Phase Diagram of 1 + 1D Abelian-Higgs Model and Its Critical Point

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We determine the phase diagram of the Abelian-Higgs model in one spatial dimension and time $(1+1\mathrm{D})$ on a lattice. We identify a line of first order phase transitions separating the Higgs region from the confined one. This line terminates in a quantum critical point above which the two regions are connected by a smooth crossover. We analyze the critical point and find compelling evidence for its description as the product of two noninteracting systems: a massless free fermion and a massless free boson. However, we find also some surprising results that cannot be explained by our simple picture, suggesting this newly discovered critical point is an unusual one.

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Introduction.—Gauge theories in $1+1$ dimensions (1D in space and time) are ideal playgrounds to characterize the effects of strong coupling between matter and gauge fields. Many of the nonperturbative aspects of $3+1$ dimensional gauge theories relevant to our understanding of particle physics, such as quark confinement and chiral symmetry breaking, have a $1+1\mathrm{D}$ analog. Furthermore, $1+1\mathrm{D}$ field theories can often be treated analytically [1,2] providing important insights to the physics of $1+1\mathrm{D}$ systems relevant also to condensed matter physics.

In this Letter, we study the lattice version of a relativistic bosonic field that interacts with a photonic field [3], the bosonic version of the Schwinger model [4–7]. In contrast to (polarized) fermions, bosons can have contact interactions that are described by the well-known Abelian-Higgs model (AHM) in $1+1\mathrm{D}$ (AHM_2) [8–12].

In AHM_2, a weak matter-field coupling limit [13] suggests that the phase diagram is shared by two phases characteristic of the Higgs mechanism [8–11]: a superfluid phase 1 with the quasicondensation of bosons (Higgs phase) and a Mott-insulating phase 2 with strong interactions. However, nonperturbative calculations show that the phenomenology in phase 1 is the same as in phase 2, and bosons are always tightly confined [13] (for a recent discussion, see [14,15]).

One can certify the presence of a phase transition in $d+1$ dimension for any $d>2$ [16,17], but due to the (boring) expectation of a single phase in the continuum, the phase diagram of AHM_2 has never been computed on the lattice in $1+1\mathrm{D}$. This work aims at filling this gap.

Our work is strongly motivated by the current prospects of simulating lattice gauge theories using cold atomic setups [18–23]. Since bosons are easier to cool down than fermions, experiments along the lines proposed in [24–30] should soon explore the phase diagram of AHM_2.

Physicists have been working hard to measure the Higgs mode in experiments with cold atoms for a long time, as reviewed in [31]. In $2+1\mathrm{D}$, an explicit particle-hole symmetry protects the decay of the Higgs mode into Goldstone modes, allowing a proper measurement of its mass. These conditions are only met at the tip of the lobe of the Mott insulator to superfluid transition in Bose-Hubbard systems [32–34], which is, unfortunately, in $1+1\mathrm{D}$ of the Berezinskii-Kosterlitz-Thouless type (see, e.g., [35–38]) and is not particle-hole symmetric. This observation seems to strengthen the picture emerging from the presence of a single phase in AHM_2 and seems to suggest that a proper Higgs mode does not exist in $1+1\mathrm{D}$.

The results we present indicate a different picture, still characterized by a single phase, but with a reach landscape of transitions.

By performing matrix product state [39,40] simulations of the Hamiltonian version of AHM_2, we confirm the presence of a single phase for all the values of the mass of the bosons μ^2/q^2 and their interaction strength λ/q^2 (in units of the bosonic charge q) in agreement with the field theoretical analysis. However, unexpectedly, for small λ/q^2 we find a line of first order quantum phase transitions (FOQPT) between the "Higgs" and the "confined" regions. This line ends, for a finite value of λ/q^2, in a critical second order quantum phase transition (SOQPT), above which the two regions are continuously connected through a smooth crossover. Close to the FOQPT line, the two regions are well separated, enabling identification of a "Higgs" mode and its analysis in the continuum limit (for a sufficiently small λ/q^2). One can indeed take a different continuum limit to the standard one by approaching, from the Higgs region, the newly discovered SOQPT.

