Distinguishing between Quantum and Classical Markovian Dephasing Dissipation

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Understanding whether dissipation in an open quantum system is truly quantum is a question of both fundamental and practical interest. We consider n qubits subject to correlated Markovian dephasing and present a sufficient condition for when bath-induced dissipation can generate system entanglement and hence must be considered quantum. Surprisingly, we find that the presence or absence of time-reversal symmetry plays a crucial role: broken time-reversal symmetry is required for dissipative entanglement generation. Further, simply having nonzero bath susceptibilities is not enough for the dissipation to be quantum. We also present an explicit experimental protocol for identifying truly quantum dephasing dissipation and lay the groundwork for studying more complex dissipative systems and finding optimal noise mitigating strategies.

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Introduction.—Open quantum systems, where a system of interest interacts with an external environment, play a central role in many areas of physics ranging from quantum information to cosmology. Their evolution is, in general, nonunitary [1], being described by, e.g., a quantum master equation or more general quantum map. A ubiquitous type of system-environment interaction is dephasing. Dephasing interactions do not change populations of energy eigenstates of the system, but rather only impact coherences, i.e., the off-diagonal elements of the density matrix in the energy eigenbasis. The role of dephasing in quantum computation has been extensively studied (see, e.g., Ref. [2–4]). Even in this simple setting, there is a fundamental, surprisingly subtle question of interest: can the environment-induced dissipation of the system be attributed to interaction with a completely classical environment, or does it necessarily require a truly quantum environment?

Answering this question is, of course, contingent on how one defines the line between classical and quantum environments. Several previous works have examined this issue (see, e.g., Ref. [5–11]), largely in terms of possibly representing dissipative quantum dynamics with an equivalent classical process. In this Letter, we take instead an operational and experimentally motivated approach and define a truly quantum environment to be one that can mediate dissipative interactions that generate system entanglement. A necessary requirement for this phenomenon is having nonzero bath response susceptibilities [12–14], manifested in asymmetric-in-frequency environmental quantum noise spectral densities. We show, surprisingly, that this is not sufficient: bath-mediated dissipative interactions can only generate system entanglement if they cannot be mimicked using local measurements and feed forward. We note that an analogous approach has been suggested to test probe whether gravitational interactions are quantum [15–18].

We focus in this Letter on setups, where a set of qubits are coupled to a generalized Markovian dephasing environment (as described by a Lindblad master equation). We show that the presence or absence of environmental time-reversal symmetry (TRS) is crucial in determining whether bath-induced dissipation is classical. In the presence of TRS, this dissipation is always equivalent to driving by classical noise, whereas without TRS this is no longer necessarily true. We provide a condition based on the Peres-Horodecki criterion [19,20] that allows one to identify truly quantum dephasing dissipation. Our Letter provides a new approach to identifying truly quantum dissipative behavior and also provides a sensitive method for detecting broken TRS in dephasing environments.

Setup.—Consider a multiqubit system, whose dephasing interaction with a stationary environment is $\hat{H}_{\text{int}} = \sum_i \hat{Z}_i \otimes \hat{B}_i$, where $\hat{Z}_i$ is the Pauli $\hat{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ operator on qubit i, and $\hat{B}_i$ is a Hermitian environment operator. Throughout the Letter, we transform to the interaction picture with respect to the internal Hamiltonians of the system (S) and environment (E), $\hat{H}_S + \hat{H}_E$, where the environment operators are given by $\hat{B}_i(t) = e^{i\hat{H}_E t}\hat{B}_i e^{-i\hat{H}_E t}$. In the Markovian limit, the evolution of the system undergoing such correlated dephasing is described by the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation $(d\hat{\rho}/dt) = \mathcal{L}(\hat{\rho})$, with the Liouvillian given by

$$\mathcal{L}(\hat{\rho}) = -i[\hat{H}_{\text{LS}}, \hat{\rho}] + \sum_{i,j} c_{ij}\left(\hat{Z}_i\hat{\rho}\hat{Z}_j - \frac{1}{2}\{\hat{Z}_i\hat{Z}_j, \hat{\rho}\}\right). \quad (1)$$

