Enhancing Gravitational Interaction between Quantum Systems by a Massive Mediator

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In 1957 Feynman suggested that the quantum or classical character of gravity may be assessed by testing the gravitational interaction due to source masses in superposition. However, in all proposed experimental realizations using matter-wave interferometry, the extreme weakness of this interaction requires pure initial states with extreme squeezing to achieve measurable effects of nonclassical interaction for reasonable experiment durations. In practice, the systems that can be prepared in such nonclassical states are limited to small masses, which in turn limits the strength of their interaction. Here we address this key challenge—the weakness of gravitational interaction—by using a massive body as an amplifying mediator of gravitational interaction between two test systems. Our analysis shows that this results in an effective interaction between the two test systems that grows with the mass of the mediator, is independent of its initial state and, therefore, its temperature. This greatly reduces the requirement on the mass and degree of delocalization of the test systems and, while still highly challenging, brings experiments on gravitational source masses a step closer to reality.

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Introduction.—In a discussion regarding the necessity of gravitational quantization at the 1957 Chapel Hill Conference on the Role of Gravitation in Physics, Richard Feynman, aiming to clarify a point made by Frederik Belinfante, presented a Gedanken experiment in which a coherent superposition of a massive particle in two different spatial locations, generated, e.g., by a particle in a coherent superposition of spin states entering a Stern-Gerlach apparatus, is allowed to interact gravitationally with another mass [1]. He pointed out that the two possibilities for treating the gravitational interaction, either via a classical or via a quantum field, result in very different quantum states and thus experimental outcomes. Notably, the particles would, respectively, emerge in a classically correlated mixture of different positions or in a coherent superposition. The latter case is, in modern quantum information parlance, referred to as an entangled state.

At the time, such a Gedanken experiment was extraordinarily far removed from the experimental technology of the day. After all, it was only in 1952 that Schrödinger wrote "... we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences [...] we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo" [2]. Owing to this evident technological gap, there has been little activity by experiment and theory to explore possible routes towards turning Feynman's Gedanken experiment into reality.

However, six decades later, the rise of advanced quantum technologies and, notably, the field of optomechanics is starting to change this perception. The increasing ability to bring particles of ever growing mass into the quantum regime and control their dynamics in a manner that leaves their coherences intact [3–13] suggests that today such an experiment may be conceivable albeit still extraordinarily challenging [14]. Indeed, by determining experimentally the entanglement gain between two gravitationally interacting parties one would be able to falsify the assumption of a classical force carrier and thereby conclude the nonclassical nature of the gravitational field between them [15,16]. This led to further proposals for experiments that probe for gravitationally induced entanglement [17–24] and add to other tests based on superpositions of source masses [25–29]. While these experiments might become feasible at some point, it is equally clear that remarkably stringent requirements on isolation from the environment, the required duration of these experiments, and the large spatial extent of the quantum superpositions that are required to achieve a measurable effect render such type of experiment extremely challenging indeed [22].

In this Letter, we show that by introducing a heavier mediator particle that interacts gravitationally with a smaller test mass and by some other stronger force with an ancillary quantum system, an effective interaction between the test mass and the ancillary system can be engineered, which grows with the mass and degree of delocalization of the mediator. Notably, at suitably chosen points in time, the mediator decorrelates from the system, leaving only the test mass and the ancillary system entangled. As a result we find that a light test mass can be made to interact with an ancillary system as if it had the much larger mass of the mediator, with the significant benefit that the heavier mass of the mediator need not be prepared in a pure state and can, thus, remain at a finite temperature. Since a key technological challenge resides in the difficulty of preparing a sufficiently heavy mass in a pure state with a large enough spatial extension, the setup described here represents a significant enhancement over existing proposals.

Concept and setup.—For the calculations in this Letter, we will assume a gravitational interaction strength that is determined by the Newtonian interaction energy, which for two bodies of mass m with their centers of mass (c.m.) located at positions x_1 and x_2 is given by $E_G = -Gm^2/|x_1 - x_2|$. Expanding this for small variations of $|x_1 - x_2|$ around a fixed separation distance d, we find that the lowest-order coupling term is linear in the positions of the two masses and has the form $Gm^2 x_1 x_2/d^3$. Under such an interaction the c.m. of the two particles entangle at a rate that grows with the extent of their spatial delocalization [15,17,20,22–24,27]. While this describes gravity as a direct interaction, ignoring any field degree of freedom that may mediate the force, it allows for the computation of the attainable amount of entanglement, the presence of which may then allow us to draw inferences regarding the classical or quantum character of gravitational interaction and the field that may be mediating it.

