m_b at m_H: The Running Bottom Quark Mass and the Higgs Boson

Javier Aparisi, Juan Fuster, Adrián Irles, Germán Rodrigo, Marcel Vos,* and Hitoshi Yamamoto†
Instituto de Física Corpuscular, CSIC-University of Valencia, Calle del Catedrático José Beltrán 2, 46980 Paterna Valencia, Spain

André Hoang and Christopher Lepenik
Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria

Michael Spira
PSI Villigen, Forschungsstrasse 111, 5232 Villigen PSI, Switzerland

Seidai Tairafune and Ryo Yonamine
Department of Physics, Tohoku University, 6-3, Aoba, Aramaki, Aoba-ku, Sendai 980-8578, Japan

(Received 1 November 2021; accepted 10 February 2022; published 21 March 2022)

We present a new measurement of the bottom quark mass in the $\overline{\rm MS}$ scheme at the renormalization scale of the Higgs boson mass from measurements of Higgs boson decay rates at the LHC: $m_b(m_H) = 2.60^{+0.36}_{-0.31}$ GeV. The measurement has a negligible theory uncertainty and excellent prospects to improve at the HL-LHC and a future Higgs factory. Confronting this result and $m_b(m_b)$ from low-energy measurements and $m_b(m_Z)$ from Z-pole data, with the prediction of the scale evolution of the renormalization group equations, we find strong evidence for the "running" of the bottom quark mass.

DOI: 10.1103/PhysRevLett.128.122001

Introduction.—Quark masses are renormalized and scheme-dependent parameters of the standard model (SM) Lagrangian. Their values must be determined experimentally through the comparison of measurements of physical observables sensitive to the mass to SM predictions in quantum chromodynamics (QCD) beyond leading-order accuracy. In the most popular renormalization scheme, the modified minimal subtraction scheme or $\overline{\rm MS}$ scheme, the strong coupling $\alpha_s(\mu)$ and the quark masses $m_q(\mu)$ are "running constants" that depend on the dimensionful renormalization scale μ.

QCD yields a precise prescription for the scale evolution: Given a value for a quark mass at a reference scale, its value can be determined at any other scale using the renormalization group equation (RGE). Determinations of the RGE for the running quark masses have by now reached the five-loop [$\mathcal{O}(\alpha_s^5)$] level [1–3] and software packages such as RUNDEC [4] and REVOLVER [5] provide access to state-of-the-art renormalization evolution and scheme conversions.

Performing several measurements at different energy scales, the renormalization scale dependence of the strong coupling and the quark masses in the $\overline{\rm MS}$ scheme can be tested experimentally. For each measurement, one can identify a typical scale of the physical process, where high-order logarithmic corrections related to renormalization group invariance are resummed, and predictions yield nicely behaved perturbative series. A large number of determinations over a broad range of energies characterizes the evolution of the strong coupling $\alpha_s(\mu)$ [6]. Experiments have also found evidence for the "running" of $\overline{\rm MS}$ quark masses for the charm quark at HERA [7] and have studied the scale evolution of the top quark at the LHC [8].

The most precise measurements of the mass of the bottom quark are performed at relatively low energy. The "world average" is given by the Particle Data Group as follows:

$$m_b(m_b) = 4.18^{+0.03}_{-0.02}\ \text{GeV}, \tag{1}$$

where the reference value of the bottom mass is quoted in the $\overline{\rm MS}$ scheme at a scale given by the mass itself [9]. Measurements at the scale of the Z-boson mass have been performed by the LEP experiments and using SLD data [10–17]. We use the following average [18] of the most precise measurements for $m_b(m_Z)$:

$$m_b(m_Z) = 2.82 \pm 0.28\ \text{GeV}. \tag{2}$$

Bottom quark mass from Higgs decay.—The discovery of the Higgs boson [48,49] and the observation of Higgs boson decay to bottom quark pairs [50,51] at the LHC

[52,53] enables an entirely new measurement of the bottom quark mass. The ATLAS [54] and CMS [55] experiments have characterized the product of Higgs boson production and decay rates in many combinations of production processes and Higgs decay channels. The measurements of the $H \to b\bar{b}$ branching ratio combining the VH, $t\bar{t}H$ associated production modes and the vector-boson-fusion mode have achieved a precision of approximately 40% in Run 1 [56] and 20% in Run 2 [57–60].

We focus on the ratio $\mathrm{BR}(H \to b\bar{b})/\mathrm{BR}(H \to ZZ)$ of the branching ratios to bottom quarks and to Z bosons. The LHC experiments present this result in terms of the signal strength $\mu = \mathrm{BR}_{\mathrm{expt}}/\mathrm{BR}_{\mathrm{SM}}$ using the SM prediction of $\mathrm{BR}(H \to b\bar{b})/\mathrm{BR}(H \to ZZ) = 22.0 \pm 0.5$. The bottom quark mass measurement presented in this Letter is based on a preliminary result by ATLAS [60] using 139 fb^{-1} at $\sqrt{s} = 13$ TeV,

$$\mu^{bb}/\mu^{ZZ} = 0.87^{+0.22}_{-0.17}(\mathrm{stat})^{+0.18}_{-0.12}(\mathrm{syst}) = 0.87^{+0.28}_{-0.21}, \quad (3)$$

and a result by CMS [57] based on 35 fb^{-1}:

$$\mu^{bb}/\mu^{ZZ} = 0.84^{+0.27}_{-0.21}(\mathrm{stat})^{+0.26}_{-0.17}(\mathrm{stat}) = 0.84^{+0.37}_{-0.27}. \quad (4)$$