Here $\hat{H}_{\text{LS}} = \frac{1}{2}\sum_{ij} h_{ij}\hat{Z}_i\hat{Z}_j$ is the so-called Lamb shift Hamiltonian [see Fig. 1(a)] and describes Hamiltonian Ising interactions mediated by the bath. The remaining

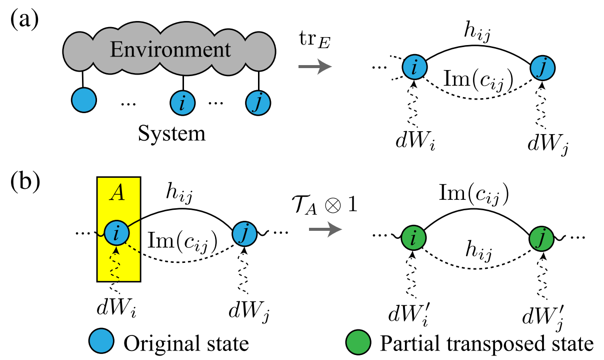

FIG. 1. (a) A system of n qubits is coupled to a Markovian dephasing environment. The resulting evolution obtained by tracing out the environment (tr_E) is a GKSL master equation (1) and can be decomposed into driving by effective classical noise dW_i with correlations $\mathrm{Re}(c_{ij})$, Hamiltonian Ising interactions h_{ij}, and dissipative Ising interactions $\mathrm{Im}(c_{ij})$. (b) The evolution of the partial transposed ($\mathcal{T}_A \otimes 1$) state with respect to subsystem A is again in GKSL form, with different classical noise dW_i', whose correlation is now $-c_{ij}$ and with the role of h_{ij} and $\mathrm{Im}(c_{ij})$ reversed.

terms describe bath-induced system dissipation and cause the net evolution to be nonunitary. The evolution generated by this equation is completely positive if and only if the matrix $C = (c_{ij})$ is positive semidefinite (PSD) [21].

Our central goal is to understand whether the bath-induced dissipation is classical or quantum; we thus set $h_{ij} = 0$ in what follows. It is tempting to think of the remaining dissipative evolution as always being equivalent to driving by external classical noise. This is not true, however, if $\mathrm{Im}(C) \neq 0$: as we show below, the imaginary part of C encodes dissipative bath-mediated interactions that are distinct from classical noise and cannot be mimicked by a Hamiltonian Ising interaction. Further, the presence of these interactions is necessary, but *not sufficient* to make the dissipation quantum, i.e., capable of generating system entanglement.

Consider first Eq. (1) with a real C and $\hat{H}_{\mathrm{LS}} = 0$. Such an evolution can always be emulated by (correlated) classical white noise [22]. Specifically, consider a system evolving under the Hamiltonian $\hat{H}_c = \sum_i b_i(t)\hat{Z}_i$, where $b_i(t)$ describe classical Gaussian fluctuations with $\langle b_i(t)\rangle = 0$. In the white noise limit, where $\langle b_i(t) b_j(t)\rangle = c_{ij}\delta(t - t')$, the average evolution of the system (over the fluctuations) reproduces the master equation of interest, with C corresponding to the covariance matrix of noise $b_i(t)$ [23,24]. Note that, because this evolution can always be emulated by a local time-dependent Hamiltonian, it cannot create entanglement in the system.