For two masses that are trapped in local harmonic potentials and cooled down to their motional ground states (GS), the amount of entanglement due to their gravitational interaction, as quantified by the logarithmic negativity [30], oscillates in time with its maximum given by $\eta = 2Gm/(\omega^2 d^3)$ at time $t = \pi/[(1-\eta)2\omega]$ [20]. To ensure that the gravitational interaction dominates over Casimir forces, the surface-to-surface distance between the interacting bodies must be kept above a certain threshold determined by the radii of the particles. Interestingly, for large particles, when the separation distance is dominated by their size, η becomes independent of the particle size. This appears to be a strong limitation, as the gravitational interaction is naturally minute, and it seems the amount of entanglement that it can generate cannot be enhanced above a certain threshold even if we would acquire the ability to cool down objects of larger size [31]. One way to avoid this limitation is to increase the spatial extent of the c.m. wave functions above that of their GSs, for example, by squeezing them [20,22–24] or by placing each system in a superposition of two spatially separated coherent states [17,22]. However, the entanglement generated will be extremely sensitive to the tiniest decoherence sources of the involved systems [22,32–34] and, in general, this sensitivity will grow with increasing delocalization of the system [22,23]. Therefore, the challenge for the observation of gravitationally induced entanglement resides in the ability to generate highly nonlocalized states of massive objects whose purity needs to be maintained over the duration of the protocol. This is a phenomenal technological challenge that increases with the size of the objects. In the remainder of this Letter, we introduce and analyze a setup where the requirement of having a heavy mass in a highly delocalized state is not imposed on the test masses that we want to entangle but is instead shifted onto a third system that serves to mediate their interaction. While the test systems require their preparation in suitable pure states, the mediator can take any pure or mixed state, and the effective interaction between the test systems can be enhanced by increasing the size of the mediator instead of that of the test systems themselves.

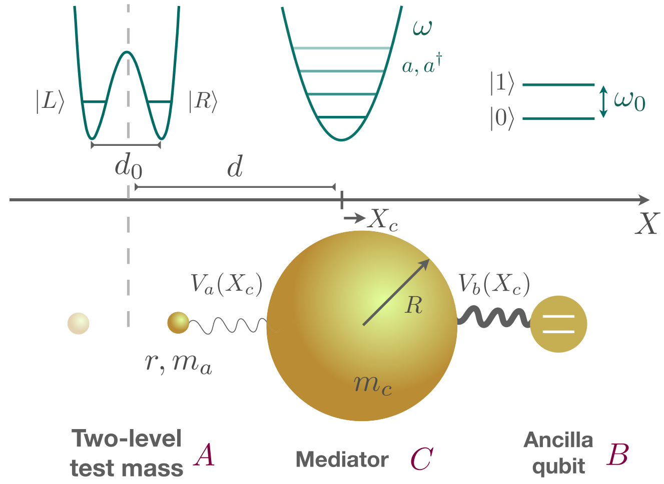

FIG. 1. Setup: A test particle (system A) of radius r and mass m_a is subject to a double-well potential with wells separated by a distance d_0 and behaves as a two-level system with states $|L\rangle$ and $|R\rangle$, which are stationary for the duration of the experiment provided that d_0 is large enough to make any tunneling negligible. A massive oscillator (system C) with frequency ω, radius R, and mass m_c has its equilibrium position at a distance d from the center of the double-well potential and acts as a mediator between the test mass and an ancillary qubit (system B) that has states $|0\rangle$ and $|1\rangle$ and bare energy splitting ω_0. The mediator is weakly coupled to the test mass through gravitational interaction with energy $\hat{V}_a(\hat{X})$ depending on the position of the oscillator, and strongly coupled to the ancillary system with a much stronger interaction energy $\hat{V}_b(\hat{X})$ of a nature other than gravitational, e.g., Casimir force. The direct interaction between systems A and B is negligible.

