Editors' Suggestion

Ultraviolet-Infrared Mixing in Marginal Fermi Liquids

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When Fermi surfaces (FSs) are subject to long-range interactions that are marginal in the renormalization-group sense, Landau Fermi liquids are destroyed, but only barely. With the interaction further screened by particle-hole excitations through one-loop quantum corrections, it has been believed that these marginal Fermi liquids (MFLs) are described by weakly coupled field theories at low energies. In this Letter, we point out a possibility in which higher-loop processes qualitatively change the picture through UV-IR mixing, in which the size of the FS enters as a relevant scale. The UV-IR mixing effect enhances the coupling at low energies, such that the basin of attraction for the weakly coupled fixed point of a $(2+1)$-dimensional MFL shrinks to a measure-zero set in the low-energy limit. This UV-IR mixing is caused by gapless virtual Cooper pairs that spread over the entire FS through marginal long-range interactions. Our finding signals a possible breakdown of the patch description for the MFL and questions the validity of using the MFL as the base theory in a controlled scheme for non-Fermi liquids that arise from relevant long-range interactions.

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Introduction.—Non-Fermi liquids (NFLs) arise ubiquitously when Fermi surfaces (FSs) are coupled to gapless collective modes that mediate long-range interactions. The physics of NFLs is central to the strange metallic behavior and/or unconventional superconductivity in various systems, including cuprates, heavy-fermion compounds, half filled Landau level, and pnictides [1–3]. However, understanding NFLs has been a long-standing challenge due to strong quantum fluctuations amplified by abundant gapless modes near FSs [4–9].

Under renormalization-group (RG) flow, most theories for $(2+1)$-dimensional NFLs flow to the strong-coupling regime at low energies, and nonperturbative methods are required to understand their universal long-distance physics [10,11]. However, there is a special class of NFLs, marginal Fermi liquids (MFLs), where interaction effects are relatively weak, i.e., marginal in the RG sense with logarithmic (log) perturbative corrections. (In this Letter, the terms "metal," "NFL," and "MFL" may refer to that of either electrons or some emergent fermions, depending on the context.) If the marginal interactions are further screened, the long-distance physics should be captured by weakly coupled theories. While the MFL was first introduced for cuprates [12], it is relevant in rather broad contexts. First, metallic states realized at the half filled Landau level and exotic Mott transitions may be related to MFLs [13,14]. Second, MFLs have been used as a foothold to gain a controlled access to strongly coupled NFLs in an expansion scheme, where the exponent with which long-range interactions decay in space is used as a control parameter [15,16].

In NFLs, fermionic quasiparticles are destroyed by scatterings that are singularly enhanced at small momenta. If large-momentum scatterings are suppressed strongly enough, one can understand physical observables that are local in momentum space (e.g., the single-particle spectral function) within local patches of FSs that include the momentum point of interest (see Fig. 1). Although this

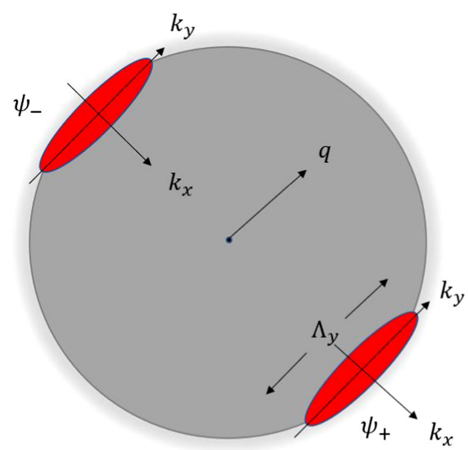


FIG. 1. In the patch theory, a FS is partitioned into multiple patches ($\sim k_F/\Lambda_y$ of them, with k_F the Fermi momentum and Λ_y the patch size). Ignoring the short-range four-fermion interactions, couplings between modes from different patches are weak unless they have almost parallel Fermi velocities. In this case, one can focus on a pair of antipodal patches that have nearly collinear Fermi velocities.

patch description becomes ultimately invalid in the presence of a pairing instability driven by short-range four-fermion couplings, which is *nonlocal* in momentum space, one may hope that the dominant physics in the normal state can be captured within the patch theory without invoking the entire FS. So the patch theory [8,9,16–18] has been widely used to describe a large class of NFLs (see Refs. [13,14,19,20] for some prominent examples).

In MFLs, however, the validity of the patch theory is questionable even before taking into account the short-range four-fermion couplings, because large-momentum scatterings are only marginally suppressed. If large-momentum scatterings create significant interpatch couplings, the patch theory fails even for the purpose of describing observables local in momentum space. In this case, the size of the FS, a UV parameter, qualitatively modifies the IR scaling behavior, showcasing UV-IR mixing.