Dependence of Higgs boson decay rates on m_b: In the limit $m_b \ll m_H$, the partial decay width can be written in the form

$$\Gamma[H \to b\bar{b}] = \frac{3G_F m_H}{4\sqrt{2}\pi} m_b(\mu)^2 (1 + \delta_{\mathrm{EW}}) \times [1 + \delta_{\mathrm{QCD}} + \delta_t + \delta_{\mathrm{mix}}], \quad (5)$$

where G_F denotes the Fermi constant, δ_{QCD} the QCD corrections related to the scalar correlator, δ_t the QCD corrections due to the interference with $H \to gg$ diagrams that start to contribute at next-to-next-to-leading order (NNLO), δ_{EW} the electroweak (EW) corrections, and finally δ_{mix} the mixed QCD EW corrections. The decay width has a quadratic dependence on the bottom quark mass and can be precisely predicted. In particular, the QCD corrections δ_{QCD} are known up to N^{4}LO [61–74], the interference term δ_t at NNLO [75–77], the EW corrections δ_{EW} at NLO [78–81], and finally the mixed corrections δ_{mix} at two-loop order [82–87].

The Higgs boson mass is the characteristic dynamical scale for the decay rate into bottom quarks. A measurement of the $H \to b\bar{b}$ partial width therefore naturally provides a measurement of the bottom quark mass at the renormalization scale of the Higgs boson mass [61,62]. This point can be illustrated by considering the convergence of the perturbative series. When the renormalization scale $\mu = m_H$ is adopted for the strong coupling and the bottom quark mass [we use $m_b(m_b) = 4.18$ GeV and $\alpha_s(m_Z) = 0.1179$ as input and obtain $m_b(m_H) = 2.790$ GeV, $\alpha_s(m_H) = 0.1125$, and $\alpha_s(m_b(m_b)) = 0.2245$ using five-loop RGEs for five active flavors], the leading QCD series for the $H \to b\bar{b}$ partial width (in the expansion in m_b^2/m_H^2) takes the following form:

$$1 + \delta_{\mathrm{QCD}} = 1 + 0.2030 + 0.0374 + 0.0019 - 0.0014. \quad (6)$$

The successive loop corrections listed as separate terms show excellent convergence. In contrast, using $\mu = m_b$, the leading perturbation series adopts the form

$$1 + \delta_{\mathrm{QCD}} = 1 - 0.5665 + 0.0586 + 0.1475 - 0.1274, \quad (7)$$

which shows very poor convergence behavior and has large perturbative uncertainties due to powers of the large logarithm $\ln(m_H/m_b)$. These large logarithmic terms are resummed to all orders in Eq. (6), which explains its much better behavior. This property supports the idea that the $H \to b\bar{b}$ partial width provides a measurement of the bottom quark mass at the renormalization scale m_H. A more detailed discussion is found in the Supplemental Material [18] provided with this Letter.

Numerical predictions: The dependence of the $H \to b\bar{b}$ partial width on the bottom quark mass is obtained with HDECAY [88,89]. The calculation of the $H \to b\bar{b}$ decay accounts for N^{4}LO corrections in QCD and includes NLO EW corrections. The full bottom quark mass effects are taken into account up to NLO and logarithmic ones up to NNLO [90]. Version 6.61 of the code is used, where the bottom quark $\overline{\mathrm{MS}}$ mass can be supplied at the scale $\mu = m_H$ of the decay process. We use $m_H = 125.1$ GeV and $\alpha_s(m_Z) = 0.1179$ [Particle Data Group (PDG) world average] throughout this Letter. For a bottom quark mass $m_b(m_H) = 2.79$ GeV [corresponding to $m_b(m_b) = 4.18$ GeV], HDECAY predicts a partial width of 2.363 MeV.

A precise prediction for the $H \to ZZ$ partial width is obtained with PROPHECY4F [92,93] (version 3.0 [94]). This package includes the full QCD and EW NLO corrections to the Higgs boson decay width to four fermions, the interference contributions between different WW and ZZ channels, and all off-shell effects of intermediate W and Z bosons. The partial width $\Gamma(H \to ZZ)$ for our choice of parameters is 0.109 MeV.

The ratio of the $b\bar{b}$ and ZZ partial widths obtained for $m_b(m_b) = 4.18$ GeV is 21.76, in agreement within the uncertainty with the reference value of 22.0 ± 0.5 from Ref. [95] used by ATLAS and CMS. The two results are fully compatible once the different input values for the Higgs boson mass are accounted for.

For our numerical analysis, the dependence of the ratio $\Gamma^{b\bar{b}}/\Gamma^{ZZ}$ on the bottom quark mass $m_b(m_H)$ is parametrized with a polynomial. The uncertainty in the fitted mass value due to the parametrization is below the per mille level over the mass range of interest. Variations of the functional form and fit range lead to negligible uncertainties.

