \eta$. Since $\eta' \neq \eta$, $\binom{n}{2}$ number of such different questions are possible. The winning condition of the game demands Bob answer all the questions correctly. Alice and Bob do not share any correlated state, but Alice can encode her message on the states of some physical system and accordingly send them to Bob. The following results provide a necessary and sufficient condition for winning the game in any GPT.

Proposition 1.—Perfect winning of the game $\mathcal{P}_D^{[n]}$ requires Alice to encode her message $\eta \in \mathcal{N}$ on a set of states $\{\omega_\eta\}_{\eta \in \mathcal{N}} \subset \Omega$ of some system $\mathcal{S} \equiv (\Omega, E)$ such that the states within the set $\{\omega_\eta\}_{\eta \in \mathcal{N}}$ are pairwise distinguishable.

Formal proof of the proposition we defer to Supplemental Material [44]. Here, we point out that the concept of information dimension of the system used by Alice to encode her messages plays a crucial role in this game, since, in each run, the question $\mathbb{Q}(\eta, \eta')$ asked to Bob will be a function of two messages, one of which has been provided to Alice.

We will now consider the situation where Alice encodes her messages on the states of multiple qubits available to her. However, depending upon the composite structures assumed to model these multiple qubits, different numbers of qubits may be required to win the same game, which leads us to one of our core results.

Theorem 1.—Four-qubit communication from Alice to Bob is required for winning the game $\mathcal{P}_D^{[12]}$ when quantum composition is considered among the elementary systems, whereas two SEP bits (i.e., two qubits in SEP composition) suffice for winning this game.

Proof (outline).—For a quantum system, the information dimension is the same as its operational dimension, which is again the same as the dimension of the associated Hilbert space [38]. Since the Hilbert space dimension of $(\mathbb{C}^2)^{\otimes 3}$ is 8, according to Proposition 1, three-qubit communication is not sufficient for winning the game $\mathcal{P}_D^{[12]}$ perfectly. However, four-qubit communication suffices as the number of distinguishable (as well as pairwise distinguishable) states, in this case, is 16. If we consider the SEP composition between two qubits, then the following 12 states $\mathcal{A} \coloneqq \{|\kappa\kappa\rangle, \kappa\bar{\kappa}\rangle, |\bar{\kappa}\kappa\rangle, |\bar{\kappa}\,\bar{\kappa}\rangle\}_{\kappa \in \{x,y,z\}}$ turn out to be pairwise distinguishable, where $|\alpha\beta\rangle \coloneqq |\alpha\rangle \otimes |\beta\rangle$ and $|\kappa\rangle(|\bar{\kappa}\rangle)$ is the eigenstate of Pauli operator σ_κ with eigenvalue $+1(-1)$, where $\kappa \in \{x, y, z\}$. While some pairs of states, such as $\{|xx\rangle, |x\bar{x}\rangle\}$, are perfectly distinguishable in quantum theory due to mutual orthogonality, some pairs, such as $\{|xx\rangle, |zz\rangle\}$, consisting of nonorthogonal states cannot be perfectly distinguished in quantum theory. However, as shown by Arai, Yoshida, and Hayashi, such states can be perfectly distinguished in SEP theory [51]. Complete analysis (along with the measurement) of pairwise distinguishability of the states $\mathcal{A}$ in SEP theory is presented in Supplemental Material [44]. The game $\mathcal{P}_D^{[12]}$, thus, can be perfectly won in SEP theory if Alice encodes her message on these states. This completes the proof of our claim. ■

Theorem 1 establishes the communication advantage of SEP composition over the other two compositions $\mathcal{Q}$ and $\overline{\text{SEP}}$, even though it is a local theory by construction. This theorem also implies a curious feature of the SEP theory, known as "dimension mismatch"—the difference between measurement dimension and information dimension [38].

Corollary 1.—SEP composition exhibits the phenomenon of dimension mismatch.

Proof.—The measurement dimension of two SEP bits is 4, which follows from Proposition 2.5 of Ref. [51], whereas our Theorem 1 establishes its information dimension to be strictly greater than 4 and completes the proof. ■

Dimension mismatch and consequently the presence of stronger-than-quantum timelike correlation in the above result arises strictly from the choice of composition. One might ask whether the advantage of two qubits in SEP composition for playing the $\mathcal{P}_D^{[n]}$ game can be made arbitrarily large. Our next result is a no-go answer to this question (proof provided in Supplemental Material [44]).

Lemma 1.—The game $\mathcal{P}_D^{[n]}$ cannot be won perfectly by communicating the encoded states chosen from the composite system $\mathbb{C}^2 \otimes_{\min} \mathbb{C}^2$ whenever $n > 12$.

However, the advantage of SEP composition over quantum theory can be increased if we start with a larger number of SEP bits initially.