Consider the setup depicted in Fig. 1 consisting of three interacting systems, A, B, and C. We denote system A as a two-level test mass (TLTM), i.e., it is a particle of mass m_a trapped in a double-well potential along dimension X and behaves as a two-level system with states $|L\rangle$ and $|R\rangle$, which correspond to the particle being located, respectively, in the left or in the right well. We assume that the wells are deep and far enough to make any tunneling term negligible, and thus, that states $|L\rangle$ and $|R\rangle$ can be treated as stationary states of the double well for the duration of the protocol. System B is an ancillary qubit (AQ) system, which may have the same or a different physical origin as system A [35,36]. We stress that the argument that we will put forward is independent of the precise physical nature of system B. Finally, C is a mediator particle of mass m_c trapped in a harmonic potential characterized by an oscillation frequency ω in the X direction, and we assume that its motion in this dimension is uncoupled from its motion in orthogonal dimensions. A similar setup, albeit without system B, has been considered in Refs. [37,38]. Here, we assume that system C interacts with both A and B, while the interaction between the latter can be neglected. Under this assumption, the setup is well described by a Hamiltonian of the form

$$\hat{H} = \hbar\omega_0\hat{\sigma}_b^z + \frac{1}{2m_c}\hat{P}^2 + \frac{1}{2}m_c\omega^2\hat{X}^2 + \sum_{\alpha=L,R}\hat{V}_{a,\alpha}(\hat{X})|\alpha\rangle\langle\alpha| + \sum_{\alpha=0,1}\hat{V}_{b,\alpha}(\hat{X})|\alpha\rangle\langle\alpha|, \tag{1}$$

where $\hat{X}$ and $\hat{P}$ are, respectively, the position and momentum operators of the mediator, and $\hat{\sigma}_b^z$ is the Pauli z operator acting on system B, with ω_0 giving its bare energy splitting. The terms $\hat{V}_{i,\alpha}$, with $i = \{a, b\}$, represent the interaction energy between system C and system i when the latter is in state α and are assumed to be a function of the position of the mediator. We are interested in the case where $\hat{V}_{a,\alpha}$ is purely of gravitational origin, while $\hat{V}_{b,\alpha} \gg \hat{V}_{a,\alpha}$ and, although typically not of gravitational origin, its specific physical origin is not relevant for the argument. In order to avoid the interaction between A and C being dominated by Casimir forces, the distance between these masses needs to be sufficiently large—the precise value depending on their masses—typically exceeding significantly the splitting d_0 of the double-well potential [39]. Hence, we can expand the gravitational potential to second order in the separation distance around the value d to find an interaction energy

$$\hat{V}_{a,\pm}(\hat{X}) = -\frac{Gm_am_c}{|d \mp \frac{d_0}{2} + \hat{X}|} \approx -\frac{Gm_am_c}{d}\left(1 + \frac{d_0^2}{4d^2} \pm \frac{d_0}{2d} - \left(1 \pm \frac{d_0}{d}\right)\frac{\hat{X}}{d} + \frac{\hat{X}^2}{d^2} + \cdots\right), \tag{2}$$

where $\hat{V}_{a,+}$ and $\hat{V}_{a,-}$ correspond, respectively, to $\hat{V}_{a,R}$ and $\hat{V}_{a,L}$, and $G = 6.67408 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the gravitational constant. The first two terms in the expansion introduce a global energy shift, the third gives an energy splitting of the TLTM, while the fourth term is responsible for a displacement of the oscillator equilibrium position and as well as for a linear interaction between mediator and the TLTM. Finally, the fifth term generates a shift in the oscillation frequency of the oscillator. Thus, putting everything together, Hamiltonian (1) can be rewritten as

$$\hat{H} = \hbar\omega_a\hat{\sigma}_a^z + \hbar\omega_b\hat{\sigma}_b^z + \hbar\tilde{\omega}\hat{a}^\dagger\hat{a} + \hbar(g_a\hat{\sigma}_z^a + g_b\hat{\sigma}_z^b)(\hat{a} + \hat{a}^\dagger), \tag{3}$$

provided that the interaction energy between systems C and B admits a similar expansion, and that $|\pm d_0/2 - \Delta_x| \ll d$, with Δ_x denoting the maximum value of the position uncertainty of the oscillator during its evolution. Here, $\hat{a}^\dagger$ and $\hat{a}$ are ladder operators of the harmonic oscillator C with modified frequency $\tilde{\omega}^2 = \omega^2 - (2Gm_a/d^3) + (2/m_c)V_b^{(2)}$, where $V_b^{(2)}$ is the coefficient of the term quadratic in $\hat{X}$ in the interaction between C and B. Furthermore, $\omega_a = Gm_am_cd_0/(2\hbar d^2)$ and $g_a = -(Gm_ad_0/d^3)\sqrt{m_c/(2\tilde{\omega}\hbar)}$, upon defining $\sigma_z^a = |L\rangle\langle L| - |R\rangle\langle R|$; and ω_b and g_b have similar expressions in terms of the specific interaction between B and C.