Impact of theory uncertainties: The theory uncertainty on the bottom quark mass extraction from the Higgs coupling measurement due to missing higher orders is estimated following Ref. [95] (and earlier work in Refs. [96,97]). The uncertainties on the predicted ratio μ^{bb}/μ^{ZZ} due to missing higher orders are estimated by varying the renormalization scale by factors between 2 and 0.5. Independent variations [98] of the scales for α_s and m_b yield a variation of ~0.3%. EW corrections beyond NLO are estimated to be ~0.5% on both partial widths [95].

The parametric uncertainty from α_s is estimated by propagating the 0.001 uncertainty on the PDG world average for $\alpha_s(m_Z)$ explicitly in HDECAY [99], which shifts the ratio of branching fractions by 0.2%.

The parametric uncertainty from the Higgs boson mass is estimated by varying the Higgs boson mass by ± 240 MeV around the nominal value $m_H = 125.1$ GeV and recalculating the partial Higgs boson decay width to Z-boson pairs with PROPHECY4F. Both the central value of the Higgs boson mass and the variations are based on the ATLAS + CMS Run 1 combination of Higgs boson mass measurements in the $\gamma\gamma$ and ZZ decay channels [100]. This leads to a variation of Γ_{ZZ} by 3% and is the dominant uncertainty on the ratio.

The linear sum of the several contributions yields a total theory uncertainty of 4.4% on the Γ^{bb}/Γ^{ZZ}, which yields an uncertainty of 60 MeV on $m_b(m_H)$. At the current experimental precision, the uncertainty on the theoretical prediction of the ratio is negligible.

Extraction of $m_b(m_H)$ from Higgs rates: We extract the bottom quark mass from the measurements of Eqs. (3) and (4). The two results [101] are combined using the CONVINO package [102], taking into account correlations among the asymmetric systematic uncertainties. The resulting value of the bottom quark mass is

$$m_b(m_H) = 2.60^{+0.36}_{-0.31}\ \text{GeV}. \tag{8}$$

This is the main result of this Letter.

The running bottom quark mass.—The new measurements of $m_b(m_H)$ based on ATLAS and CMS measurements of Higgs decay rates (indicated with open red markers) and the average of both measurements (red star) are presented together with the existing results for the bottom quark mass in Fig. 1. The PDG world average of $m_b(m_b)$ is indicated with a green star and the measurements of the bottom quark $\overline{\text{MS}}$ mass at m_Z by the LEP experiments and SLD with blue, open markers.

The measurements at different scales are connected by the predicted RG evolution of $m_b(Q)$ in QCD. The evolution of the PDG world average for $m_b(m_b)$ to higher scales is given by the black curve, using the REVOLVER code [5] at five-loop precision. The dark gray error band indicates the uncertainty on $m_b(Q)$ within the SM, with the dominant uncertainties stemming from the parametric

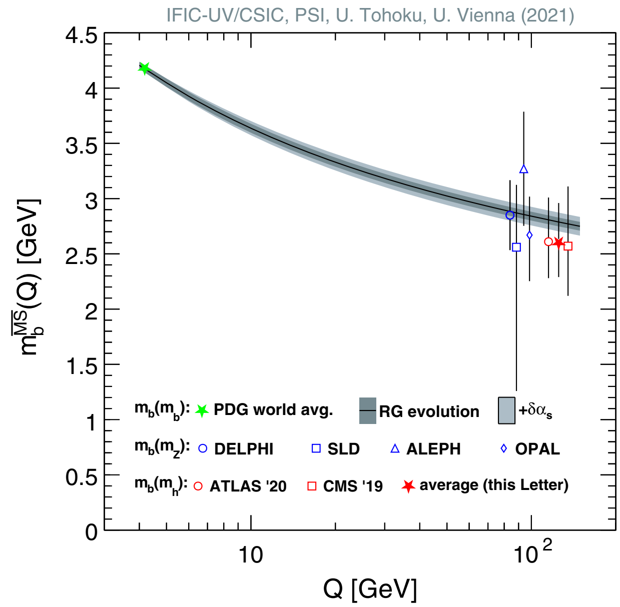

FIG. 1. The scale evolution of the bottom quark $\overline{\text{MS}}$ mass. The measurements include the PDG world average for $m_b(m_b)$ determined at a typical scale of the bottomonium mass, the measurements of $m_b(m_Z)$ from jet rates at the Z pole at the LEP and SLC, and the measurement of $m_b(m_H)$ from Higgs boson branching fractions. The prediction of the evolution of the mass is calculated from the world average for $m_b(m_b)$ at five-loop precision with REVOLVER [5]. The inner dark gray error band includes the effect of missing higher orders and the parametric uncertainties from $m_b(m_b)$ and α_s from the PDG averages. The outer band with a lighter shading includes additionally the effect of a ± 0.004 variation of $\alpha_s(m_Z)$.

$m_b(m_b)$ and α_s uncertainties [6]. The impact of higher-order uncertainties estimated as half the difference between the three-loop and four-loop prediction is negligible in comparison.

