Causality Constraints on Gravitational Effective Field Theories

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(Received 23 December 2021; accepted 7 March 2022; published 1 April 2022)

We consider the effective field theory of gravity around black holes, and show that the coefficients of the dimension-8 operators are tightly constrained by causality considerations. Those constraints are consistent with—but tighter than—previously derived causality and positivity bounds and imply that the effects of one of the dimension-8 operators by itself cannot be observable while remaining consistent with causality. We then establish in which regime one can expect the generic dimension-8 and lower order operators to be potentially observable while preserving causality, providing a theoretical prior for future observations. We highlight the importance of "infrared causality" and show that the requirement of "asymptotic causality" or net (sub)luminality would fail to properly diagnose violations of causality.

DOI: 10.1103/PhysRevLett.128.131102

Introduction.—General relativity (GR) should be thought as the leading order term in an effective field theory (EFT) that includes an infinite number of higher-dimension operators [1–5]. If we are interested in gravity below some energy scale Λ, we may integrate out all particles with masses above that scale. Assuming a tree level weakly coupled completion, such as a string theory, the effective action is

$$\mathcal{L}_{\rm EFT} = \frac{M_{\rm Pl}^2}{2}\left(R + \frac{\mathcal{L}_{D4}}{\Lambda^2} + \frac{\mathcal{L}_{D6}}{\Lambda^4} + \frac{\mathcal{L}_{D8}}{\Lambda^6} + \cdots\right), \quad (1)$$

where $\mathcal{L}_{Dn}$ denotes a linear combination of all possible dimension-n operators built out of the Riemann (or Weyl) curvature and its covariant derivatives, see Supplemental Material (SM) [6]. The higher-dimension operators capture the effects of the heavy fields that have been integrated out at tree level, i.e., particles of spin ≥ 2. One would expect the scale Λ to be the mass of the lightest higher spin state ($s \geq 2$). Motivated by the recent detections of gravitational waves (GWs), there has been a surge of interest in establishing whether these operators could be probed assuming a very low Λ. Such operators could indicate the presence of new physics beyond the standard model and potentially connect us with the dark sector. A formalism for probing those operators with inspiraling GWs was proposed in Ref. [7]. Finite size effects of black holes (BHs) have also been investigated in the presence of dimension-8 operators [8] and dimension-6 operators [9,21]. Interestingly, LIGO and Virgo constraints on the dimension-8 operators were explored in Ref. [10]. For related works see Refs. [11,12,23,24].

While we may be on the edge of constraining gravitational EFTs using GW observations, theoretical considerations also have significant impact. For instance, requiring the low-energy EFTs to be embeddable in a local Wilsonian, unitary, Lorentz invariant and causal high energy completion like string theory imposes a set of positivity constraints on these EFTs [25,26]. In parallel, it is well known that in gravitational EFTs, the sound speed can appear to be superluminal [9,27–36], and by demanding the local group velocity of GWs to be (sub)luminal, it was shown in Ref. [37] that the coefficients of the dimension-8 operators ought to be sign definite. In this Letter, we shall complement the state of the art by further investigating the constraints set by causality. Our requirements for preserving causality are similar to the ones indicated in Refs. [13,14,38] but differ from the notion of "asymptotic causality" or net (sub)luminality which is sometimes postulated in the literature. As we shall see, the asymptotic causality condition, while necessary is not sufficient for preserving causality and fails to identify situations which are known to be in tension with causality as inferred for instance from positivity bounds.

GWs in dimension-8 EFT.—Motivated by the findings of Ref. [10] we shall start with the following dimension-8 operator,

$$S_{D8}^{(1)} = \int d^4x\sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left[R + \frac{c_1}{\Lambda^6}(R_{abcd}R^{abcd})^2\right], \quad (2)$$

with $c_1 = \pm 1$. Considering a BH of mass M, the metric slightly deviates from the Schwarzschild one with a magnitude proportional to the dimensionless parameter $\mu = (GM\Lambda)^{-6}$, where $G = 1/(8\pi M_{\rm Pl}^2)$ is Newton's constant, see SM for details [6].