Interpretation of complex C.—As shown above, driving a system with classical noise will never generate a nonzero $\mathrm{Im}(C)$. To understand the physical origin of $\mathrm{Im}(C) \neq 0$, we revisit the general microscopic quantum bath model and $\hat{H}_{\mathrm{int}}$. Making use of the standard Born-Markov approximation [1], we can relate $\mathrm{Im}(C)$ to spatial asymmetries in the environment's response properties: $\mathrm{Im}(c_{jk}) = \frac{1}{2}\{\mathrm{Re}[\chi_{jk}(\omega = 0)] - \mathrm{Re}[\chi_{kj}(\omega = 0)]\}$. Here, $\chi_{jk}(\omega) \equiv -i\int_0^\infty dt e^{i\omega t}\langle[\hat{B}_j(t), \hat{B}_k(0)]\rangle$ are standard linear response susceptibilities, which describe how a bath operator expectation value $\langle \hat{B}_j(t)\rangle$ changes due to a weak perturbation that couples to $\hat{B}_k$ [12]. In contrast, $\mathrm{Re}(C)$ is not related to bath response; for a generic quantum bath, $\mathrm{Re}(C)$ are given by the symmetrized quantum noise spectra, which play the role of classical noise [12]. It is important to note that, if the bath Hamiltonian $\hat{H}_E$ has TRS and the bath operators all transform under TRS with the same parity, we may invoke Onsager-type reciprocity relations to show that $\mathrm{Im}(c_{jk}) = 0$ [25]. Conversely, for environments with broken TRS, which is typically the case for driven-dissipative environments, there is no fundamental reason to expect that $\mathrm{Im}(C)$ should vanish.

An alternate way to understand the bath-induced dissipation and $\mathrm{Im}(C)$ is to realize that it also describes a situation in which there is no environment, but where the system evolves because of continuous measurement and feed forward [26]. We first write the dissipative part of Eq. (1) as

$$\mathcal{L}_{\mathrm{diss}}(\hat{\rho}) = \sum_k \gamma_k \left(\hat{L}_k\hat{\rho}\hat{L}_k^\dagger - \frac{1}{2}\{\hat{L}_k^\dagger\hat{L}_k, \hat{\rho}\}\right), \qquad (2)$$

where γ_k and $\hat{L}_k$ are found by diagonalizing C [1]. Further, each $\hat{L}_k$ can be written as $\hat{L}_k = \hat{A}_k + i\hat{B}_k$, where $\hat{A}_k$ and $\hat{B}_k$ are Hermitian system operators. While such dissipators arise in many contexts [26–29], we focus on the connection to continuous measurement and feed forward of Ref. [30]. As shown in the Supplemental Material [24], each term k in Eq. (2) describes unconditional system evolution under a two-way measurement and feed forward scheme. One half of this scheme is a weak continuous measurement of $\hat{A}_k$ [26], with the measurement record used to set the amplitude of a drive applied to $-\hat{B}_k$. The other half is the reverse process, i.e., measuring $\hat{B}_k$ and feed forwarding the result to drive $\hat{A}_k$.

This equivalence immediately lets us make general statements on the properties of the dissipative evolution. In general, the measurement and feed forward realization involves nonlocal operations, indicating that a complex C might create system entanglement. However, there are notable exceptions. First, emulating a purely real C in this way does not have any feed forward driving. In this case, measurement results are discarded, and the evolution is just due to measurement backaction (which is equivalent to classical noise). No entanglement is generated. Even if $\mathrm{Im}(C) \neq 0$, the measurements and feed forward could all be purely local processes. Again, in this case, no entanglement generation is possible, and the environment dissipation would be categorized as being classical.

We also comment on a third way to realize a restricted class of processes with a complex C using classically stochastic time-dependent system Hamiltonians [8,31]. Suppose our system is coupled to classical noise whose time integral is a Poisson process with the rate ν. Averaging over this noise leads to a term $\nu(\hat{L}_k^\dagger \hat{\rho} \hat{L}_k - \hat{\rho})$ in the master equation, where $\hat{L}_k$ is a unitary operator reflecting the coupling of the noise to the system. Combined with the previously discussed classical white noise processes that generate Hermitian $\hat{L}_k$, we now have an approach for realizing any master equation with Hermitian or unitary Lindblad operators [8,31]. In our case, a process that can be expressed in this way can have nonzero $\mathrm{Im}(C)$, but can never generate system entanglement. The reverse statement is, however, surprisingly, not true: there exist master equations with nonzero $\mathrm{Im}(C)$ that are not equivalent to the above classical noise model, but nonetheless are unable to generate entanglement (see Supplemental Material [24]).