The measurements at high scales are in good agreement with the evolution predicted by the SM.

The anomalous mass dimension: The QCD scale evolution of the $\overline{\text{MS}}$ quark masses $m_q(\mu)$ can be written in terms of the anomalous dimension γ_m and the scale-dependent strong coupling $\alpha_s(\mu)$:

$$\frac{\partial m_q(\mu)}{\partial \log(\mu^2)} = \gamma_m[\alpha_s(\mu)]m_q(\mu). \tag{9}$$

Focusing on the first term in the expansion $\gamma_m[\alpha_s] = \gamma_0(\alpha_s/\pi) + \mathcal{O}(\alpha_s^2)$, we obtain, in leading-log (LL) approximation:

$$\gamma_0 = -\beta_0 \log\left(\frac{m_q(\mu^2)}{m_q(\mu_0^2)}\right) / \log\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right). \tag{10}$$

In the SM, $\gamma_0 = -1$ and β_0 is given as a function of the (effective) number of flavors n_f. Adopting the five-flavor scheme, $\beta_0 = (33 - 2n_f)/12 = 23/12$. The LL approximation is expected to be accurate to approximately 12%, where the uncertainty is given by the size of the next-to-leading-log correction within the five-flavor scheme.

Combining the values of $m_b(m_b)$, $m_b(m_Z)$, and $m_b(m_H)$ of Eqs. (1), (2), and (8), one can determine γ_0 experimentally. We perform a χ^2 minimization to obtain the following best-fit value for the ratio

$$\gamma_0/\beta_0 = -0.64 \pm 0.12(\mathrm{expt}) \pm 0.08(\mathrm{theor}) \pm 0.03(\alpha_s). \tag{11}$$

The uncertainty due to α_s is evaluated by propagating the experimental uncertainties on $\alpha_s(m_b)$, $\alpha_s(m_Z)$, and $\alpha_s(m_H)$. To reduce the SM bias, a conservative uncertainty of 0.004 is assigned to the α_s values at m_Z and m_H. This covers the envelope of experimental measurements of α_s at high scale from deep-inelastic scattering and parton distribution function fits as well as EW precision fits based on the preaveraging quoted in Ref. [6].

With $\beta_0 = 23/12$, we find the following value for the anomalous mass dimension:

$$\gamma_0 = -1.23 \pm 0.22(\mathrm{expt}) \pm 0.14(\mathrm{theor}) \pm 0.06(\alpha_s), \tag{12}$$

in good agreement with the SM result.

The leading-log approximation is found to be sufficiently accurate for the current measurement precision. A combined analysis of the evolution of the strong coupling and the bottom quark mass can disentangle the running of α_s and m_b, and may be an interesting diagnostic tool for new physics effects that impact their scale evolution in different ways.

Testing the running hypothesis: With the independent determinations of the bottom quark mass at different scales, we can test the hypothesis of the running of the bottom quark mass. To avoid a SM bias, we again relax the assumption that the evolution of the strong coupling is given by the RGE prediction, increasing the uncertainty on α_s at high scales to 0.004. The impact of this additional uncertainty is shown as a second, light gray band in Fig. 1. Even with this additional uncertainty, the total uncertainty on the prediction for $m_b(m_H)$ is 90 MeV, still more than 3 times smaller than the experimental uncertainty.

We test the running hypothesis with the following parametrization adapted from Ref. [8]:

$$m(\mu; x, m_b(m_b)) = x[m_b^{\mathrm{RGE}}(\mu, m_b(m_b)) - m_b(m_b)] + m_b(m_b), \tag{13}$$

where $m_b^{\mathrm{RGE}}(\mu, m_b(m_b))$ describes the RGE evolution expected in the SM for a reference mass $m_b(m_b)$, and x is a multiplicative factor that adjusts the scale dependence, interpolating smoothly between the no-running scenario ($x = 0$) and the SM ($x = 1$).

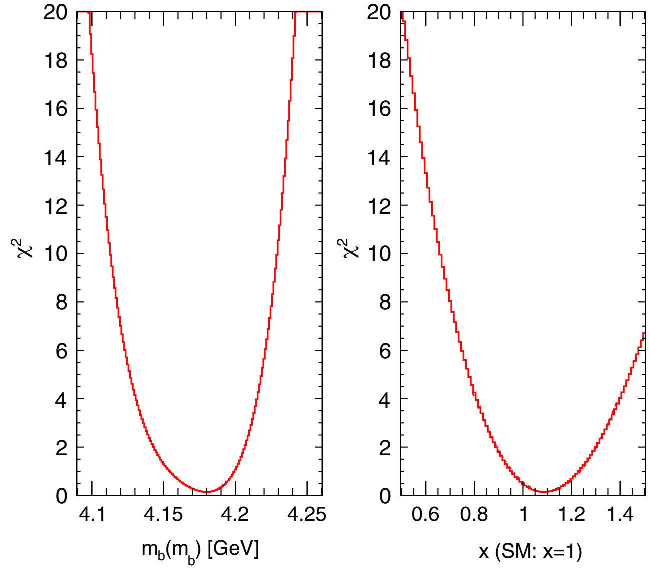

FIG. 2. The χ^2 of the fit of Eq. (13) to the measurements of $m_b(m_b)$, $m_b(m_Z)$, and $m_b(m_H)$, as a function of the reference bottom quark mass $m_b(m_b)$ and the factor x that multiplies the RGE evolution to higher scale. The factor x interpolates smoothly between the no-running scenario ($x = 0$) and the RGE evolution predicted by QCD ($x = 1$).