GWs can be decomposed into odd and even parity metric perturbations $h_{\mu\nu}^{\pm}$ propagating independently on the Schwarzschild-like background. Expressed in spherical harmonics with multipole ℓ, the radial dependence of each mode can be captured by the master variables $\Psi^{\pm}_{\omega\ell}(r)$, where ω denotes the frequency. Including the dimension-8 operators perturbatively, the master variable satisfies the modified Regge-Wheeler-Zerilli equation [8,39]

$$\frac{d^2\Psi^{\pm}_{\omega\ell}}{dr_*^2} = -[\omega^2 - V^{\pm}_{\rm GR}(r;\ell) - c_1\mu V^{\pm}(r;\ell,\omega)]\Psi^{\pm}_{\omega\ell}, \tag{3}$$

where r_* is the tortoise coordinate, $V^{\pm}_{\rm GR}$ are the GR potentials, and $V^{\pm}$ the leading-order EFT correction (see SM for the technical details [6]). Parity ensures that both modes decouple and we shall omit the $\pm$ indices unless relevant.

For the EFT to remain valid when scattering GWs on a BH, the Riemann curvature ought to be small as compared to the cutoff at the impact parameter $r_b = (\ell + 1/2)/\omega$, meaning $r_b\Lambda \gg 1$ and $GM/r_b^3 \ll \Lambda^2$. Moreover, we also require that the description of the GWs is under control as discussed in Refs. [13,14] (see SM [6]), meaning that their asymptotic energy ω should be bounded by

$$\omega \ll \Lambda^2 r_b. \tag{4}$$

With this in mind, the background is then automatically under control if $\mu \lesssim 1$ and $r_b > GM$.

Scattering phase shift and time delay.—When considering the scattering of GWs on a Schwarzschild-like BH in model (2), the EFT corrections manifest themselves in the scattering phase shift and time delay, which can be inferred from solving Eq. (3) in the Wentzel-Kramers-Brillouin (WKB) approximation. For practical reasons, we shall focus on GWs with $\omega^2 < \max(|V_{\rm GR}|)$, in which case the desired WKB solution is the one that decays exponentially at the horizon (tortoise coordinate $r_* \to -\infty$). At infinity, the corresponding solution asymptotes to [13],

$$\Psi_\ell \propto e^{2i\delta_\ell} e^{i\omega r_*} - (-1)^\ell e^{-i\omega r_*}, \tag{5}$$

with the phase shift

$$\delta_\ell = \int_{r_*^T}^{\infty} dr_* (\sqrt{\omega^2 - V_{\rm GR} - c_1\mu V} - \omega) - \omega r_*^T + \frac{\pi}{2}\left(\ell + \frac{1}{2}\right), \tag{6}$$

where r_*^T is the turning point defined by $\omega^2 - V_{\rm GR} - c_1\mu V = 0$. The scattering time delay is then given in terms of the phase shift by $T_\ell = 2\partial\delta_\ell(\omega)/\partial\omega$. As compared to the GR answer $T_\ell^{\rm GR}$, the total time delay T_ℓ acquires an additional EFT contribution δT_ℓ,

$$T_\ell = T_\ell^{\rm GR} + \delta T_\ell + \mathcal{O}(\mu^2). \tag{7}$$

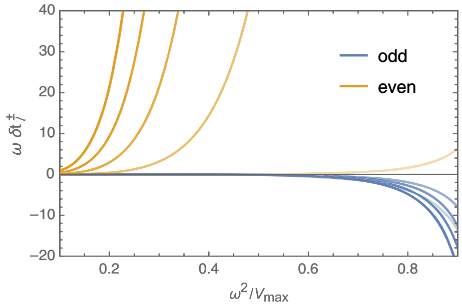

FIG. 1. EFT corrections on the scattering time delay of the odd (blue) and even (orange) modes in the dimension-8 EFT (2). From light to dark, the curves show $\omega\delta t_\ell$ with $\ell = 2$, 22, 42, 62, and 82. The EFT contribution to the time delay is given by $\delta T_\ell = c_1\mu\delta t_\ell$, so the odd modes enjoy a time advance when $c_1 = 1$, and the even ones when $c_1 = -1$.

Writing $\delta T_\ell = c_1\mu\delta t_\ell$, Fig. 1 shows $\delta t_\ell^{\pm}$ as a function of ω for various values of ℓ (see SM [6]). Interestingly, δt_ℓ^- and δt_ℓ^+ always have an opposite sign, so that there is always a time advance for one of the GW polarizations for any choice of $c_1 = \pm 1$.