While such UV-IR mixing does not show up at low orders in the perturbative expansion [16,18], a systematic understanding of higher-order effects is still lacking. This issue is also pertinent to multiple experimentally relevant problems. First, in the context of quantum Hall physics, it is debated whether the Halperin-Lee-Read theory [13] and Son's recently proposed theory [21] describe the same universal physics of the composite Fermi liquid (CFL). Second, in the continuous Mott transitions reported in κ-$(\mathrm{ET})_2\mathrm{Cu}_2(\mathrm{CN})_3$ [22,23] and moiré materials [24,25], the observed phenomena appear to be compatible with the predictions of the patch theory [14], but some specific critical properties seem to disagree [24]. To resolve these issues, it is crucial to understand the behaviors of the corresponding MFL theories. Moreover, understanding higher-order effects in NFLs, in general, may provide new insight into the nature of quantum phase transitions associated with sudden jumps of the FS size [26,27] and deconfined metallic quantum criticality [28–31].

In this Letter, we study the higher-order behaviors of a theory of N flavors of two-dimensional FSs coupled to a dynamical U(1) gauge field, whose kinetic energy scales as $k_y^{1+\epsilon}$. For the marginal exponent ($\epsilon = 0$), we indeed find potential UV-IR mixing in four-loop processes that renormalize the gauge coupling (see Fig. 2), with a strength logarithmically singular in the FS size. This is caused by gapless *virtual* Cooper pairs that manage to explore the entire FS, assisted by large-momentum scatterings that are only marginally suppressed.

Model and regularization scheme.—We denote the low-energy fermion fields near a pair of antipodal patches by ψ_{ip}, with $p = \pm$ the patch index and $i = 1, \ldots, N$ the flavor index. a represents the gauge field (see Fig. 1). Because of the kinematic constraints, the most important interactions occur between fermionic modes and the gauge bosons with momenta nearly perpendicular to their Fermi velocity [4,17]. The Euclidean action for the patch theory is [16]

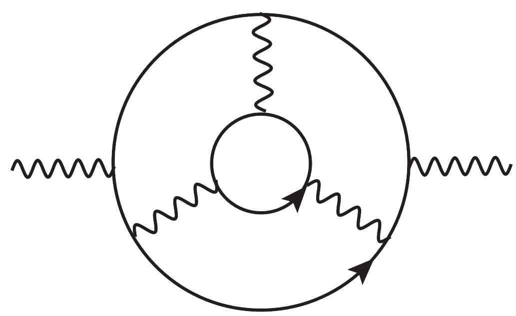
$$S = S_\psi + S_{\mathrm{int}} + S_a, \tag{1}$$

where

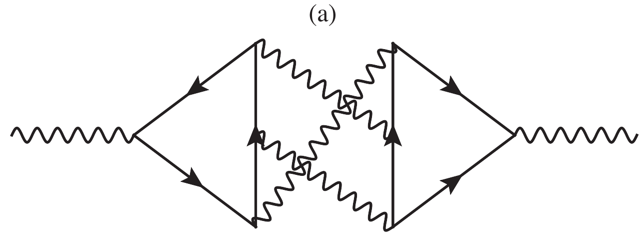
$$\begin{aligned} S_\psi &= \int [dk] \sum_{i,p} \psi^\dagger_{ip}(k)(-ik_\tau + p k_x + k_y^2)\psi_{ip}(k), \\ S_{\mathrm{int}} &= \int [dk_1][dk_2] \sum_{i,p} \lambda_p a(k_1) \psi^\dagger_{ip}(k_1 + k_2)\psi_{ip}(k_2), \\ S_a &= \int [dk] \frac{N}{2e^2} |k_y|^{1+\epsilon} a(-k) a(k), \end{aligned} \tag{2}$$

with $[dk] = \{[dk_\tau dk_x dk_y]/(2\pi)^3\}$ and $\lambda_\pm = \pm 1$. The reason for the opposite signs of $\lambda_\pm$ is because a couples to the currents of the fermions, and fermions from the two patches have opposite Fermi velocities. By power counting, the coupling e^2 is marginal (relevant) if $\epsilon = 0$ ($\epsilon > 0$). Physically, with decreasing ϵ, the fermion-boson coupling gets weaker at small momenta, but large-momentum scatterings become stronger, which increases the "risk" of UV-IR mixing. Below we primarily focus on the marginal case with $\epsilon = 0$. Note that a large N is still useful in organizing the calculations when $\epsilon = 0$.