Entanglement generation.—We have defined the environment-induced dissipation in (1) as being quantum if it can generate entanglement within the system. To study this condition quantitatively, we now employ the Peres-Horodecki (PH) criterion [19,20]. It ensures that a state whose partial transpose (PT) has a negative eigenvalue is entangled. Thus, to see if environmental dissipation can create entanglement, we could, in principle, evolve arbitrary initial product states and check whether the PH condition is violated for *any* $t > 0$. We do not require steady-state entanglement generation (unlike, e.g., reservoir engineering protocols [32]). This would appear to be a formidable task. Fortunately, we can greatly simplify this problem. First, we find the exact evolution of the partial transposed state of the system with respect to a chosen subsystem A, i.e., $\hat{\rho}^{T_A} = (\mathcal{T}_A \otimes 1)(\hat{\rho})$ [see Fig. 1(b)]. Surprisingly, the evolution $(d\hat{\rho}^{T_A}/dt) = \tilde{\mathcal{L}}(\hat{\rho}^{T_A})$ is still in the GKSL form $\tilde{\mathcal{L}}(\hat{\rho}^{T_A}) = -i[\hat{H}_{\mathrm{PT}}, \hat{\rho}^{T_A}] + \tilde{\mathcal{L}}_{\mathrm{diss}}(\hat{\rho}^{T_A})$ [24], with $\hat{H}_{\mathrm{PT}} = \frac{1}{2}\sum_{ij} \tilde{h}_{ij} \hat{Z}_i \hat{Z}_j$, and

$$\tilde{\mathcal{L}}_{\mathrm{diss}}(\hat{\rho}^{T_A}) = \sum_{i,j} \tilde{c}_{ij} \left(\hat{Z}_i \hat{\rho}^{T_A} \hat{Z}_j - \frac{1}{2}\{\hat{Z}_i \hat{Z}_j, \hat{\rho}^{T_A}\} \right). \quad (3)$$

In this partial transposed equation of motion, the coefficients (for $i < j$) of the Hamiltonian and the dissipator are given by

$$(\tilde{h}_{ij}, \tilde{c}_{ij}) = \begin{cases} (\mathrm{Im}(c_{ij}), -\mathrm{Re}(c_{ij}) + i h_{ij}) & i \in A \text{ and } j \notin A, \\ (-h_{ij}, c_{ji}) & i \in A \text{ and } j \in A, \\ (h_{ij}, c_{ij}) & \text{otherwise.} \end{cases} \quad (4)$$

The coefficients for $i > j$ can be inferred from the symmetries $\tilde{c}_{ij} = \tilde{c}_{ji}^*$ and $\tilde{h}_{ij} = \tilde{h}_{ji}$.

Note that $\hat{H}_{\mathrm{PT}}$ is Hermitian; hence the only way that $\tilde{\mathcal{L}}$ can generate nonpositive states is through the dissipative part $\tilde{\mathcal{L}}_{\mathrm{diss}}$ specified by $\tilde{C}$. This is, in general, possible, as $\tilde{C}$ is Hermitian, but not necessarily PSD. If our original master equation had a nonzero Ising Hamiltonian $h_{ij} \neq 0$, we see clearly from Eq. (4) that $\tilde{C}$ could be non-PSD. This simply reflects the fact that Hamiltonian Ising interactions can create entanglement. We are, however, interested in the effects of the dissipative evolution alone, i.e., $h_{ij} = 0$. As we will see, in this case too, $\tilde{C}$ can fail to be positive. Note that there is a small subtlety here: to show the possibility of entanglement generation, we need to show that the evolution generated by $\tilde{\mathcal{L}}_{\mathrm{diss}}$ is not positive. However, a negative eigenvalue of $\tilde{C}$ only implies, in general, that the evolution is not completely positive [33]. Luckily, in our case, we can show that these two notions coincide (see Supplemental Material [24]).