Infrared causality.—As has been established for QED [38] and for other gravitational theories [13], a time advance compared to GR, i.e., $\delta T_\ell < 0$, does not necessarily indicate acausality. It violates causality only if the time advance, calculable within the validity regime of the EFT, is resolvable. For causality to be respected, the front velocity should be luminal, meaning that the infinite frequency limit of the phase velocity should be luminal as dictated by the geometry seen by those high-frequency modes. As unitarity and analyticity (derived from causality) dictate that the phase velocity cannot decrease with frequency, this implies that low-frequency modes should necessarily be subluminal with respect to the local background geometry. Indeed, the equivalence principle implies that the high-frequency modes can only be sensitive to the local inertial frame, so causality is fixed by the background geometry seen by the high-frequency modes. At the level of a low-energy EFT, causality therefore demands that low-energy modes be (sub)luminal as compared to the background geometry, which in terms of observables requires that any support outside the light cone determined by the geometry be unresolvable, see [14] for more details. In other words, the statement of "infrared causality" is violated if

$$-\delta T_\ell \gtrsim 1/\omega \quad \text{(infrared acausality)}. \tag{8}$$

Note that infrared acausality necessarily implies the absence of a standard and causal high energy completion, however, respecting infrared causality does not necessarily guarantee the presence of a consistent UV embedding, it only is a necessary condition. Translating this back into the parameters of the model (2), we infer that a wave with

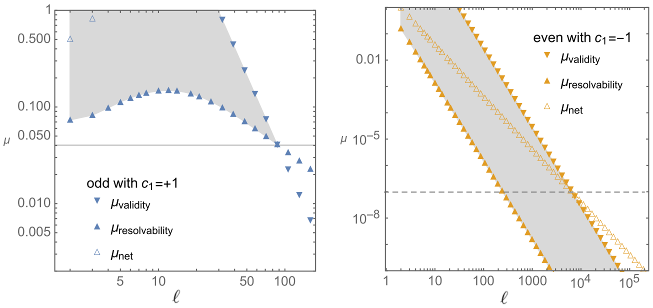

FIG. 2. Parameter space (shaded gray) of causal violating time advance in the dimension-8 EFT (2). $\mu_{\rm validity}$ and $\mu_{\rm resolvability}$ are the upper and lower bound of μ in condition (9), and $\mu_{\rm net}$ is the lower bound in condition (11).

frequency ω and multipole ℓ scattered about a BH of mass M in the EFT (2) violates causality whenever

$$\frac{1}{-c_1\omega\delta t_\ell} \lesssim \mu \ll \left(\frac{\ell+1/2}{\omega^2 G^2 M^2}\right)^3. \tag{9}$$

Introducing the parameter γ defined as $\omega^2 = \gamma V_{\rm max}$, with $V_{\rm max}$ being the maximum of $V_{\rm GR} \sim \ell^2 (GM)^{-2}$, the condition (9) only depends on the BH mass via μ. Naturally, the effect of the dimension-8 operator increases with ω as illustrated in Fig. 1, however ω should be smaller than $V_{\rm max}$ for the phase shift to be well approximated by (6) [40]. For these reasons, we consider $\gamma = 0.9$, so that $\omega \propto \ell$ at large ℓ, and the impact parameter is constant. We compute the condition (9) numerically and present the results in Fig. 2 which indicate that the EFT (2) violates causality in the odd sector if $c_1 = +1$ and $\mu \gtrsim 0.04$.

Crucially we see that if $c_1 = -1$, the even sector always violates the notion of infrared causality. In that case, the even modes lead to a time advance with δt_ℓ^+ increasing quadratically with the multipole ℓ, while ω increases linearly as $\ell \to \infty$. Therefore, both sides of the inequality (9) decrease as ℓ^{-3} at large ℓ. Explicit calculation shows that the left hand side of the inequality (9) is smaller than its right hand side (cf. Fig. 2). This implies that no matter how small μ is, for sufficiently large ℓ the time advance will always be resolvable when $c_1 = -1$, hence violates causality [41].