We fit the predicted scale evolution of the bottom quark $\overline{\mathrm{MS}}$ mass m with a χ^2 minimization [103] using Eq. (13) and the averages of $m_b(m_b)$, $m_b(m_Z)$, and $m_b(m_H)$ given in Eqs. (1), (2), and (8). The resulting χ^2 values are shown in Fig. 2 as a function of the two fit parameters. The best-fit value for the reference mass is $m_b(m_b) = 4.18^{+0.03}_{-0.02}$ GeV, compatible with the PDG world average. The fit yields $x = 1.08 \pm 0.15(\mathrm{expt}) \pm 0.05(\alpha_s)$, where the first uncertainty corresponds to a propagation of the uncertainty on the mass measurements and the second to an uncertainty of ± 0.004 on the value of $\alpha_s(m_Z)$ used in the RG evolution. The best-fit value of x is compatible with the SM prediction ($x = 1$), within 1σ and differs by nearly 7 standard deviations from the no-running scenario ($x = 0$). A fit of $m_b(m_b)$ and $m_b(m_Z)$, without the Higgs data, yields a value of x of 1.03 ± 0.21, just below 5 standard deviations.

Discussion and outlook.—Caveat: When the Higgs decay rates are used for a determination of the bottom quark mass, we must assume that physics beyond the SM has a negligible impact. The procedure followed by the ATLAS and CMS experiments is quite robust against certain new physics effects. The contribution of unknown "invisible decays" to the Higgs width cancels in the ratio and other assumptions, e.g., on the Higgs boson production cross sections, can be tested to good precision. A shift of the bottom quark Yukawa coupling (and none of the other Higgs couplings) would, however, lead to a bias in the mass measurement. The results in this Letter are strictly valid only for a SM bottom quark Yukawa coupling.

Prospects: Future improvements of the Higgs branching fraction measurements are expected to rapidly reduce the uncertainties of this method. Projections for the HL-LHC [104] envisage a measurement of $\mathrm{BR}(H \to b\bar{b})$ with a precision of 4.4%, reducing the experimental uncertainty on $m_b(m_H)$ to ± 60 MeV.

The recoil mass analysis at a future electron-positron "Higgs factory" can reach subpercent precision for Higgs boson couplings [105–107], with minimal assumptions on the total width or the production rates. The ratio μ^{bb}/μ^{WW} of the Higgs branching fractions (which is preferred over the μ^{bb}/μ^{ZZ} ratio because of the larger branching fraction) is expected to be measured with 0.86% precision for the 250 GeV stage of the International Linear Collider (ILC) and 0.47% for the complete $250 + 500$ GeV program [108,109] corresponding to an uncertainty on $m_b(m_H)$ of ± 12 and ± 6 MeV, respectively. Future e^+e^- colliders furthermore offer opportunities to improve the precision of $m_b(m_Z)$, either with a dedicated high-luminosity run at the Z pole or with radiative-return events, and to extend the analysis to $m_b(250\ \mathrm{GeV})$ [110].

Summary: In this Letter, we have presented a new method to determine the bottom quark mass from the Higgs boson decay rate to bottom quarks and have used it to perform the first measurement of the bottom quark $\overline{\mathrm{MS}}$ mass at the renormalization scale of the Higgs boson mass. Combining ATLAS and CMS Run 2 results, we obtain $m_b(m_H) = 2.60^{+0.36}_{-0.31}$ GeV, in good agreement with the value $2.79^{+0.03}_{-0.02}$ GeV expected from evolving the world average for $m_b(m_b)$ to the Higgs mass scale. The extraction of the mass from Higgs decay rates has several advantages over previous analyses. The scale at which the bottom quark mass is measured is unambiguously identified with the Higgs boson mass. Theory uncertainties due to higher-order effects and the impact of the running strong coupling are negligible at the current precision, and the HL-LHC and a future Higgs factory offer excellent prospects to reduce the experimental uncertainties.

Combining our result with the world average for $m_b(m_b)$ and the determination of $m_b(m_Z)$ by the LEP and SLC experiments, we can test the running of the bottom quark mass. The observed scale evolution is compatible within errors with the RGE evolution predicted in QCD, and the data strongly disfavor the no-running scenario.

The idea for this analysis goes back to the LEP days and was revived in a DESY-IFIC workshop organized by K. Lipka. We thank Junping Tian for his inputs on the ILC potential and Stefano Catani for valuable comments. The authors acknowledge support from Projects No. FPA2015-65652-C4-3-R, No. PID2020-114473GB-I00, and No. PGC2018-094856-B-100 (MCIN/AEI/10.13039/501100011033), support from the U. Valencia and CSIC for H. Y., Grants No. PROMETEO/2021/071, No. PROMETEO-2018/060, and No. CIDEGENT/2020/21 (Generalitat Valenciana), iLINK Grant No. LINKB20065 (CSIC), and the FWF Austrian Science Fund Project No. P28535-N27 and the FWF Austrian Science Fund Doctoral Program "Particles and Interactions" No. W1252-N27.