We precisely identify the position and the nature of the new critical point. By assuming Lorentz invariance at the critical point and then using the machinery of conformal field theories (CFTs), we can understand the critical point as the direct sum of two noninteracting fields: a free fermionic field describing the Higgs mode and a free bosonic field, a collective mode of the Goldstone modes and the gauge field. Still, the complete characterization of this critical point remains an outstanding challenge as some results do not fit the above picture.

The model.—Following [3], we discretize AHM_2 on a finite 1D lattice with L sites (with spacing a) (see [41] for details), arriving at the Hamiltonian (with open boundary conditions)

$$\hat{H} = \sum_j \left[\hat{L}_j^2 + 2x\hat{\Pi}_j^\dagger\hat{\Pi}_j + \left(4x - \frac{2\mu^2}{q^2}\right)\hat{\phi}_j^\dagger\hat{\phi}_j + \frac{\lambda}{q^2}(\hat{\phi}_j^\dagger)^2\hat{\phi}_j^2 - 2x(\hat{\phi}_{j+1}^\dagger\hat{U}_j\hat{\phi}_j + \mathrm{H.c.}) \right], \qquad (1)$$

with $x = 1/a^2q^2$. The matter fields $\{\hat{\phi}_j, \hat{\phi}_j^\dagger, \hat{\Pi}_j, \hat{\Pi}_j^\dagger\}$ operators act in Hilbert space at sites j, while the gauge-field $\{\hat{L}_j, \hat{U}_j, \hat{U}_j^\dagger\}$ objects act in Hilbert space defined on the bond linking sites j and $j+1$. The operators fulfill the standard commutation relations $[\hat{\phi}_j, \hat{\Pi}_k] = [\hat{\phi}_j^\dagger, \hat{\Pi}_k^\dagger] = i\delta_{jk}$, $[\hat{L}_j, \hat{U}_j] = -\hat{U}_j$, and $[\hat{L}_j, \hat{U}_j^\dagger] = \hat{U}_j^\dagger$.

The usual continuum limit is $x \to \infty$. Here, we fix $x = 2$ and characterize the phase diagram on the lattice.

We can define creation and annihilation operators for particles "a" and antiparticles "b" as $\hat{a}_j$ and $\hat{b}_j$ fulfilling $[\hat{a}_j, \hat{a}_k^\dagger] = [\hat{b}_j, \hat{b}_k^\dagger] = \delta_{jk}$ [55]. We use the density matrix renormalization group algorithm [39,40,56–59] to find the ground state of the Eq. (1) Hamiltonian. Specifically, we employ a strictly single-site variant of the density matrix renormalization group with subspace expansion [60].

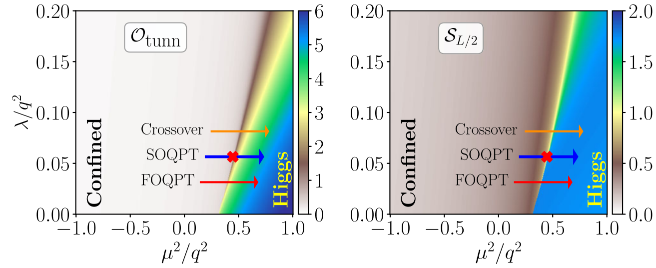

FIG. 1. Phase diagram of the AHM_2 (1) in the $(\mu^2/q^2, \lambda/q^2)$ plane for a system of size $L = 60$. At small couplings, the system occupies two qualitatively different regions, a confined region and a Higgs region, separated by a line of FOQPT as witnessed by the average tunneling amplitude $\mathcal{O}_{\mathrm{tunn}}$ (left panel) (effectively zero in the confined region and finite in the Higgs region) and the entanglement entropy $\mathcal{S}_{L/2}$ measured at the center of the chain (right panel) (small in the confined region and large in the Higgs region). The line of FOQPT ends at a SOQPT, above which the two regions are smoothly connected representing different aspects of a single phase.

For numerics we limit the occupations of bosonic modes to at most $n_0^a = n_0^b = 10$ [41].