The main implication of our findings is that the EFT defined in (2) can only ever be causal if $c_1 = 1$ and if $(GM\Lambda)^{-6} \lesssim 0.04$ for any BH. Given that the smallest known BH has $3\,M_\odot$ [42–44], the causality constraint translates into a lower bound on the cutoff scale enforcing $\Lambda \gtrsim 7\times 10^{-11}$ eV. Within the current state of the art, the EFT (2) with a cutoff of order $\Lambda \sim 10^{-13}$ eV was shown to lead to observable effects [10]. While such a low cutoff can lead to a potentially interesting phenomenology, it also comes hand in hand with violations of causality. We can push those bounds further by considering BHs with arbitrarily small mass, which would lead to a constraint on c_1/Λ^6 to be arbitrarily small. In particular, BHs with radii as small as the fundamental scale of quantum gravity $M_{\rm fund}$ would force the scale Λ to be associated to that scale $\Lambda \sim M_{\rm fund}$ in the case where no other dimension-6 and dimension-8 operators are considered.

Our conclusion on the sign of the coefficient of the dimension-8 operator is entirely consistent with expectations on the low-energy operators derived in type II string theory [15] after compactification [16]. It also has been shown that the sign of c_1 can be fixed by demanding the local group velocity of high-frequency GWs to be (sub) luminal [37]. The dimension-8 operator $c_1(R_{abcd}R^{abcd})^2$ is, in spirit, the gravitational analog of the $c(\partial\phi)^4$ operator that enters generic Goldstone EFTs. In that case the existence of a standard Wilsonian completion, manifests itself via positivity bounds, which have been shown to be directly linked with the sign of the coefficient c [45–47]. Within the low-energy EFT, the sign of coefficient c is also directly linked to resolvability of time advances and hence to causality [13]. Applying similar types of positivity bounds to gravitational EFTs have been shown to impose $c_1 > 0$ [25,26]. Not only are our conclusions fully consistent with those results, they also allow us to derive a lower bound of the cutoff of the EFT considered in (2) when $c_1 = 1$.

Asymptotic causality.—The time delay $T_\ell^{\rm GR}$ introduced in Eq. (7), is the one perceived by the freely propagating modes following null geodesics on that background. In this sense, $T_\ell^{\rm GR}$ represents what the high-energy modes (the modes with energy well above Λ and well below $M_{\rm Pl}$) are subject to on that very background. See, for instance, Ref. [38] for an analog discussion in the case of QED.

On the other hand, δT_ℓ represents the additional time delay of the low-energy modes on that same background, arising from interactions with the heavy fields (whose effects are precisely encapsulated by the inclusions of the higher dimension operators). Since causality demands that low-energy modes do not travel outside the light cone set by the high-energy modes, what matters in setting causality is the sign of δT_ℓ and not the net T_ℓ. Only a time advance in addition to the GR contribution, i.e., a negative δT_ℓ, would signal that the retarded propagator has support outside the light cone set by the high-energy modes [13,17,38,48–50].

Positivity of the net T_ℓ, is referred to as asymptotic causality, and violating it requires

$$-T_\ell \gtrsim 1/\omega, \qquad \text{(asymptotic acausality)}, \tag{10}$$

leading to a different lower bound in the inequality (9),

$$\frac{(T_\ell^{\rm GR} + \omega^{-1})}{-c_1\delta t_\ell} < \mu \ll \left(\frac{\ell+1/2}{\omega^2 G^2 M^2}\right)^3. \tag{11}$$

From Eq. (6), we see that $T_\ell^{\rm GR}$ approaches to a constant at large ℓ, while $\delta t_\ell^- \sim \ell^0$ and $\delta t_\ell^+ \sim \ell^2$, whereas the upper bound in the inequality (11) still scales as ℓ^{-3}. Therefore, irrespective of the parameters of the EFT, sufficiently high multipoles that remain within the regime of validity of the EFT always enjoy a positive net time delay, as depicted in Fig. 2. Comparing to the criteria used previously, the statement of asymptotic causality while necessary is not sufficient and by itself would always leads to much weaker constraints on the EFT. For the EFT considered in (2), the statement of asymptotic causality would allow for a negative $c_1 = -1$ so long as $\mu < 10^{-7}$, i.e., so long as $\Lambda > 15/GM$. Stated differently, around a BH of mass $M = 3\ M_\odot$, the net time delay remains positive for all polarizations even if $c_1 = -1$ and Λ taken to be as low as $10^{-37} M_{\rm Pl}$. Yet we know from positivity bounds that such a situation would be in direct tension with causal and unitary requirements. This illustrates how the statement of asymptotic causality fails to properly diagnose violations of causality. These considerations further show how insisting instead on an unresolvability of the EFT time advance is precisely what is linked with causality considerations in known situations.