Dynamics and entanglement.—The unitary-evolution operator associated to Hamiltonian (3) can be conveniently expressed in the interaction picture as [39]

$$\hat{U}(t) = \exp\left\{(g_a\hat{\sigma}_z^a + g_b\hat{\sigma}_z^b)(-\hat{a}\alpha_t + \hat{a}^\dagger\alpha_t^*)\right\} \times \exp\left\{-i\frac{2g_ag_b}{\tilde{\omega}}\hat{\sigma}_z^a\hat{\sigma}_z^b\left(t - \frac{\sin\tilde{\omega}t}{\tilde{\omega}}\right)\right\}, \tag{4}$$

with $\alpha_t = [(e^{-i\tilde{\omega}t} - 1)/\tilde{\omega}]$. The first term generates a time-dependent displacement of the mediator in phase space conditional on the states of the TLTM and the AQ. The second term gives a second order interaction between the two lateral systems with an effective coupling $g_{\rm eff} = 2g_ag_b/\tilde{\omega}$. Remarkably, at times $t_n = 2\pi n/\tilde{\omega}$ that are a natural period of the mediator frequency, $\alpha_{t_n} = 0$ and the first term vanishes, leaving an effective interaction between the TLTM and the AQ which is independent of the state of the oscillator, with $\hat{U}(t_n) = \exp\{-ig_{\rm eff}\hat{\sigma}_z^a\hat{\sigma}_z^bt_n\}$. Therefore, at these points in time the mediator is decorrelated from the rest of the system, while entanglement is retained between the TLTM and the AQ. Thus, provided that the TLTM and the AQ are initialized in suitable states, and that the gravitational interaction is able to mediate quantum correlations, entanglement will grow between the TLTM and AQ. This entanglement can then be detected by standard methods making local measurements on the 2-qubit system [40,41]. The principle that gives rise to the interaction is the same as that of the phase gates employed in trapped-ion platforms to entangle their internal degrees of freedom mediated by their collective motion [42–45]. Here, we use it as an amplification mechanism of the gravitational interaction. Notice, that the interaction strength between the TLTM and the AQ grows with the mass of the mediator as $\sqrt{m_c}$ and can be enhanced by a factor g_b/ω over the strength of the gravitational interaction g_a. The latter occurs because during the evolution the mediator will be displaced in phase space in opposite directions conditionally on the states of the TLTM and AQ, with this displacements reaching values of $(g_a + g_b)/\tilde{\omega}$, see Fig. 2(a). Thus, with increasing coupling of the AQ to the mediator, this grows into states with larger spatial delocalization, which in turn enhance the interaction between the TLTM and the mediator.