To summarize, we have found a sufficient condition for an environment to be entangling and hence be truly quantum. To check this condition, we need to find $\tilde{C}$ using the recipe in Eq. (4) and examine its eigenvalues for all the possible choices of subsystem A. If there exists a negative eigenvalue in any of these cases, it indicates that the dissipative evolution is entangling. We note that, however, the absence of a negative eigenvalue does not rule out entanglement generation for $n > 2$, as there are entangled states with a positive partial transpose [34].

Case studies.—To provide further intuition, we now analyze three special cases of Markovian correlated dephasing on n qubits. Let $\{\mathbf{e}_i\}_{i=0}^{n-1}$ denote the standard basis of $\mathbb{C}^n$. Additionally, define the Fourier basis $\{\mathbf{f}_k\}_{k=0}^{n-1}$, such that $\mathbf{f}_k = (1/\sqrt{n}) \sum_{j=0}^{n-1} \omega^{jk} \mathbf{e}_j$, where $\omega = \exp(2\pi i/n)$. As our first example, consider a purely real correlation matrix $C^{(1)} = \mathbf{f}_0 \mathbf{f}_0^\dagger$, whose entries are $c_{ij}^{(1)} = 1/n$ for all i's and j's. As mentioned earlier, this process can always be emulated by classically correlated fluctuations of $\hat{\sigma}_z$ terms in the Hamiltonian and is not entangling. Using our procedure, $\tilde{C}^{(1)}$ stays PSD under any bipartition. This is because, for any real C, Eq. (4) is equivalent to mapping $\mathbf{e}_i \to -\mathbf{e}_i$ for all i's in one of the partitions. This is a unitary transformation and does not change the eigenvalues of the originally PSD matrix C.

Next, we consider a correlation matrix $C^{(2)}$, whose elements above (below) the diagonal are all i $(-i)$. The diagonal elements are all set equal to a constant $\gamma = n - 1$, chosen so that $C^{(2)}$ is PSD. This C matrix corresponds to an environment with broken TRS and with vanishing classical noise correlations (a situation where one might expect the bath to be maximally quantum). For concreteness, we take subsystem A to be the first m of our n qubits. Under Eq. (4), $\tilde{C}^{(2)}$ is a block diagonal matrix. It is obtained from $C^{(2)}$ by setting the off-diagonal blocks to zero, i.e., $\tilde{c}_{ij}^{(2)} = 0$ if $i \in A$ ($i \notin A$) and $j \notin A$ ($j \in A$), and transposing the block corresponding to A. Because the original matrix $C^{(2)}$ is PSD and noting that principal submatrices of a PSD matrix are also PSD [35], we conclude that $\tilde{C}^{(2)}$ is PSD. We note that the dynamics corresponding to $C^{(2)}$ can be fully realized using measurement and feed forward that is local with respect to the A/B bipartition. Hence, despite not being equivalent to classical noise, this environment cannot generate system entanglement and would be deemed classical under our classification.

Finally, we analyze a rank-1 complex correlation matrix $C^{(3)}$ that is impossible to emulate with a local measurement and feed forward strategy. Specifically, we choose $C^{(3)} = \mathbf{f}_1 \mathbf{f}_1^{\dagger}$. For $n \geq 3$, we have $\mathrm{Im}(C^{(3)}) \neq 0$, corresponding to a bath with broken TRS that mediates nonzero dissipative interactions. To show that the corresponding evolution is capable of generating entanglement, it suffices to find one bipartition such that $\tilde{C}^{(3)}$ has a negative eigenvalue. Choosing the first qubit as one of the partitions results in a rank-3 $\tilde{C}^{(3)}$. As shown in the Supplemental Material [24], the low-rank nature of $\tilde{C}^{(3)}$ allows us to analytically calculate $|\tilde{C}^{(3)}|_{+} = [(2-n)/4n^2]$ (the pseudo-determinant of $\tilde{C}$), which implies that $\tilde{C}^{(3)}$ has at least one negative eigenvalue for all $n > 2$. Note that, in general, the evolution of an initial product state will at most result in transient entanglement [36] (but no steady-state entanglement) (see Supplemental Material [24]).