One can introduce two cutoffs. Λ denotes the energy cutoff [35], and Λ_y is the cutoff of y momentum. The former is the usual UV cutoff, while the latter represents the size of the patch. We take $\Lambda \to \infty$ for simplicity; i.e., the



(a)



(b)

FIG. 2. Equation (3) comes from these two diagrams, together with their cousins with both internal fermion loops flipped in direction, which are not shown here. Note that only when the two fermion loops in each diagram run in the same direction do they contribute to double-log divergence. See Sec. III C 2 in the Supplemental Material [32] for all diagrams at this order, where (a) and (b) are dubbed Benz diagram and 3-string diagram, respectively.

theory is regularized by Λ_y only. Crucially, Λ_y also serves as IR data that measure the number of gapless modes near the FS, and there is *a priori* no guarantee that low-energy observables are insensitive to Λ_y. If the Λ_y dependence cannot be removed in low-energy observables by renormalization, the theory has UV-IR mixing, and the patch description fails.

UV-IR mixing.—We consider the photon self-energy $\Pi(k)$, which is $O(N^1)$ to the leading order. To order N^0, $\Pi(k)$ is finite as $\Lambda_y \to \infty$ [9,16], due to a kinematic constraint that is nevertheless absent at higher orders [see Eq. (30) of the Supplemental Material [32] or Ref. [36]]. At order N^{-1}, by exact calculations we find a *double*-log divergence in the diagram in Fig. 2(a): $\Pi_0(k_\tau = 0, k_y) = \frac{1}{N}(|k_y|/e^2)[(2\alpha^4)/3\pi^2][\ln(\Lambda_y/k_y)]^2$, with $\alpha \equiv e^2/(4\pi)$ (see Sec. III C 3 in the Supplemental Material [32]). Other diagrams are harder to compute explicitly. However, under reasonable assumptions, we argue that the only other net contribution to the double-log divergence is from Fig. 2(b), whose contribution is also Π_0 at $k_\tau = 0$ (see Sec. III C 4 in [32]). Double-log divergences are usually from divergences in subdiagrams, which can then be canceled by diagrams with counterterms. However, the present double-log divergences are not due to this, since the only divergent subdiagram in Fig. 2 is the three-loop vertex correction, but the corresponding counterterm does not contribute to the renormalization of the boson kinetic term to order N^{-1}. Taking all diagrams together (including the ones with counterterms), the total double-log divergence to order N^{-1} is

$$\Pi(k_\tau = 0, k_y) \sim \frac{1}{N}\frac{|k_y|}{e^2}\frac{4\alpha^4}{3\pi^2}\left[\ln\left(\frac{\Lambda_y}{k_y}\right)\right]^2. \tag{3}$$

To better understand this result, first consider the usual renormalizable field theories without a FS, e.g., $3+1$-dimensional ϕ^4 theory. In such theories, given a UV cutoff Λ, quantities like $d\Pi/d\ln\Lambda$ are analytic in the external momentum $k \ll \Lambda$, since this derivative measures the contribution of *high-energy* modes in the energy window $[\Lambda, \Lambda + d\Lambda)$ (see Fig. 3). Consequently, the nonanalyticity in Π can at most take the form of $k^2\ln(\Lambda/k)$, and the Λ dependence of the results can then be eliminated by local counterterms, allowing any observable at a scale $k_1 \ll \Lambda$ to be expressed solely in terms of renormalized quantities measured at another scale $k_2 \ll \Lambda$, and the IR physics is insensitive to the UV physics.

However, the present theory has another short-distance scale Λ_y, which measures the number of *gapless* modes near the FS. Low-energy observables, in general, can depend on Λ_y in a sensitive manner. Especially, $d\Pi/d\ln\Lambda_y$ does not have to be analytic in k (see Fig. 3). Gapless modes can not only renormalize the existing nonlocal term through $|k_y|\ln(\Lambda_y/|k_y|)$, but also generate stronger nonanalyticity in the quantum effective action, such as $|k_y|\ln^n(\Lambda_y/|k_y|)$ with $n > 1$ [$n = 2$ in Eq. (3)]. In this case, the Λ_y dependence cannot be removed in low-energy observables through renormalization of the existing terms (local or not) in the action, signaling UV-IR mixing.