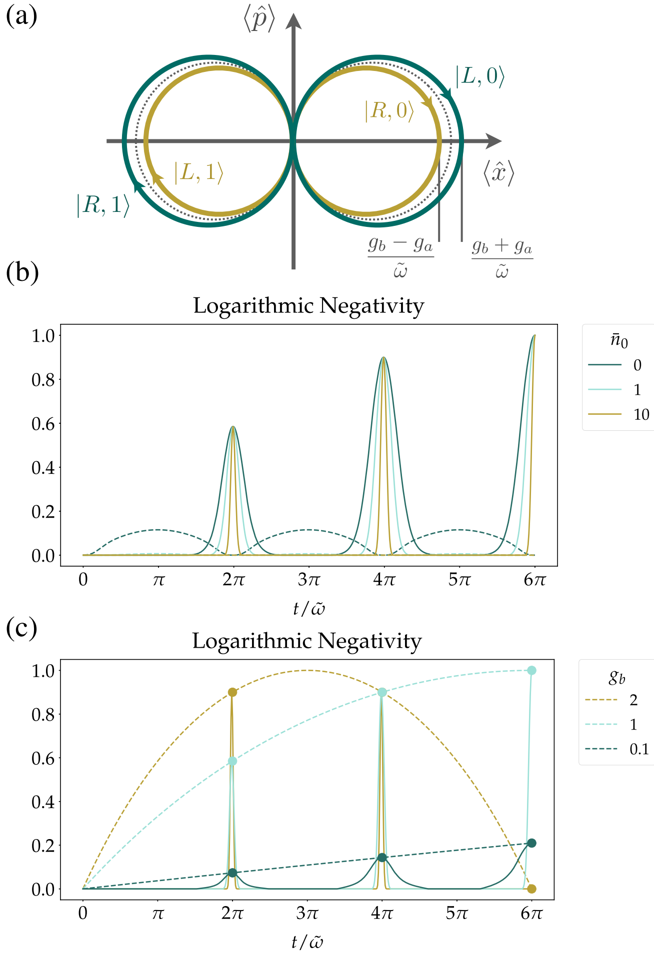

FIG. 2. System dynamics: In (a) we show the evolution in phase space of the four components of the mediator state correlated with each of the four states of the TLTM and the AQ. Here, $\{\langle\hat{x}\rangle, \langle\hat{p}\rangle\}$ are dimensionless position and momentum quadratures of the redefined oscillator, with shifted frequency and displaced equilibrium position. (b) The evolution of the entanglement, as quantified by the logarithmic negativity, between the TLTM and the AQ in continuous lines and between the TLTM and the mediator in dashed lines, for different temperatures of the mediator. Here, $g_a = 1/48\tilde{\omega}$ and $g_b = \tilde{\omega}$. Continuous lines in (c) display the entanglement between the TLTM and the AQ for different values of the coupling g_b expressed in units of $\tilde{\omega}$. For this simulation we initialize the mediator in a thermal state with mean phonon occupation $\langle n\rangle_0 = 10$, and set g_a as in (b). Dots indicate the value of the entanglement at the decoupling times t_n. Dashed lines correspond to the evolution of entanglement between two generic qubits governed by $\hat{H} = (2g_a g_b/\tilde{\omega})\hat{\sigma}_z\hat{\sigma}_z$, which at times t_n has a unitary-evolution operator equivalent to that of the full-system Hamiltonian, see Eq. (4).

In practice, the tolerable delocalization of the mediator will be limited by the distance that preserves the linear approximation in the expansion of the gravitational potential that we did in Eq. (2); that is $\Delta_x \approx \sqrt{\hbar/(2m\tilde{\omega})}\sqrt{\langle\bar{n}\rangle} \ll d$, where Δ_x and $\langle\bar{n}\rangle$ correspond, respectively, to the maximum values of the position uncertainty and the mean phonon occupation number of the mediator during the evolution. The time-dependent phonon occupation number $\langle n\rangle_t$ can be exactly calculated for an initial state with the TLTM and the AQ in even superpositions of the type $(|L/0\rangle + |R/1\rangle)/\sqrt{2}$ and the mediator in a thermal state with mean phonon occupation number $\bar{n}_0$. It is given by

$$\langle n\rangle_t = \bar{n}_0 + 4\frac{g_a^2 + g_b^2}{\tilde{\omega}^2}\sin^2\frac{\tilde{\omega}t}{2}. \quad (5)$$

This sets a limit on the strength of the coupling of the AQ to the mediator

$$g_b/\tilde{\omega} \approx \frac{1}{2}\sqrt{\frac{2m_c\tilde{\omega}\Delta_x^2}{\hbar} - \bar{n}_0} \ll \frac{1}{2}\sqrt{\frac{2m_c\tilde{\omega}d^2}{\hbar} - \bar{n}_0}, \quad (6)$$

where we have assumed $g_b \gg g_a$. We now consider the entanglement dynamics between the TLTM and the AQ, which we quantify in terms of the logarithmic negativity $LN = \max(0, \log_2\|\rho_{ab}^{T_b}\|_1)$, with $\|\cdot\|_1$ the trace norm and where T_b represents the partial transpose with respect to subsystem B. For a closed system ruled by Hamiltonian (3) an exact expression can be found at the times when the mediator is decoupled from the system:

$$LN(t_n) = \max\{0, \log_2[1 + |\sin(\phi_m)|]\}, \quad (7)$$

with

$$\phi_m = \frac{4g_b g_a}{\tilde{\omega}}t_n \approx \frac{2Gm_c m_a}{\hbar d^3}\Delta_x d_0 t_n, \quad (8)$$