[*]marcel.vos@ific.uv.es
[†]On leave from Tohoku University, Sendai, Japan.

[1] J. A. M. Vermaseren, S. A. Larin, and T. van Ritbergen, Phys. Lett. B **405**, 327 (1997).
[2] K. G. Chetyrkin, Phys. Lett. B **404**, 161 (1997).
[3] P. A. Baikov, K. G. Chetyrkin, and J. H. Kühn, J. High Energy Phys. 10 (2014) 076.
[4] F. Herren and M. Steinhauser, Comput. Phys. Commun. **224**, 333 (2018).
[5] A. H. Hoang, C. Lepenik, and V. Mateu, Comput. Phys. Commun. **270**, 108145 (2022).
[6] P. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[7] A. Gizhko *et al.*, Phys. Lett. B **775**, 233 (2017).
[8] CMS Collaboration, Phys. Lett. B **803**, 135263 (2020).
[9] Throughout this Letter, we define the $\overline{\mathrm{MS}}$ bottom quark mass in the $n_f = 5$ flavor scheme.
[10] P. Abreu *et al.* (DELPHI Collaboration), Phys. Lett. B **418**, 430 (1998).
[11] G. Rodrigo, A. Santamaria, and M. S. Bilenky, Phys. Rev. Lett. **79**, 193 (1997).
[12] K. Abe *et al.* (SLD Collaboration), Phys. Rev. D **59**, 012002 (1998).
[13] A. Brandenburg, P. N. Burrows, D. Muller, N. Oishi, and P. Uwer, Phys. Lett. B **468**, 168 (1999).
[14] R. Barate *et al.* (ALEPH Collaboration), Eur. Phys. J. C **18**, 1 (2000).
[15] G. Abbiendi *et al.* (OPAL Collaboration), Eur. Phys. J. C **21**, 411 (2001).
[16] J. Abdallah *et al.* (DELPHI Collaboration), Eur. Phys. J. C **46**, 569 (2006).
[17] J. Abdallah *et al.* (DELPHI Collaboration), Eur. Phys. J. C **55**, 525 (2008).
[18] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.122001 for a discussion of the previous measurements of $m_b(m_Z)$ and of the natural renormalization scale for the $H \to b\bar{b}$ decay, which includes Refs. [83–111]. Details of the averaging method are given in the Supplemental Material
[19] S. Narison, Phys. Lett. B **802**, 135221 (2020).
[20] C. Peset, A. Pineda, and J. Segovia, J. High Energy Phys. 09 (2018) 167.
[21] Y. Kiyo, G. Mishima, and Y. Sumino, Phys. Lett. B **752**, 122 (2016); **772**, 878(E) (2017).
[22] A. A. Penin and N. Zerf, J. High Energy Phys. 04 (2014) 120.
[23] A. Alberti, P. Gambino, K. J. Healey, and S. Nandi, Phys. Rev. Lett. **114**, 061802 (2015).
[24] M. Beneke, A. Maier, J. Piclum, and T. Rauh, Nucl. Phys. **B891**, 42 (2015).