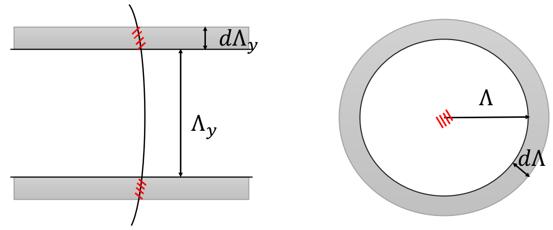


FIG. 3. The gray regions illustrate modes that are integrated out *if* we tune Λ_y in MFL (left) or Λ in a usual field theory without FS (right). The latter has gapless modes only at a single point in the momentum space (shown in red), while the former has gapless modes overlapping with the gray regions. In MFL, $\Pi(k)$ calculated at a *fixed* Λ_y can have IR singularities stronger than $\ln(\Lambda_y/k)$.

UV-IR mixing is known to arise in metals. (We note that UV-IR mixing with different origins is also proposed in other setups [37–41].) First, the FS size k_F, a UV parameter, becomes relevant at low energies when a critical boson is coupled with a FS whose dimension is greater than 1, as a boson can decay into particle-hole pairs along the "great circle" of FS whose tangent space includes the boson momentum [42,43]. Second, the FS size is important in the presence of pairing instabilities driven by short-range four-fermion interactions, via which Cooper pairs residing on the FS with zero total momentum can be scattered throughout the entire FS without violating momentum or energy conservation [18,44,45].

The origin of the UV-IR mixing we find here is related to the second one, but different. The contribution in Eq. (3) comes from virtual Cooper pairs (VCPs), represented by the two fermion loops that come from opposite patches and run in the same direction in Fig. 2. Via the *marginal* long-range interactions mediated by the gauge field, these VCPs spread over the entire FS, which enjoys a large phase space for scattering and can have singular contributions [see Eq. (86) in the Supplemental Material [32]]. Indeed, the double-log divergence disappears if either of the VCPs are absent (e.g., by taking the fermion loops in Fig. 2 to be in the same patch and/or run in opposite directions) or large-momentum scatterings are further suppressed (e.g., by taking $\epsilon > 0$) [46]. This is reminiscent of the enhanced quasiparticle decay rate due to VCPs in Fermi liquids [49].

Consequences of UV-IR mixing.—Equation (3) forces us to view Λ_y as another "coupling constant" of the theory [42]. In particular, $\tilde{\Lambda}_y \equiv \Lambda_y/\mu$ plays the role of a relevant coupling as the size of the FS blows up relative to the decreasing scale μ. The beta functions of the theory are

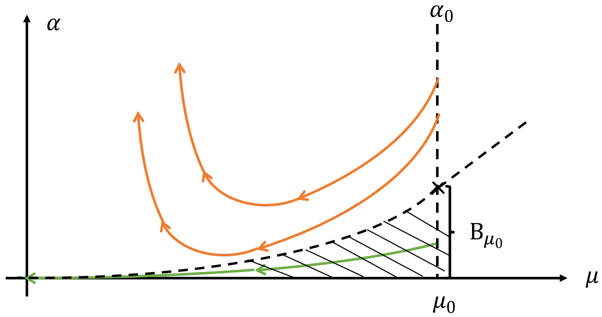


FIG. 4. The flow of $\alpha = e^2/(4\pi)$ with initial condition $\alpha = \alpha_0$ at $\mu = \mu_0$. For each μ_0, there is a critical value $\alpha^* \approx N/\ln(\Lambda_y/\mu_0)$ (the dashed curve): when $\alpha_0 < \alpha^*$ the gauge coupling flows to zero at low energies (green), while when $\alpha > \alpha^*$ it flows to infinity (orange).

(see Secs. I and II in the Supplemental Material [32] for details)

$$\frac{d\tilde{\Lambda}_y}{d\ln\mu} = -\tilde{\Lambda}_y, \qquad \frac{d\alpha}{d\ln\mu} = \frac{2\alpha^2}{\pi N} - \frac{8\alpha^5}{3\pi^2 N^2}\ln\tilde{\Lambda}_y. \quad (4)$$