In absence of external charges, the local $\mathbb{U}(1)$ symmetry implies the Gauss law $\hat{G}_j = 0, \ \forall\ j$, where the generators are [3] $\hat{G}_j = \hat{L}_j - \hat{L}_{j-1} - \hat{Q}_j$, and $\hat{Q}_j = \hat{a}_j^\dagger\hat{a}_j - \hat{b}_j^\dagger\hat{b}_j$ encodes the density of dynamical charges. Using the Gauss law, we can integrate out the gauge fields in a chain with open-boundary conditions in favor of a long-range potential for the matter fields [4].

Confined and Higgs: two shades of the same phase.—The long-range interactions among bosons destroy the phases of the standard 1 + 1D Bose-Hubbard model (see the phase diagram in Fig. 1). For $\lambda/q^2 \geq 0$, the system is in the confined region as far as $\mu^2/q^2 \leq 0$. In this region, the model has a finite mass gap, and the elementary excitations are mesons, bound pairs of particle and antiparticle. The gauge bosons are in the lowest eigenstate of $\hat{L}_j$, so that the variance $\sigma^2(\hat{L}_j) = \langle\hat{L}_j^2\rangle - \langle\hat{L}_i\rangle^2 \approx 0$.

For $\mu^2/q^2 \gg 0$, the system enters the "gapped" Higgs region where the variance $\sigma^2(\hat{L}_j)$ becomes large. The effective gauge-field mediated tunneling amplitude $\mathcal{O}_{\mathrm{tunn}} = (1/2L)\sum_j \langle\hat{\phi}_{j+1}^\dagger\hat{U}_j\hat{\phi}_j + \mathrm{H.c.}\rangle$. increases, so that it can distinguish the Higgs region from the confined one.

We can also characterize the two regions by considering the behavior of the entanglement entropy of a block made of l constituents starting from the boundary, defined as

$$\mathcal{S}_l = -\mathrm{Tr}[\rho_l \ln \rho_l], \qquad (2)$$

where $\rho_l = \mathrm{Tr}_{l+1,l+2,\ldots,L}|\psi\rangle\langle\psi|$ is the reduced density matrix. In the Higgs region, the entanglement entropy is systematically larger than in the confined region. However,

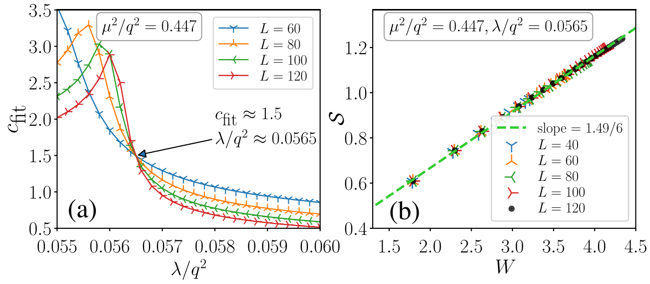

FIG. 2. Entropy scaling of the AHM_2. (a) The fitted central charge $c_{\rm fit}$ according to Eq. (3) for fixed $\mu^2/q^2 = 0.447$ and different system sizes. Curves for different system sizes cross each other at $\lambda/q^2 \approx 0.0565$ and $c_{\rm fit} \approx 3/2$. (b) The scaling of the entanglement entropy at the critical point for different system sizes yields the central charge of the critical theory as $c = 1.49(1)$.

since both phases are gapped, the entropy follows area-law scaling with respect to the bipartition size (see [41]).

For sufficiently small λ/q^2, the two regions are separated by a FOQPT line characterized by discontinuous jumps, both in the tunneling amplitude $\mathcal{O}_{\rm tunn}$ and in $\mathcal{S}_{L/2}$ measured across the central bond [41]. This line terminates at a critical SOQPT at a finite value of λ_c/q^2 and μ_c^2/q^2, identified by the red cross in Fig. 1. We discuss the precise location and characterization of this critical point below. Above the SOQPT, the two regions are smoothly connected, as revealed by smooth changes in all the physical quantities while moving from one region to the another.