[25] B. Dehnadi, A. H. Hoang, and V. Mateu, J. High Energy Phys. 08 (2015) 155.
[26] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **88**, 056011 (2013).
[27] S. Bodenstein, J. Bordes, C. A. Dominguez, J. Penarrocha, and K. Schilcher, Phys. Rev. D **85**, 034003 (2012).
[28] A. Laschka, N. Kaiser, and W. Weise, Phys. Rev. D **83**, 094002 (2011).
[29] K. G. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser, and C. Sturm, Phys. Rev. D **80**, 074010 (2009).
[30] A. Bazavov, C. Bernard, N. Brambilla, N. Brown, C. DeTar *et al.* (Fermilab Lattice, MILC, and TUMQCD Collaborations), Phys. Rev. D **98**, 054517 (2018).
[31] B. Colquhoun, R. J. Dowdall, C. T. H. Davies, K. Hornbostel, and G. P. Lepage, Phys. Rev. D **91**, 074514 (2015).
[32] F. Bernardoni, B. Blossier, J. Bulava, M. D. Morte, P. Fritzsch, N. Garron, A. Gérardin, J. Heitger, G. von Hippel, H. Simma, and R. Sommer, Phys. Lett. B **730**, 171 (2014).
[33] A. J. Lee, C. Monahan, R. R. Horgan, C. T. H. Davies, R. J. Dowdall, and J. Koponen (HPQCD Collaboration), Phys. Rev. D **87**, 074018 (2013).
[34] P. Dimopoulos *et al.* (ETM Collaboration), J. High Energy Phys. 01 (2012) 046.
[35] S. Aoki *et al.* (Flavour Lattice Averaging Group), Eur. Phys. J. C **80**, 113 (2020).
[36] H. Abramowicz *et al.* (H1, ZEUS Collaborations), Eur. Phys. J. C **78**, 473 (2018).
[37] C. Schwanda *et al.* (Belle Collaboration), Phys. Rev. D **78**, 032016 (2008).
[38] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **81**, 032003 (2010).
[39] M. S. Bilenky, G. Rodrigo, and A. Santamaria, Nucl. Phys. **B439**, 505 (1995).
[40] M. Bilenky, S. Cabrera, J. Fuster, S. Marti, G. Rodrigo, and A. Santamaria, Phys. Rev. D **60**, 114006 (1999).
[41] G. Rodrigo, M. S. Bilenky, and A. Santamaria, Nucl. Phys. **B554**, 257 (1999).
[42] W. Bernreuther, A. Brandenburg, and P. Uwer, Phys. Rev. Lett. **79**, 189 (1997).
[43] A. Brandenburg and P. Uwer, Nucl. Phys. **B515**, 279 (1998).
[44] P. Nason and C. Oleari, Phys. Lett. B **407**, 57 (1997).
[45] P. Nason and C. Oleari, Nucl. Phys. **B521**, 237 (1998).
[46] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, and G. Heinrich, Phys. Rev. Lett. **100**, 172001 (2008).
[47] R. Nisius, SoftwareX **11**, 100468 (2020).
[48] ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012).
[49] CMS Collaboration, Phys. Lett. B **716**, 30 (2012).
[50] ATLAS Collaboration, Phys. Lett. B **786**, 59 (2018).
[51] CMS Collaboration, Phys. Rev. Lett. **121**, 121801 (2018).
[52] O. Bruning *et al.*, Report No. CERN-2004-003, 2004, 10.5170/CERN-2004-003-V-1.
[53] L. Evans and P. Bryant, J. Instrum. **3**, S08001 (2008).
[54] ATLAS Collaboration, J. Instrum. **3**, S08003 (2008).
[55] CMS Collaboration, J. Instrum. **3**, S08004 (2008).
[56] ATLAS and CMS Collaborations, J. High Energy Phys. 08 (2016) 045.
[57] CMS Collaboration, Eur. Phys. J. C **79**, 421 (2019).
[58] ATLAS Collaboration, Phys. Rev. D **101**, 012002 (2020).
[59] CMS Collaboration, Report No. CMS-PAS-HIG-19-005, 2020, https://cds.cern.ch/record/2706103.
[60] ATLAS Collaboration, Report No. ATLAS-CONF-2020-027, 2020, https://cds.cern.ch/record/2725733.
[61] E. Braaten and J. P. Leveille, Phys. Rev. D **22**, 715 (1980).
[62] N. Sakai, Phys. Rev. D **22**, 2220 (1980).
[63] T. Inami and T. Kubota, Nucl. Phys. **B179**, 171 (1981).
[64] M. Drees and K.-I. Hikasa, Phys. Lett. B **240**, 455 (1990); **262**, 497(E) (1991).
[65] M. Drees and K.-I. Hikasa, Phys. Rev. D **41**, 1547 (1990).
[66] S. G. Gorishnii, A. L. Kataev, and S. A. Larin, Yad. Fiz. **40**, 517 (1984) [Sov. J. Nucl. Phys. **40**, 329 (1984)], https://lib-extopc.kek.jp/preprints/PDF/1983/8311/8311112.pdf.
[67] S. G. Gorishnii, A. L. Kataev, S. A. Larin, and L. R. Surguladze, Mod. Phys. Lett. A **05**, 2703 (1990).
[68] S. G. Gorishny, A. L. Kataev, S. A. Larin, and L. R. Surguladze, Phys. Rev. D **43**, 1633 (1991).
[69] A. L. Kataev and V. T. Kim, Mod. Phys. Lett. A **09**, 1309 (1994).
[70] L. R. Surguladze, Phys. Lett. B **341**, 60 (1994).
[71] K. G. Chetyrkin, Phys. Lett. B **390**, 309 (1997).
[72] K. Melnikov, Phys. Rev. D **53**, 5020 (1996).
[73] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **96**, 012003 (2006).
[74] F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, J. High Energy Phys. 08 (2017) 113.
[75] K. G. Chetyrkin and A. Kwiatkowski, Nucl. Phys. **B461**, 3 (1996).
[76] S. A. Larin, T. van Ritbergen, and J. A. M. Vermaseren, Phys. Lett. B **362**, 134 (1995).
[77] A. Primo, G. Sasso, G. Somogyi, and F. Tramontano, Phys. Rev. D **99**, 054013 (2019).
[78] J. Fleischer and F. Jegerlehner, Phys. Rev. D **23**, 2001 (1981).
[79] D. Yu. Bardin, B. M. Vilensky, and P. K. Khristova, Yad. Fiz. **53**, 240 (1991) [Sov. J. Nucl. Phys. **53**, 152 (1991)], https://lib-extopc.kek.jp/preprints/PDF/1990/9008/9008076.pdf.
[80] A. Dabelstein and W. Hollik, Z. Phys. C **53**, 507 (1992).
[81] B. A. Kniehl, Nucl. Phys. **B376**, 3 (1992).
[82] A. L. Kataev, JETP Lett. **66**, 327 (1997).
[83] B. A. Kniehl and M. Spira, Nucl. Phys. **B432**, 39 (1994).
[84] A. Kwiatkowski and M. Steinhauser, Phys. Lett. B **338**, 66 (1994); **342**, 455(E) (1995).
[85] K. G. Chetyrkin, B. A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **78**, 594 (1997).
[86] L. Mihaila, B. Schmidt, and M. Steinhauser, Phys. Lett. B **751**, 442 (2015).
[87] E. Chaubey and S. Weinzierl, J. High Energy Phys. 05 (2019) 185.
[88] A. Djouadi, J. Kalinowski, M. Mühlleitner, and M. Spira, Comput. Phys. Commun. **238**, 214 (2019).