Causality in the generic EFT of gravity.—We now generalize the previous argument to more generic gravitational EFTs, and show that the causal requirement $c_1 > 0$ in model (2) could have been drawn by focusing on the high multipole limit. To keep the discussion general, we consider the EFT corrections on the potential to scale as $V \sim \ell^n$ at large ℓ, and extend the definition of μ to $\mu = (GM\Lambda)^{-2m}$, where n and m are integers determined by the leading operators present in the EFT. Again, we write $\omega^2 = \gamma V_{\rm max}$. As $\ell \to \infty$, $V_{\rm max} \to \ell^2/27G^2M^2$ and hence $\omega \sim \ell$. Focusing on the scaling in ℓ, the condition (9) reduces to

$$\ell^{-n+1} < \mu \ll 27^m \ell^{-m}, \quad \text{for } \ell \gg 1, \tag{12}$$

where the lower bound is the resolvability condition, and the upper bound ensures the EFT is under control.

From the condition (12) we see that a resolvable time advance at infinitely large ℓ can only be trusted when $n \geq m + 1$. In this case, for any μ there exists a large enough ℓ, such that the resulting multipole would necessarily violate causality for a particular sign choice of the higher dimensional operator coefficient. This is exactly the case for the even modes in model (2), which have $n = 4$ and $m = 3$. It explains why the sign of c_1 has to be definite. On the other hand the odd modes have $n = 2$, and causality only imposes an upper bound on μ.

This argument can be directly applied to other higher-dim operators. In particular, the corrected Regge-Wheeler-Zerilli equations in the presence of dimension-6 and dimension-8 parity-preserving operators takes a similar form as Eq. (3) [8,9]. Up to field redefinitions, there are two additional dimension-8 operators beside the one in Eq. (2). While one of them is parity violating and is beyond the scope of this Letter, another operator $c_2(\epsilon^{ab}{}_{ef} R_{abcd} R^{efcd})^2$ only affects the odd modes with $V^- \sim \ell^4$. In this case, the odd modes exhibit a time advance when $c_2 < 0$, and the previous argument indicates that causality demands $c_2 > 0$, which again is fully consistent with the causality requirements inferred in Ref. [37] and with the low-energy EFT arising from type II string theory compactification [15,16].

The constraints on the dimension-8 EFT implicitly assume that dimension-6 ones are subdominant, however, up to field redefinitions, the generic EFT of gravity could also include the dimension-6 operator $b_1 R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}$ (see SM [6]). In this case, the EFT corrections are suppressed by $\mu = (GM\Lambda)^{-4}$, with $V \sim \ell^2$ at large ℓ. Performing the same analysis, we find that consistency with causality depends on the sign of the coefficient b_1. For $b_1 = -1$, both modes will always exhibit a resolvable time advance and violate causality whenever $\Lambda < 10^{-11}$ eV $(M_\odot/M)$ (see SM [6]). However, whenever $b_1 = +1$ neither mode presents a time advance. The statement of infrared causality (imposed by consistency and causality of the UV completion) thus implies that a low-energy EFT of the form (1) can only enjoy a standard causal high-energy completion when the coefficient of the Riemann^3 operator is positive or is sufficiently suppressed that it can be ignored as compared to higher order operators. This is precisely consistent with known explicit string theory realizations. Indeed, for maximally supersymmetric and heterotic string theory that coefficient vanishes while it is positive in bosonic string theory [51]. Note, however, that this result is now proven to be generic for any consistent tree level weakly coupled UV completion, independently of the details of the specific realization.