Nature of the critical point.—We can precisely locate and characterize the critical point assuming its Lorentz invariance, which implies an applicability of a CFT at low energies. In a CFT, the finite-size scaling of the entanglement entropy of a block of first l consecutive sites in a chain with open boundary conditions and length L is

$$\mathcal{S}(l, L) = \frac{c}{6} W + b', \tag{3}$$

where c is the central charge of the corresponding CFT, b' is a nonuniversal constant, and the chord length W is a function of both L and l: $W(l, L) = \ln[(2L/\pi)\sin(\pi l/L)]$ [61–63].

We pinpoint the SOQPT by adapting the idea of the phenomenological renormalization group [64] to the scaling of the entropy in Eq. (3) as explained in [65,66]. At the critical point, the c value should be independent of the system's size. For each L, we obtain $c_{\rm fit}$ by fitting Eq. (3) to our numerical data for $\mathcal{S}(l, L)$. The extracted values in the $(\mu^2/q^2, \lambda/q^2)$ plane depend on L and become independent of the system size only at the critical point. Our data suggest that the L dependent central charges collapse to a single value at $[\mu^2/q^2 = 0.447(1), \lambda/q^2 = 0.0565(1)]$ (see Fig. 2). The central charge at the critical point $(\mu_c^2/q^2 = 0.447,\ \lambda_c/q^2 = 0.0565)$ is found to be $c = 1.49(1)$ [67].

The value of the central charge mentioned above suggests that we are not dealing with a minimal model. However, we want to argue here that we are in the presence of the direct sum of two different minimal models, each contributing to a piece of the total central charge: a $c_f = 1/2$ for a free Majorana fermion and a $c_b = 1$ for free boson. This scenario is strongly motivated by the standard Higgs mechanism. The complex Higgs field separates into its amplitude and its phase. The amplitude mode is effectively described by a real $\lambda\phi^4$ theory that undergoes the standard Ising phase transition (the $c = 1/2$ part). The phase, on the other hand, provides the longitudinal degree of freedom to the photon field. The latter becomes massless at the transition and provides the $c = 1$ free bosonic part. The value of $c = 1.5$ furthermore suggests, based on the c theorem, that the two parts should be noninteracting [70].

In order to confirm this scenario, we compute the entanglement spectrum that is also known to encode the central charge of the theory [71]. The entanglement spectrum, denoted by ε_s, is the spectrum of the entanglement Hamiltonian $H_l = -\log(\rho_l)$. By assuming a factorized ground state, we should observe that the smallest eigenvalue of H_l, ε_0 diverges logarithmically. In particular, we should see that [71]

$$\varepsilon_0 = (\varepsilon_0^{\rm Ising} + \varepsilon_0^{\rm boson}) \propto \frac{(c_f + c_b)}{12} W + O(1/W). \tag{4}$$

By fitting our numerical data to Eq. (4), we observe a perfect collapse on the functional form predicted by CFT, but the numerical result for of $c_{\rm eff} = 1.20(1)$ is not compatible with 1.5 [see Fig. 3(a)]. This disagreement between the scaling of the entanglement entropy and that for the entanglement ground state already suggests that the critical point is unusual and exotic in nature.

We thus turn to analyze the operator content of the model by studying the correlation functions of local operators. We should be able to identify a set of primary operators by studying the large distance two point correlations function that should decay algebraically as $\phi(0)\phi(r) \sim 1/r^{\Delta_\phi}$. The presence of gauge symmetry, however, strongly reduces the set of operators we can consider. Most of the candidates that should couple to primary operators are either trivial (due to the low dimensionality of the system) or vanishing since they are not gauge invariant. The only nonvanishing operators are indeed Wilson lines terminating on a boson-antiboson pair and electric field correlations. We also have access to local operators such as $\hat{\phi}^\dagger\hat{\phi}$ and $\hat{\Pi}^\dagger\hat{\Pi}$ that couple both to the real part and the phase of the field. By assuming we are dealing with a CFT, we can use the conformal map that maps the profile of local operators to two points correlation functions on the full plane (see, e.g., [72]).