Let us analyze these beta functions in the weak-coupling regime with $\alpha \lesssim 1$ and low-energy limit with $\mu \ll \Lambda_y$, together with a large but finite N. The first term in $d\alpha/d\ln\mu$ is the lowest-order term in $1/N$ and $\tilde{\Lambda}_y$ independent. As the scale μ is lowered, it makes the gauge coupling decrease logarithmically through screening. If the initial coupling α_0 defined at energy scale μ_0 satisfies $(\alpha_0^3/N)\ln(\Lambda_y/\mu_0) \lesssim 1$, this term dominates and the gauge coupling flows to zero at low energies. On the other hand, for $(\alpha_0^3/N)\ln(\Lambda_y/\mu_0) \gtrsim 1$, the second term dominates, which tends to enhance the coupling at low energies. In this case, we can ignore the first term to the leading order. Then the gauge coupling grows as $\alpha = \alpha_0\{1 - [16/(3\pi^2N^2)]\alpha_0^4[\ln^2(\mu/\Lambda_y) - \ln^2(\mu_0/\Lambda_y)]\}^{-1/4}$. This solution shows a divergence of the gauge coupling with decreasing μ, although it cannot be trusted in the strong-coupling regime. For theories defined at scale μ_0, the basin of attraction for the $\alpha = 0$ fixed point is given by $\mathcal{B}_{\mu_0} \equiv \{\alpha_0 | \alpha_0^3 < cN/\ln(\Lambda_y/\mu_0)\}$, with c an $O(1)$ constant (see the shaded region in Fig. 4). The salient feature is that $\mathcal{B}_{\mu_0}$ shrinks to a measure-zero set in the low-energy limit (i.e., $\mu_0 \ll \Lambda_y$), due to the scale dependence in the beta function. The fact that the beta function explicitly depends on Λ_y is a hallmark of UV-IR mixing.

In the presence of the UV-IR mixing, the FS size cannot be dropped in low-energy physical observables. For example, the single-fermion spectral function takes the form of $\mathcal{A}(\omega, \boldsymbol{k}, T) = \omega^\Delta f[(\omega/k_\parallel^z), (k_\parallel/k_F^{z'}), (\omega/T)]$, where k_F is the FS size, $k_\parallel$ is the distance of $\boldsymbol{k}$ away from the FS, T is the temperature, Δ, z, and z' are critical exponents [$z' = 2$ from Eq. (3)], and f is a universal function [42]. It is interesting to test this in CFLs at various filling factors that can be realized in Chern bands [29].

The UV-IR mixing in MFLs also has implications for the ϵ-expansion scheme [15,16], which has been used to approach NFLs with $\epsilon = 1$ from MFLs with $\epsilon = 0$ perturbatively in ϵ. To see it, we examine how the UV-IR mixing in the base theory with $\epsilon = 0$ affects the perturbative ϵ expansion. In theories with $\epsilon > 0$, the UV-IR mixing disappears since the diagrams in Fig. 2 are no longer divergent in Λ_y, as large-momentum scatterings are further suppressed. Instead, the double-log in Eq. (3) is translated to a double pole in ϵ as

$$\Pi(k_\tau = 0, k_y) \sim \frac{1}{N}\frac{|k_y|}{\tilde{e}^2}\frac{8\tilde{\alpha}^4}{\pi^2} \times \left(\frac{1}{27\epsilon^2} + \frac{1}{9\epsilon}\ln(\mu/|k_y|) + \frac{1}{6}[\ln(\mu/|k_y|)]^2\right), \quad (5)$$

where $\tilde{e}^2 = e^2\mu^{-\epsilon}$ is the dimensionless coupling and $\tilde{\alpha} = \tilde{e}^2/(4\pi)$. (See Ref. [50] for a different but related calculation at $\epsilon = 1$.) Since $1/\epsilon$ poles cannot be absorbed by terms already present in the action, the naive perturbative expansion appears ill defined. Moreover, this singular self-energy suggests that ϵ is renormalized to a larger value, further indicating that the ϵ expansion may break down. This calls for alternative control schemes for NFLs. See Refs. [51–53] for the dimensional regularization scheme that has no UV-IR mixing and Refs. [54–57] for other proposals.

Summary and discussion.—We provide strong evidence that a $(2+1)$-dimensional MFL exhibits UV-IR mixing, caused by virtual Cooper pairs that spread over the entire FS due to large-momentum scatterings. Our finding suggests the breakdown of the patch theory for MFLs and a potential issue in the ϵ expansion that uses MFLs as the base theory for NFLs.

We conclude with a few final remarks. First, the UV-IR mixing identified in $(2+1)$-dimensional MFLs can be extended to more general cases. Consider metals with m-dimensional FS (e.g., a spherical or cylindrical FS has $m = 2$ and a Weyl nodal line has $m = 1$) coupled to a critical boson whose kinetic energy goes as $k_y^{1+\epsilon}$. The contribution of virtual Cooper pairs to loop corrections scales as $\sim \int[(d^m k_y)/(k_y^{1+\epsilon})]$, which suggests that, for $m \geq 1+\epsilon$, there exist UV divergences associated with the extended size of FSs, and UV-IR mixing can arise. So we expect virtual-Cooper-pair-induced UV-IR mixing in $(3+1)$-dimensional gauge theories with $m = 2$ and $\epsilon = 1$ (on top of the UV-IR mixing identified in Ref. [42]). This is relevant to quantum spin liquids [58,59] and mixed-valence insulators [60]. (Since $\epsilon = 1$ corresponds to the local kinetic term for gauge boson, we expect that UV-IR mixing identified here does not originate from the nonanalyticity of the kinetic term of the gauge boson.)