Observability and outlook.—With the growing interests in probing gravity with GWs, our study provides a theoretical prior from causality considerations for all constraints on EFTs of gravity. Remarkably, for the EFT (2), the regime of parameters which was found to be disfavored by the GW events GW151226 [52] and GW170608 [53] in Ref. [10] could have been ruled out on causality considerations alone, assuming a tree level UV completion. This also implies that the current GW observations are not able to test the model (2) against GR as causality priors require the cutoff of this EFT to be bounded by at least $\Lambda \gtrsim 7 \times 10^{-11}$ eV (possibly much higher). We emphasize that the lower bound on Λ is imposed only for the particular dimension-8 model (2). General dimension-8 EFTs usually involve both c_1 and c_2 operators, and there will be no lower bound on Λ from infrared causality considerations if both c_1 and c_2 are positive and if $c_2/c_1 \sim \mathcal{O}(1)$ (see SM [6]), in which case the cutoff of the EFT can be as low (or even lower) as that considered in Ref. [10] and the dimension-8 EFTs could be probed or constrained with the current GW observations without being in conflict with infrared causality. Note that if $c_2 = 0$, and consider BHs with arbitrarily low mass then causality forces c_1/Λ^6 to be arbitrarily close to zero.

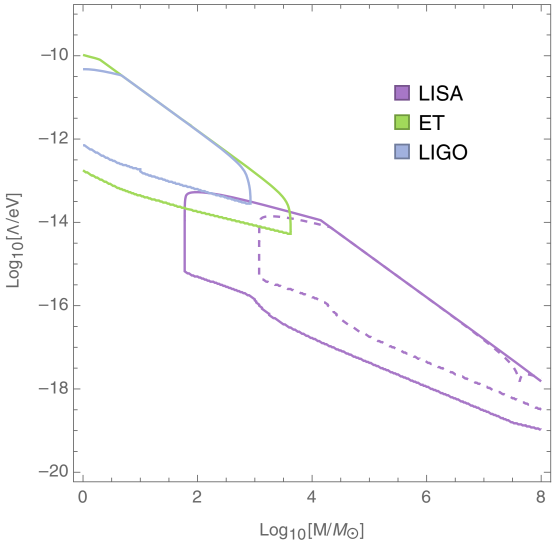

FIG. 3. Observability of generic dimension-6 operators with inspiraling GWs. One could expect to constrain (or observe) dimension-6 operators when they enter at a scale Λ in the enclosed contours. We consider inspiraling GWs from equal mass binaries BHs with total masses shown in the horizontal axis. For LIGO and Einstein Telescope (ET), we assume the binaries are at 300 Mpc, while for LISA, we consider binaries at 3 Gpc (solid line) and 26 Gpc (dashed line).

Moreover, probing the EFT of gravity with GWs is even more promising when dimension-6 operators are included. The dimension-6 operators typically dominate over the dimension-8 ones in the EFT expansion and could contribute to inspiral waveforms at lower post-Newtonian (PN) order. While dimension-6 operators could start contributing to the inspiral waveform at 5PN order [11], 3 orders lower than the dimension-8 operators (see SM [6]), we can still use the constraints obtained in Ref. [10] as a conservative estimation of constraints on the dimension-6 EFT corrections. This implies that GW events like GW151226 and GW170608 could already probe the EFT of gravity with dimension-6 operators for a cutoff $\Lambda \in [10^{-13}, 10^{-12}]$ eV or even a wider range. Future GW detectors like Einstein Telescope and LISA are expected to measure the PN coefficients with fractional accuracies of 10% [54,55] or better. Figure 3 shows the potential detectability of generic dimension-6 operators with future GW detectors. Specifically, we assume the dimension-6 operators are detectable if their corrections on the phase of observed inspiraling GWs, calculated within the EFT validity regime, is greater than $\mathcal{O}(1)$. For a given GW source, the EFT corrections are proportional to Λ^{-4} and accumulate during inspiral. Therefore, the total dephasing could be less than $\mathcal{O}(1)$, if Λ is too large or is so small that there are not enough GW data available within the EFT validity regime, leading to an upper bound and a lower bound on Λ for the dimension-6 operators to be detectable. Remarkably, it is possible to probe the dimension-6 operators of the EFT of gravity while remaining consistent with causality at a cutoff in the range $\Lambda \in [10^{-14}, 10^{-11}]$ eV if we observe binary BHs of 20 $M_\odot$ inspiraling at 300 Mpc with the Einstein Telescope. That range can then be lowered to $\Lambda \in [10^{-17}, 10^{-15}]$ eV if we observe two 10^5 $M_\odot$ BHs inspiraling at 3 Gpc with LISA.