where we have assumed for simplicity that $n_0 = 0$. This expression is upper bounded due to the constraint $\Delta_x \ll d$. In Fig. 2(b) we show the dynamics of entanglement between the different subsystems, for various temperatures of the mediator. We see that when the mediator starts in the GS the logarithmic negativity between the TLTM and the mediator oscillates with the period of the mediator frequency and vanishes completely at times t_n. At these times the mediator is decoupled while the entanglement between the TLTM and the AQ reaches a maximum. While the logarithmic negativity between the mediator and the TLTM decreases with increasing temperature, the entanglement between the TLTM and the AQ at the rephasing times t_n remains unaffected. This is observed in the form of peaks centered at positions t_n, which get narrower with increasing temperature of the mediator. In Fig. 2(c) we illustrate the enhancement of the entanglement between the TLTM and the AQ as the coupling of the AQ to the mediator is increased. We observe that with increasing coupling strength the peaks of entanglement between the TLTM and the AQ become higher and narrower.

To understand the degree of amplification that such a setup can introduce, we compare it to the case without a mediator.

We consider two gravitationally interacting TLTMs separated by a distance D, whose double-well potentials have a separation d_0. For the setup featuring a mediator we consider a TLTM with the same separation distance d_0, located a distance d away from the mediator. The distance d will in general be larger than D by an amount given by the difference between the radii of the mediator, R, and the TLTM, r, that is $d = D + \Delta R - d_0/2 + \Delta_x/2$, with $\Delta R = R - r$. In this way we make sure that the distances between the surfaces of the gravitationally interacting bodies is the same in both setups, and thus avoid the appearance of Casimir-Polder forces between the mediator and the TLTM. We find that in the case of two directly interacting TLTMs the logarithmic negativity evolves as in Eq. (7) with the argument of the sine given by [39]

$$\phi_d = \frac{Gm_a^2}{\hbar D}\frac{(d_0/D)^2 t}{1-(d_0/D)^2}. \tag{9}$$

Thus, the enhancement of the mediated setup over the setup with directly interacting masses can be expressed as the ratio

$$\phi_m/\phi_d = 2\frac{m_c\Delta_x}{m_a d_0}\frac{1}{(1+\frac{\Delta R}{D})^3}, \tag{10}$$

where we have assumed $d_0/D \ll 1$. To quantitatively illustrate such an enhancement we examine the following example. Consider a particle of silica with radius $r = 70$ nm (recently, particles of this size have been placed in their motional GS [9,46]) in a double-well potential with $d_0 = 500$ nm. If we impose that the gravitational interaction energy has to exceed the Casimir interaction energy by a factor of 10, we find that the minimum distance between their surfaces must be kept above 166 μm. This holds for all silica particles with radii below 166 μm [39]. Thus, we fix $D = 166$ μm and consider a mediator with a radius that is α times lager than that of the TLTM, that is $R = \alpha r$. Assuming a frequency for the mediator of $\tilde{\omega} = 100$ Hz, and that both mediator and TLTM are silica particles, with mass density $\rho \approx 2400$ Kg/m^3, this gives an enhancement of

$$\phi_m/\phi_d \approx \frac{\sqrt{\alpha^3}}{[1+(\alpha-1)4\times 10^{-4}]^3}10^{-3}\frac{g_b}{\tilde{\omega}}, \tag{11}$$

Thus, we see that, for example, a mediator particle of radius $R = 7$ μm, corresponding to $\alpha = 100$, would provide an enhancement on the order of $\phi_m/\phi_d \approx g_b/\tilde{\omega} \ll 10^8$, where the upper bound is imposed by the relation in Eq. (6).

Conclusion.—The detection of gravitationally mediated entanglement would represent a remarkable experimental result with far-reaching consequences for our understanding of physics. Although this is an outstanding technological challenge that will require the quantum control of heavier and heavier systems, rapid developments and recent experimental breakthroughs in the field of optomechanics suggest that the consideration of this question is pertinent and timely. In this spirit, we propose an enhancement of the experimental design with respect to existing proposals, which rely on the direct gravitational interaction between heavy test masses. In our design, we shift the large mass requirement to a mediator system, while keeping the test systems, where the entanglement is to be detected, at scales more friendly for their quantum control. While these smaller test systems would not show detectable amounts of entanglement were they to interact directly, in the mediated design they show an effective interaction that grows with the mass of the mediator. Remarkably, the required degree of controllability on the heavy mediator mass is considerably lower than that of the test systems in the directly interacting case, such that the mediator can remain in a thermal state. This paves the way for experimental tests of the gravitational interaction between masses that are significantly larger than those that can be prepared in pure states.

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