Second, our result is obtained within the standard patch theory. To understand the full consequences of the UV-IR mixing caused by large-momentum scatterings, one should consider a general theory that keeps track of how the boson-fermion coupling is renormalized at *large* boson momenta. For this, instead of a coupling constant, one should take into account a "momentum-dependent coupling function," reminiscent of the familiar form factors in the interaction vertices in various settings [61]. Moreover, the four-fermion couplings, which should also be described by a coupling function, are not considered here, but they should, in principle, be studied on equal footing as the gauge coupling. Whether there is UV-IR mixing can depend on the microscopic details of the physical system. What we have shown is the presence of a UV-IR mixing in systems where the two effects above are negligible. In the future, it will be of great interest to understand whether such UV-IR mixing exists in systems where these effects are significant and should be incorporated into the theory. In any case, our results suggest that the physics of MFLs is richer than originally expected and mandates a qualitative improvement of the current theoretical understanding.

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[1] T. Senthil, On non-Fermi liquid quantum critical points in heavy fermion metals, Ann. Phys. (Amsterdam) **321**, 1669 (2006).
[2] Sung-Sik Lee, Recent developments in non-Fermi liquid theory, Annu. Rev. Condens. Matter Phys. **9**, 227 (2018).
[3] Erez Berg, Samuel Lederer, Yoni Schattner, and Simon Trebst, Monte Carlo studies of quantum critical metals, Annu. Rev. Condens. Matter Phys. **10**, 63 (2019).
[4] Joseph Polchinski, Effective field theory and the Fermi surface, arXiv:hep-th/9210046.
[5] B. L. Altshuler, L. B. Ioffe, and A. J. Millis, Low-energy properties of fermions with singular interactions, Phys. Rev. B **50**, 14048 (1994).
[6] Yong Baek Kim, Patrick A. Lee, and Xiao-Gang Wen, Quantum Boltzmann equation of composite fermions interacting with a gauge field, Phys. Rev. B **52**, 17275 (1995).
[7] Ar. Abanov and A. Chubukov, Anomalous Scaling at the Quantum Critical Point in Itinerant Antiferromagnets, Phys. Rev. Lett. **93**, 255702 (2004).
[8] Sung-Sik Lee, Low-energy effective theory of Fermi surface coupled with U(1) gauge field in $2+1$ dimensions, Phys. Rev. B **80**, 165102 (2009).
[9] Max A. Metlitski and Subir Sachdev, Quantum phase transitions of metals in two spatial dimensions. I. Ising-nematic order, Phys. Rev. B **82**, 075127 (2010).
[10] Shouvik Sur and Sung-Sik Lee, Chiral non-Fermi liquids, Phys. Rev. B **90**, 045121 (2014).
[11] Andres Schlief, Peter Lunts, and Sung-Sik Lee, Exact Critical Exponents for the Antiferromagnetic Quantum Critical Metal in Two Dimensions, Phys. Rev. X **7**, 021010 (2017).
[12] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phenomenology of the Normal State of CU-O High-Temperature Superconductors, Phys. Rev. Lett. **63**, 1996 (1989).
[13] B. I. Halperin, Patrick A. Lee, and Nicholas Read, Theory of the half-filled landau level, Phys. Rev. B **47**, 7312 (1993).
[14] T. Senthil, Theory of a continuous Mott transition in two dimensions, Phys. Rev. B **78**, 045109 (2008).
[15] Chetan Nayak and Frank Wilczek, Non-Fermi liquid fixed point in $2+1$ dimensions, Nucl. Phys. **B417**, 359 (1994).
[16] David F. Mross, John McGreevy, Hong Liu, and T. Senthil, Controlled expansion for certain non-Fermi-liquid metals, Phys. Rev. B **82**, 045121 (2010).
[17] Sung-Sik Lee, Stability of the U(1) spin liquid with a spinon Fermi surface in $2+1$ dimensions, Phys. Rev. B **78**, 085129 (2008).
[18] Max A. Metlitski, David F. Mross, Subir Sachdev, and T. Senthil, Cooper pairing in non-Fermi liquids, Phys. Rev. B **91**, 115111 (2015).
[19] Patrick A. Lee and Naoto Nagaosa, Gauge theory of the normal state of high-Tc superconductors, Phys. Rev. B **46**, 5621 (1992).
[20] Sung-Sik Lee and Patrick A. Lee, U(1) Gauge Theory of the Hubbard Model: Spin Liquid States and Possible Application to κ-$(\mathrm{BEDT\text{-}TTF})_2\mathrm{Cu}_2(\mathrm{CN})_3$, Phys. Rev. Lett. **95**, 036403 (2005).
[21] Dam Thanh Son, Is the Composite Fermion a Dirac Particle?, Phys. Rev. X **5**, 031027 (2015).
[22] Y. Kurosaki, Y. Shimizu, K. Miyagawa, K. Kanoda, and G. Saito, Mott Transition from a Spin Liquid to a Fermi Liquid in the Spin-Frustrated Organic Conductor κ-$(\mathrm{ET})_2\mathrm{Cu}_2(\mathrm{CN})_3$, Phys. Rev. Lett. **95**, 177001 (2005).
[23] Tetsuya Furukawa, Kazuya Miyagawa, Hiromi Taniguchi, Reizo Kato, and Kazushi Kanoda, Quantum criticality of Mott transition in organic materials, Nat. Phys. **11**, 221 (2015).
[24] Tingxin Li, Shengwei Jiang, Lizhong Li, Yang Zhang, Kaifei Kang, Jiacheng Zhu, Kenji Watanabe, Takashi Taniguchi, Debanjan Chowdhury, Liang Fu, Jie Shan, and Kin Fai Mak, Continuous Mott transition in semiconductor moiré superlattices, Nature (London) **597**, 350 (2021).
[25] Augusto Ghiotto, En-Min Shih, Giancarlo S. S. G. Pereira, Daniel A. Rhodes, Bumho Kim, Jiawei Zang, Andrew J. Millis, Kenji Watanabe, Takashi Taniguchi, James C. Hone, Lei Wang, Cory R. Dean, and Abhay N. Pasupathy, Quantum criticality in twisted transition metal dichalcogenides, Nature (London) **597**, 345 (2021).