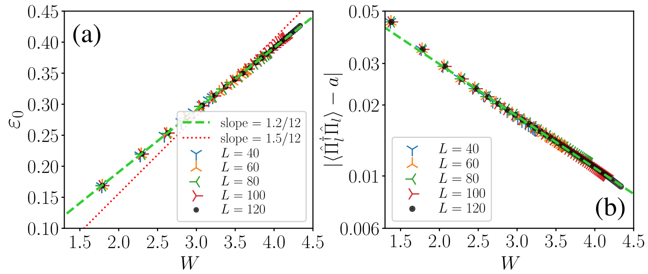

FIG. 3. (a) The scaling of the entanglement ground state ε_0 matches perfectly the functional form suggested by the CFT analysis, reported in the text. However, the numerical value for the central charge deviates by around 20% from the value we extract from the scaling of the entanglement entropy as the best fit suggests $c_{\mathrm{eff}} = 1.20(1)$. The red dotted line depicts the fit assuming $c = 1.5$. (b) The scaling of $\langle\hat{\Pi}_l^\dagger\hat{\Pi}_l\rangle$ according to the CFT prediction [Eq. (5)]. It couples both to the identity operator and one primary with scaling dimension Δ that comes out to be $\Delta = 0.51(2)$ from the fit, with a being 0.5474(2).

At first we analyze the behavior of $\langle\hat{L}_l^2\rangle$ as function of the chord coordinate W. The numerical results show that $\langle\hat{L}_l^2\rangle$ diverges linearly as a function of W, unveiling that $\hat{L}^2$ behaves as a free-bosonic field. Furthermore, the slope of such linear scaling is found to be $1.20(4)/12$, matching that of the entanglement ground state energy. Turning to analyzing the profile of $\langle\hat{\Pi}_l^\dagger\hat{\Pi}_l\rangle$ as a function of W, we find that

$$\langle\hat{\Pi}_l^\dagger\hat{\Pi}_l\rangle \simeq a + b[\exp(W)]^{-\Delta}, \qquad (5)$$

where a and b encode the overlap of the above expectation value with the identity operator and one of the primaries. The numerical data [Fig. 3(b)] suggest that $\Delta \simeq 0.5$, the conjugate operator to the one that would match to the derivative of the electric field.

Unfortunately, we do not find any operator that couples to the primary of the Ising part of the CFT. Summarizing, our data seem to confirm that we have one part of the system that behaves as a free boson and suggest that $\partial_x\hat{L}^2$ should have a large overlap with the primary operator (the derivative of the free-bosonic field), while $\hat{\Pi}^\dagger\hat{\Pi}$ should have a strong overlap with the conjugate primary operator.

Now moving away from the critical point, we can use the standard scaling hypothesis to extract the exponent ν from the collapse of the fitted central charge as

$$c_{\mathrm{fit}}(L) = f[(\mu^2/\lambda - \mu_c^2/\lambda_c)L^{1/\nu}], \qquad (6)$$

where $f(.)$ is a continuous function and ν is the corresponding critical exponent. Performing the data collapse according to Eq. (6) in the neighborhood of the critical point μ_c^2/λ_c (see Fig. 4), we find the critical exponent to be $\nu = 1/2 \pm 0.02$, which matches the value observed in the transition from polarized to critical phase in the XX model

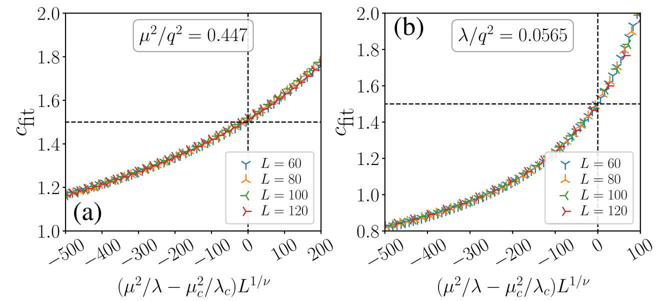

FIG. 4. The collapse of c_{fit} according to the scaling hypothesis [Eq. (6)] in the neighborhood of the critical point $\mu_c^2/\lambda_c = 0.447/0.0565$ for fixed $\mu^2/q^2 = 0.447$ (left) and for fixed $\lambda/q^2 = 0.0565$ (right). Here, we vary (a) λ/q^2 in the range [0.056, 0.058] and (b) μ^2/q^2 in the range [0.435, 0.45] for the data collapses. In both cases, the critical exponent is found to be $\nu = 0.5 \pm 0.02$ from data collapses.