[89] A. Djouadi, J. Kalinowski, and M. Spira, Comput. Phys. Commun. **108**, 56 (1998).

[90] The complete calculation to NNLO accuracy was performed recently [91].

[91] W. Bernreuther, L. Chen, and Z.-G. Si, J. High Energy Phys. 07 (2018) 159.

[92] A. Bredenstein, A. Denner, S. Dittmaier, and M. M. Weber, J. High Energy Phys. 02 (2007) 080.

[93] A. Bredenstein, A. Denner, S. Dittmaier, and M. M. Weber, Phys. Rev. D **74**, 013004 (2006).

[94] A. Denner, S. Dittmaier, and A. Mück, Comput. Phys. Commun. **254**, 107336 (2020).

[95] D. de Florian *et al.* (LHC Higgs Cross Section Working Group), Report No. CERN-2017-002-M, 2017, 10.23731/CYRM-2017-004.

[96] J. R. Andersen *et al.* (LHC Higgs Cross Section Working Group), Report No. CERN-2013-004, 2013, 10.5170/CERN-2013-004.

[97] A. Denner, S. Heinemeyer, I. Puljak, D. Rebuzzi, and M. Spira, Eur. Phys. J. C **71**, 1753 (2011).

[98] The independent variations yield a slightly larger uncertainty compared to the 0.2% reported in Ref. [95].

[99] The relatively small impact of the α_s uncertainty reported here can be understood. The larger effect reported in Ref. [95] stems mainly from the evolution of the bottom quark mass from $\mu = m_b$ to $\mu = m_H$. This evolution is avoided altogether when $m_b(m_H)$ is adopted as the input parameter.

[100] ATLAS and CMS Collaborations, Phys. Rev. Lett. **114**, 191803 (2015).

[101] The intermediate result based on the ATLAS measurement [60] is

$$m_b(m_H) = 2.61^{+0.32}_{-0.27}(\mathrm{stat})^{+0.26}_{-0.19}(\mathrm{syst})\ \mathrm{GeV}, \tag{14}$$

and that based on the CMS measurement [57] is

$$m_b(m_H) = 2.57^{+0.39}_{-0.35}(\mathrm{stat})^{+0.37}_{-0.28}(\mathrm{syst})\ \mathrm{GeV}. \tag{15}$$

[102] J. Kieseler, Eur. Phys. J. C **77**, 792 (2017).

[103] The χ^2 is given by

$$\chi^2(x, m_b(m_b)) = \sum_{\mu_i} [m(\mu_i; x, m_b(m_b)) - m_b^{\mathrm{expt}}(\mu_i)]^2/(\sigma_i)^2, \tag{16}$$

where $m(\mu_i; x, m_b(m_b))$ is given by Eq. (13), and the experimental values $m_b^{\mathrm{expt}}(\mu_i)$ are the averages for $m_b(m_b)$, $m_b(m_Z)$, and $m_b(m_H)$ given in Eqs. (1), (2), and (8), respectively. The uncertainties σ_i include the experimental uncertainties listed in Eqs. (1), (2), and (8), and the parametric uncertainty due to α_s in the evolution.

[104] M. Cepeda *et al.*, CERN Yellow Rep. Monogr. **7**, 221 (2019).

[105] J. de Blas, M. Cepeda, J. D'Hondt, R. K. Ellis, C. Grojean, B. Heinemann, F. Maltoni, A. Nisati, E. Petit, R. Rattazzi, and W. Verkerke, J. High Energy Phys. 01 (2020) 139.

[106] T. Barklow, K. Fujii, S. Jung, R. Karl, J. List, T. Ogawa, M. E. Peskin, and J. Tian, Phys. Rev. D **97**, 053003 (2018).

[107] H. Abramowicz *et al.*, Eur. Phys. J. C **77**, 475 (2017).

[108] K. Fujii *et al.* (LCC Physics Working Group), arXiv:1908.11299.

[109] P. Bambade *et al.*, arXiv:1903.01629.

[110] J. Fuster, A. Irles, S. Tairafune, G. Rodrigo, M. Vos, H. Yamamoto, and R. Yonamine, Report No. ILD-PHYS-PUB–2021-001, 2021, https://confluence.desy.de/download/attachments/42357928/ILD-PHYS-PUB-2021-001.pdf.

[111] S. Kluth, Rep. Prog. Phys. **69**, 1771 (2006).