[26] Hiroaki Shishido, Rikio Settai, Hisatomo Harima, and Yoshichika Onuki, A drastic change of the Fermi surface at a critical pressure in cerhin5: dHvA study under pressure, J. Phys. Soc. Jpn. **74**, 1103 (2005).

[27] Yawen Fang, Gael Grissonnanche, Anaelle Legros, Simon Verret, Francis Laliberte, Clement Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Fermi surface transformation at the pseudogap critical point of a cuprate superconductor, arXiv:2004.01725.

[28] Liujun Zou and Debanjan Chowdhury, Deconfined metallic quantum criticality: A U(2) gauge-theoretic approach, Phys. Rev. Research **2**, 023344 (2020).

[29] Liujun Zou and Debanjan Chowdhury, Deconfined metal-insulator transitions in quantum Hall bilayers, Phys. Rev. Research **2**, 032071(R) (2020).

[30] Ya-Hui Zhang and Subir Sachdev, From the pseudogap metal to the Fermi liquid using ancilla qubits, Phys. Rev. Research **2**, 023172 (2020).

[31] Ya-Hui Zhang and Subir Sachdev, Deconfined criticality and ghost Fermi surfaces at the onset of antiferromagnetism in a metal, Phys. Rev. B **102**, 155124 (2020).

[32] See Supplementary Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.106402 for additional details of the analysis and additional references, including Refs. [33,34].

[33] R. Shankar, Renormalization-group approach to interacting fermions, Rev. Mod. Phys. **66**, 129 (1994).

[34] L. G. Aslamazov and A. I. Larkin, Effect of fluctuations on the properties of a superconductor above the critical temperature, in *30 Years of the Landau Institute, Selected Papers*, edited by Isaak M. Khalatnikov and Vladimir P. Mineev (World Scientific, Singapore, 1996), Vol. 11, p. 23.

[35] UV cutoff associated with high-energy modes can be imposed on the x momentum relative to the FS as well, yet this cutoff violates an emergent gauge invariance [16].

[36] Tobias Holder and Walter Metzner, Fermion loops and improved power-counting in two-dimensional critical metals with singular forward scattering, Phys. Rev. B **92**, 245128 (2015).

[37] Sedigh Ghamari, Sung-Sik Lee, and Catherine Kallin, Renormalization group analysis of a neck-narrowing Lifshitz transition in the presence of weak short-range interactions in two dimensions, Phys. Rev. B **92**, 085112 (2015).

[38] Nathan Seiberg and Shu-Heng Shao, Exotic U(1) symmetries, duality, and fractions in $3+1$-dimensional quantum field theory, SciPost Phys. **9**, 046 (2020).