in a magnetic field [73–75]. The same transition can be understood in terms of free bosons that pass from their Fock vacuum to the superfluid regime as the chemical potential exceeds the width of the first band. In our case, the strange thing is that there is no superfluid regime but just a single critical point where the gauge boson condenses, while away from the critical point our system passes from vacuum to a Mott insulator phase. Now using the standard scaling hypothesis, once we have figured out that $\nu = 1/2$ we can deduce that $\langle\hat{\Pi}^\dagger\hat{\Pi}\rangle \simeq (\mu^2/\lambda - \mu_c^2/\lambda_c)$, meaning that $\beta = 1$.

It is worth pointing out that $\nu = 1/2$ seems inconsistent with a CFT, where we would expect that $d - 1/\nu = \Delta$, with Δ the thermal critical exponent and d being 2 for the $1 + 1$D quantum system, while we find $d - 1/\nu = 0$. However, all our results so far have been obtained by assuming a full conformal invariance in mapping the correlation function of our finite system to the ones of an infinite plane by means of a conformal transformations.

The appearance of $\nu = 1/2$, together with the failure to identify a local operator that couples to the primary field of the Ising part of the CFT, contrasts with the factorization on the critical point. However, by repeating a similar analysis in a $\mathbb{Z}_3$ gauge theory coupled to bosonic matter we find a $c \simeq 0.8 + 0.5$ [76]. As a result, we still believe that the factorization hypothesis is correct, but it requires further analysis to be appropriately confirmed. In particular, it would be interesting to analyze the system under periodic boundary conditions which, however, is impractical at current computational capabilities using matrix product states ansatz but may become a possibility using next-generation tensor network algorithms.

Discussion and conclusions.—We have analyzed the phase diagram of AHM_2 on a discrete lattice in $1 + 1$D. We have found two distinct regions, the confined and Higgs regions, that are separated by line of FOQPT that terminates at a SOQPT. Beyond the SOQPT, the two regions are connected by a smooth crossover. The presence of a SOQPT allows one to construct an unorthodox continuum limit of the theory that should be described by free fermions and free bosons that do not interact.

This would result in a CFT with central charge $c = 3/2$, compatible with our numerical result and would have a compelling interpretation in terms of the standard Higgs mechanism—the real part of the complex field undergoes an Ising transition (the $c = 1/2$ part), while the phase of it provides the transverse degree of freedom to the photon that becomes dynamical and massless (the $c = 1$ part).

However, further numerical analyses unveil surprising pieces of the puzzle that do not fit our interpretation. We did not find a local operator that couples to the $c = 1/2$ part of the CFT. The scaling of the entanglement ground state should follow a similar law to the one of the entanglement entropy. The numerical value of the central charge that we extract from it is $c = 1.20(1)$. We also obtain $\nu = 1/2$ analyzing the collapse of the data for the entanglement entropy close to the critical point, in contrast to the expected $\nu = 1$.

Are we actually observing a Lorentz invariant critical point where the Higgs and photon mode factorize? We believe this is the case as also supported by the presence of a linear dispersion relation witnessed by the nonzero "sound velocity" extracted from a finite-size scaling analysis of the ground state energy [41]. Still our study leaves some questions unanswered. We strongly believe that this Letter will open the debate, and that together with the broader scientific community we will soon have a final picture of the mechanism behind this newly observed critical point.

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$$\hat{\phi}_j = \frac{1}{\sqrt{2}}(\hat{a}_j + \hat{b}_j^\dagger), \qquad \hat{\Pi}_j = \frac{i}{\sqrt{2}}(\hat{a}_j^\dagger - \hat{b}_j),$$
$$\hat{\phi}_j^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_j^\dagger + \hat{b}_j), \qquad \hat{\Pi}_j^\dagger = \frac{i}{\sqrt{2}}(\hat{b}_j^\dagger - \hat{a}_j),$$

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