We would like to thank Simon Caron-Huot for useful discussions. The work of A. J. T. and C. d. R. is supported by STFC Grants No. ST/P000762/1 and No. ST/T000791/1. C. d. R. thanks the Royal Society for support at ICL through a Wolfson Research Merit Award. C. d. R. and J. Z. are supported by the European Union Horizon 2020 Research Council Grant No. 724659 MassiveCosmo ERC2016COG. C. d. R. is also supported by a Simons Foundation Grant ID 555326 under the Simons Foundation Origins of the Universe initiative, Cosmology Beyond Einstein's Theory and by a Simons Investigator Grant No. 690508. J. Z. is also supported by scientific research starting Grant No. 118900M061 from University of Chinese Academy of Sciences. A. J. T. thanks the Royal Society for support at ICL through a Wolfson Research Merit Award.

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[1] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[2] J. F. Donoghue, in *Advanced School on Effective Theories* (World Scientific Pub Co Inc., Almunecar, Spain, 1997), arXiv:gr-qc/9512024.
[3] C. P. Burgess, Living Rev. Relativity **7**, 5 (2004).
[4] J. F. Donoghue, in Proceedings of the 6th International School on Field Theory and Gravitation (ISFTG 2012): Petropolis, Rio de Janeiro, Brazil, 2012 [AIP Conf. Proc. **1483**, 73 (2012)].
[5] S. Weinberg, Int. J. Mod. Phys. A **31**, 1630007 (2016).
[6] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.131102 for the technical details related to some conventions and derivations used in the Letter, which includes Refs. [7–20].
[7] S. Endlich, V. Gorbenko, J. Huang, and L. Senatore, J. High Energy Phys. 09 (2017) 122.
[8] V. Cardoso, M. Kimura, A. Maselli, and L. Senatore, Phys. Rev. Lett. **121**, 251105 (2018).
[9] C. de Rham, J. Francfort, and J. Zhang, Phys. Rev. D **102**, 024079 (2020).
[10] N. Sennett, R. Brito, A. Buonanno, V. Gorbenko, and L. Senatore, Phys. Rev. D **102**, 044056 (2020).
[11] M. Accettulli Huber, A. Brandhuber, S. De Angelis, and G. Travaglini, Phys. Rev. D **103**, 045015 (2021).
[12] P. A. Cano, K. Fransen, T. Hertog, and S. Maenaut, Phys. Rev. D **105**, 024064 (2022).
[13] C. de Rham and A. J. Tolley, Phys. Rev. D **102**, 084048 (2020).
[14] C. Y. R. Chen, C. de Rham, A. Margalit, and A. J. Tolley, J. High Energy Phys. 03 (2022) 025.