Random environments.—The above examples suggest that both the imaginary and the real parts of the off diagonals of C are necessary to create entanglement. This expectation is corroborated by examining three-qubit random Liouvillians with $C = ww^{\dagger}$, where w is drawn from a complex Ginibre ensemble [37] (see Supplemental Material [24] for details). Analogous ensembles of random Liouvillians have been considered previously, in a different context, to understand spectral properties of random open quantum systems [38]. Interestingly, the results reported in Ref. [38] show no sensitivity to the choice of real and complex ensembles, whereas in our Letter the latter corresponds to broken TRS, which is necessary for entanglement generation. In Fig. 2, we observe that $\lambda_{\min}$, the minimum eigenvalue of $\tilde{C}$ [39], is non-negative when C is purely real [$\|\mathrm{Im}(C)\|_{\mathrm{fro}} = 0$] or has a purely imaginary off-diagonal part [$\|\mathrm{Re}(C) - \mathrm{diag}(C)\|_{\mathrm{fro}} = 0$]. However, when the norm of the imaginary and real off-diagonal parts are comparable, the fraction f of the samples that are entangling ($\lambda_{\min} < 0$) is maximized.

Our simple examples also raise another question: can dissipative interactions only generate entanglement if C is a low-rank matrix? Physically, this corresponds to a situation where the bath couples to the system via only a small number of delocalized system operators. To answer this question, we again consider random $C = ww^{\dagger}$ and vary system size. We examine the minimum eigenvalue of $\tilde{C}$ acting on the partial transposed state of the system with respect to the first qubit. To quantify the rank of C, we introduce $\mathrm{tr}(C^2)/\mathrm{tr}(C)^2$ that measures variance of C eigenvalues. We observe that the entangling behavior is not a property of rank-1 matrices [for which $\mathrm{tr}(C^2)/\mathrm{tr}(C)^2 = 1$] and is common in the ensemble of random C's we considered (see Fig. 3). Note that the minimum eigenvalue of C in this case scales inversely with n [40]. Hence, the transformation to $\tilde{C}$ is more likely to create a negative eigenvalue (see Supplemental Material [24]).

Experimental implementation.—When combined with the ability to measure C, our results can serve as a probe for fundamental symmetries and the nature of the environment. The evolution generated by the Lindbladian $\mathcal{L}$ in Eq. (1) can be decomposed into a decaying part from $\mathrm{Re}(C)$ and a phase evolution from $\mathrm{Im}(C)$ and $\hat{H}_{\mathrm{LS}}$ [41]. Reference [41] presents a compressed sensing protocol

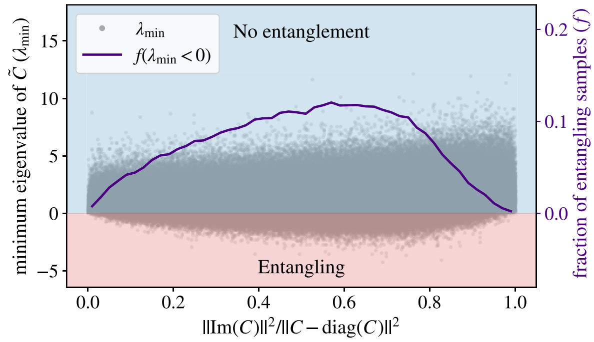

FIG. 2. The distribution of the minimum eigenvalue of $\tilde{C}$ for 10^6 random three-qubit dephasing environments. The solid purple curve (right axis) shows the fraction (over bins of size 0.02) of samples where $\tilde{C}$ has a negative eigenvalue (entangling). A C matrix with purely real or purely imaginary off diagonals is not entangling.