Theorem 2.—$2k$ number of SEP bits are sufficient for winning the game $\mathcal{P}_D^{[12^k]}$ perfectly, whereas it requires $2k + \lceil k \log_2 3 \rceil$ number of qubits, with $k \in \mathbb{Z}_+$.

Proof is provided in Supplemental Material [44]. While deriving this theorem, we use the fact that Bob addresses at most two SEP bits together while decoding Alice's message. The gap between the number of SEP bits and qubits might increase further if Bob addresses all the SEP bits together. However, we leave this question open for future research. We rather move to a possible experimental implication of our study.

Novel experimental proposals bring adequate physical reasoning to the 'mathematical fiction': of Hilbert space formulation of quantum theory. For instance, the experimentally observed algebraic relationship among the coherent cross sections of scattering amplitudes in a triple-slit experiment constitutes a test for complex versus quaternion quantum theory [52]. The experiment by Sinha *et al.* that rules out multiorder interference in quantum mechanics is worth mentioning at this point [53]. In a similar spirit, a pertinent question to ask is which particular composite structure between two elementary qubits must be preferred [54,55]. At this point, one might wish to postulate a particular composite structure. Schrödinger, for instance, found quantum composition "rather discomforting" [56] due to the peculiarity of quantum entanglement as demonstrated in the Einstein-Podolsky-Rosen gedanken experiment [57]. The SEP composition, which does not contain these "discomforting" features, is immediately ruled out due to the seminal experiment by Aspect and collaborators [58] and the recent *loophole-free* Bell tests [59–61] which validate the presence of nonlocal correlations in nature. There are still a number of different bipartite compositions, such as $\text{SEP} + \delta_{EP}$, $\overline{\text{SEP}} - \xi_{EP}$, and $\overline{\text{SEP}}$, that, like the quantum composition $\mathcal{Q}$, incorporate nonlocal correlations and, hence, cannot be excluded from the Bell test's results. Furthermore, no such model can contain any spacelike correlation which is not available in quantum theory [18]. At this point, our $\mathcal{P}_D^{[12]}$ game starts playing a crucial role. Perfect success of this game with communication of less than four qubits assures the presence of beyond quantum timelike correlation which indicates a departure from the composition rule adopted in quantum theory. In fact, our next result proposes a generic test in this direction.

Proposition 2.—For $n > 2^k$, the game $\mathcal{P}_D^{[n]}$ cannot be won with k qubits communication from Alice to Bob if the composition rule is quantum.

The proof simply follows from Proposition 1 and the fact that the information dimension of a quantum system is the same as its Hilbert space dimension. A non-null result in Proposition 2, i.e., successful completion of the task $\mathcal{P}_D^{[>2^k]}$ with k qubits communication, will indicate a departure from the quantum composition rule, whereas null result builds confidence toward quantum composition.

Discussions.—The present Letter initiates a novel paradigm toward experimental tests for derivation of the composition rule from quantum mechanics. Importantly, unlike the seminal Bell tests that deal with spacelike correlations, our proposal is based on timelike correlations. In this regard, the recent works in Refs. [62,63] are worth mentioning. In particular, the authors in Ref. [62] provide examples of stronger-than-quantum timelike correlations while considering two elementary systems. However, the elementary systems considered there are postquantum in nature (particularly the *square bits*), which itself can generate stronger timelike correlation than a qubit as established through our $\mathcal{P}_D^{[n]}$ task. The possibility of stronger-than-quantum timelike correlations in this present work, therefore, emerges strictly from the choices of composition between the qubits. From a technical point of view, the authors in Ref. [62] utilize a concept called "signaling dimension" which is motivated from the study made in Ref. [64], whereas, in our case, the concept of information dimension plays a crucial role. On the other hand, the approach in Ref. [63] involves calculating entropic quantities, which requires particular structure in a theory to be well defined [65,66]. Our approach, however, involves a very intuitive notion of pairwise distinguishability. A more elaborate discussion regarding novelty of our method in comparison to the existing approaches is deferred to Supplemental Material [44].

Our study also motivates a number of questions that might be interesting for further exploration. First of all, dimension mismatch studied in Ref. [38] for a square bit model suggests several exotic implications. For instance, it can result in collapse of communication complexity and can also empower the Maxwell's demon, indicating a violation of the second law of thermodynamics. Similar studies will be interesting in our case as our Corollary 1 implies a dimension mismatch arising strictly from a compositional aspect of local quantum systems. It will also be interesting from a complexity theory perspective to answer the question mentioned after Theorem 2. Finding the implications of different composition rules is worth exploring for higher-dimensional elementary quantum systems. Similar questions might also be reframed in field theoretic formalism [67,68]. On the other hand, studying the implication of such stronger timelike correlations within the framework of generalized probabilistic theory might also provide fundamentally new insights regarding the structure of spacetime. The toy models of polygonal theory that have been studied extensively in the recent past [40–42,69–71] might be a starting point toward this endeavor.

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