[15] D. J. Gross and E. Witten, Nucl. Phys. **B277**, 1 (1986).
[16] R. Metsaev and A. Tseytlin, Phys. Lett. B **185**, 52 (1987).
[17] C. de Rham and A. J. Tolley, Phys. Rev. D **101**, 063518 (2020).
[18] M. Ruhdorfer, J. Serra, and A. Weiler, J. High Energy Phys. 05 (2020) 083.
[19] P. A. Cano and A. Ruipérez, J. High Energy Phys. 05 (2019) 189; 03 (2020) 187(E).
[20] R. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Phys. Rev. D **103**, 122002 (2021).
[21] Motivated by arguments presented in Ref. [22], when arising from a weakly coupled tree-level completion, the scale of the dimension-6 operators is related to the mass of a tower of higher-spin states, which to date has not been observed. Such arguments have motivated focusing instead on dimension-8 operators. However, it should be pointed out that the precise same arguments equally apply to dimension-8 operators and as an EFT, dimension-8 operators seldom appear without the emergence of dimension-6 operators at the same scale, unless supersymmetry were preserved.
[22] X. O. Camanho, J. D. Edelstein, J. Maldacena, and A. Zhiboedov, J. High Energy Phys. 02 (2016) 020.
[23] A. Brandhuber and G. Travaglini, J. High Energy Phys. 01 (2020) 010.
[24] W. T. Emond and N. Moynihan, J. High Energy Phys. 12 (2019) 019.
[25] B. Bellazzini, C. Cheung, and G. N. Remmen, Phys. Rev. D **93**, 064076 (2016).
[26] Z. Bern, D. Kosmopoulos, and A. Zhiboedov, J. Phys. A **54**, 344002 (2021).
[27] I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22**, 343 (1980).
[28] R. Lafrance and R. C. Myers, Phys. Rev. D **51**, 2584 (1995).
[29] G. Shore, Nucl. Phys. **B646**, 281 (2002).
[30] T. J. Hollowood and G. M. Shore, Phys. Lett. B **655**, 67 (2007).
[31] T. J. Hollowood and G. M. Shore, Nucl. Phys. **B795**, 138 (2008).
[32] T. J. Hollowood and G. M. Shore, J. High Energy Phys. 12 (2008) 091.
[33] K. Benakli, S. Chapman, L. Darmé, and Y. Oz, Phys. Rev. D **94**, 084026 (2016).
[34] G. Goon and K. Hinterbichler, J. High Energy Phys. 02 (2017) 134.
[35] M. Accettulli Huber, A. Brandhuber, S. De Angelis, and G. Travaglini, Phys. Rev. D **102**, 046014 (2020).
[36] J. D. Edelstein, R. Ghosh, A. Laddha, and S. Sarkar, J. High Energy Phys. 09 (2021) 150.
[37] A. Gruzinov and M. Kleban, Classical Quantum Gravity **24**, 3521 (2007).
[38] T. J. Hollowood and G. M. Shore, J. High Energy Phys. 03 (2016) 129.
[39] Owing to the spherical symmetry of the background, there is no dependence on the second spherical harmonic quantum number m.
[40] Violating $\omega^2 < V_{\rm max}$ does not indicate a failure of the EFT in any way, it simply indicates that the WKB boundary conditions used to derive (6) are no longer valid. In principle we could accommodate for waves with $\omega^2 \geq V_{\rm max}$, but such a situation is unlikely to have resolvable time advance within the validity regime of the EFT (see SM [6]) and is not relevant to our argument.
[41] Noting that $V^+_{\rm GR} \sim \ell^2$ while $V^+ \sim \ell^4$, one may worry that the EFT corrections on the Regge-Wheeler-Zerilli equations might no longer remain perturbative at large ℓ. However, this is not necessary to be the case for having causal violating time advance. The time advance is resolvable as long as $-\delta T_\ell \sim \mu\ell^2$ is larger than $1/\omega \sim 1/\ell$, in which case the fractional EFT corrections $V^+/V^+_{\rm GR} \sim \mu\ell^2 > 1/\ell$ can remain small as $\ell \to \infty$.
[42] T. A. Thompson *et al.*, Science **366**, 637 (2019).
[43] R. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Astrophys. J. Lett. **896**, L44 (2020).
[44] T. Jayasinghe *et al.*, Mon. Not. R. Astron. Soc. **504**, 2577 (2021).
[45] T. N. Pham and T. N. Truong, Phys. Rev. D **31**, 3027 (1985).
[46] B. Ananthanarayan, D. Toublan, and G. Wanders, Phys. Rev. D **51**, 1093 (1995).
[47] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, J. High Energy Phys. 10 (2006) 014.
[48] G. M. Shore, Nucl. Phys. **B460**, 379 (1996).
[49] G. M. Shore, Nucl. Phys. **B605**, 455 (2001).
[50] T. J. Hollowood, G. M. Shore, and R. J. Stanley, J. High Energy Phys. 08 (2009) 089.
[51] G. D'Appollonio, P. Di Vecchia, R. Russo, and G. Veneziano, J. High Energy Phys. 05 (2015) 144.
[52] B. P. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Phys. Rev. Lett. **116**, 241103 (2016).
[53] B. P. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Astrophys. J. Lett. **851**, L35 (2017).
[54] C. K. Mishra, K. G. Arun, B. R. Iyer, and B. S. Sathyaprakash, Phys. Rev. D **82**, 064010 (2010).
[55] K. G. Arun, B. R. Iyer, M. S. S. Qusailah, and B. S. Sathyaprakash, Classical Quantum Gravity **23**, L37 (2006).