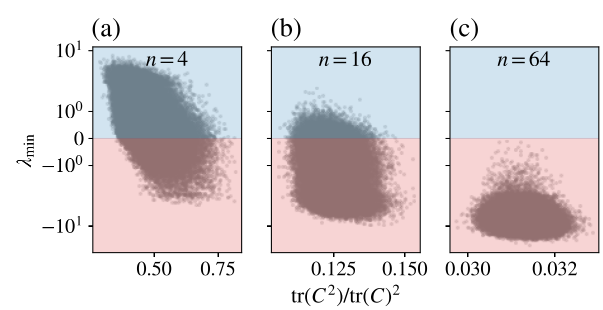

FIG. 3. Typicality of entangling C's. The distribution of the minimum eigenvalue $\lambda_{\min}$ of $\tilde{C}$ for 10^5 random C's as a function of $\mathrm{tr}(C^2)/\mathrm{tr}(C)^2$ for (a) $n = 4$, (b) $n = 16$, and (c) $n = 64$ qubits. To obtain $\tilde{C}$, we choose the first qubit as a subsystem. Entangling C's become more common as the system size increases.

that uses randomized measurement to extract both the real and imaginary part of c_{ij} and also the Lamb shift terms h_{ij}. Here, we present a less efficient but simpler scheme. Our measurement protocol relies on preparing and measuring Bell states $|\phi_{ij}\rangle = (1/\sqrt{2})(|0\rangle_i|0\rangle_j + |1\rangle_i|1\rangle_j)$ on pairs of qubits with $j > i$ (the omitted qubits are assumed to be in the $|0\rangle$ state). Let $\hat{\rho}(t)$ denote the state of the system after some time t that is initially prepared in $|\phi_{ij}\rangle$. The matrix element $\langle 0|_i\langle 0|_j\hat{\rho}(t)|1\rangle_i|1\rangle_j$ evolves as $(1/2)\exp[(i\Omega_{ij} - \Gamma_{ij})t]$. Therefore, by measuring this matrix element at different times and finding its decay rate and oscillation frequency, we can find both Ω_{ij} and Γ_{ij}. The real part of c_{ij} can then be directly extracted from $\Gamma_{ij} = 2(c_{ii} + c_{jj} + c_{ij} + c_{ji})$ as shown in [42]. The analysis of Ω_{ij}, however, is more subtle. First, unlike Γ_{ij}, Ω_{ij} contains linear contributions from $\mathrm{Im}(c_{km})$'s, where k and m are not just restricted to $\{i, j\}$. Therefore, we need to solve a linear system to find $\mathrm{Im}(C)$. Second, there might be other sources of phase evolution in addition to $\mathrm{Im}(C)$, e.g., the Lamb shift term, in the experiment. In particular, it is impossible to distinguish Lamb shift terms from $\mathrm{Im}(C)$ using only the above measurements.

However, performing an additional set of measurements using $|\bar{\phi}_{ij}\rangle = (\otimes_{k=1}^{i}|1\rangle_k) \otimes (1/\sqrt{2})(|0\rangle_j + |1\rangle_j)$ with $j > i$, and where qubits with omitted index are in $|0\rangle$, provides enough information to distinguish h_{km}'s contributions from $\mathrm{Im}(c_{km})$'s [24]. Determining the nature of such coherent phase errors is also helpful in a broader context. In the context of error mitigation, where correlated noise processes severely impact the performance of the device [2,43–45], it is important to correctly identify the source of noise to combat it. For example, coherent phase errors originating from parasitic ZZ couplings (see, e.g., Ref. [46,47]) can be simply canceled by an offset Hamiltonian, whereas a simple offset cannot help with the errors coming from $\mathrm{Im}(C)$.

Discussion.—Our study reveals that the presence or absence of TRS has a profound effect on dephasing dissipation, even in the Markovian limit: broken TRS is needed for the dissipation to be entanglement generating (and hence quantum). Our Letter thus provides a concrete experimental protocol for detecting the presence of broken environmental TRS. Note that, to check entanglement generation between all possible bipartitions, our protocol requires a time that scales exponentially with the system size. It is intriguing to ask whether this is a fundamental limitation or whether more efficient schemes are possible. It would be extremely interesting to apply our ideas to systems with nonlinear and nonlocal coupling to an environment and going beyond the Markovian limit and, more generally, study the role of TRS in more general kinds of dissipative dynamics (e.g., baths that couple transversely to the qubits).

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