[39] Yizhi You, Julian Bibo, Taylor L. Hughes, and Frank Pollmann, Fractonic critical point proximate to a higher-order topological insulator: How does UV blend with IR?, arXiv:2101.01724.

[40] Henry Shackleton, Alex Thomson, and Subir Sachdev, Deconfined criticality and a gapless Z_2 spin liquid in the square-lattice antiferromagnet, Phys. Rev. B **104**, 045110 (2021).

[41] Zheng Zhou, Xue-Feng Zhang, Frank Pollmann, and Yizhi You, Fractal quantum phase transitions: Critical phenomena beyond renormalization, arXiv:2105.05851.

[42] Ipsita Mandal and Sung-Sik Lee, Ultraviolet/infrared mixing in non-Fermi liquids, Phys. Rev. B **92**, 035141 (2015).

[43] Ipsita Mandal, UV/IR mixing in non-Fermi liquids: Higher-loop corrections in different energy ranges, Eur. Phys. J. B **89**, 278 (2016).

[44] Leon N. Cooper, Bound electron pairs in a degenerate Fermi gas, Phys. Rev. **104**, 1189 (1956).

[45] Huajia Wang, Srinivas Raghu, and Gonzalo Torroba, Non-Fermi-liquid superconductivity: Eliashberg approach versus the renormalization group, Phys. Rev. B **95**, 165137 (2017).

[46] Our double-log divergence appears similar to the Sudakov double pole [47,48] in QED and effective theories of QCD. Both of them originate from gapless degrees of freedom. However, the Sudakov double-log originates from small momentum modes, while our double-log divergence is from modes with large y momentum.

[47] Michael E. Peskin and Daniel V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, PA, 1995).

[48] Iain Stewart, 8.851 Effective Field Theory. Spring 2013. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu/courses/physics/8-851-effective-field-theory-spring-2013.

[49] Dimitri Pimenov, Alex Kamenev, and Andrey V. Chubukov, One-dimensional scattering of two-dimensional fermions near quantum criticality, Phys. Rev. B **103**, 214519 (2021).

[50] Tobias Holder and Walter Metzner, Anomalous dynamical scaling from nematic and U(1) gauge field fluctuations in two-dimensional metals, Phys. Rev. B **92**, 041112(R) (2015).

[51] Denis Dalidovich and Sung-Sik Lee, Perturbative non-Fermi liquids from dimensional regularization, Phys. Rev. B **88**, 245106 (2013).

[52] Shouvik Sur and Sung-Sik Lee, Quasilocal strange metal, Phys. Rev. B **91**, 125136 (2015).

[53] Peter Lunts, Andres Schlief, and Sung-Sik Lee, Emergence of a control parameter for the antiferromagnetic quantum critical metal, Phys. Rev. B **95**, 245109 (2017).

[54] Jeremias Aguilera Damia, Shamit Kachru, Srinivas Raghu, and Gonzalo Torroba, Two-Dimensional Non-Fermi Liquid Metals: A Solvable Large-N Limit, Phys. Rev. Lett. **123**, 096402 (2019).

[55] Erik E. Aldape, Taylor Cookmeyer, Aavishkar A. Patel, and Ehud Altman, Solvable theory of a strange metal at the breakdown of a heavy Fermi liquid, arXiv:2012.00763.

[56] Jaewon Kim, Ehud Altman, and Xiangyu Cao, Dirac fast scramblers, Phys. Rev. B **103**, L081113 (2021).

[57] Ilya Esterlis, Haoyu Guo, Aavishkar A. Patel, and Subir Sachdev, Large-N theory of critical Fermi surfaces, Phys. Rev. B **103**, 235129 (2021).

[58] Yi Zhou, Patrick A. Lee, Tai-Kai Ng, and Fu-Chun Zhang, $Na_4Ir_3O_8$ as a 3D Spin Liquid with Fermionic Spinons, Phys. Rev. Lett. **101**, 197201 (2008).

[59] Michael J. Lawler, Arun Paramekanti, Yong Baek Kim, and Leon Balents, Gapless Spin Liquids on the Three-Dimensional Hyperkagome Lattice of $Na_4Ir_3O_8$, Phys. Rev. Lett. **101**, 197202 (2008).

[60] Debanjan Chowdhury, Inti Sodemann, and T. Senthil, Mixed-valence insulators with neutral Fermi surfaces, Nat. Commun. **9**, 1766 (2018).

[61] Siddharth A. Parameswaran, Rahul Roy, and Shivaji L. Sondhi, Fractional quantum Hall physics in topological flat bands, C.R. Phys. **14**, 816 (2013).