Reverse Bootstrapping: IR Lessons for UV Physics

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S-matrix bootstrap and positivity bounds are usually viewed as constraints on low-energy theories imposed by the requirement of a standard UV completion. By considering graviton-photon scattering in the standard model, we argue that the low-energy theory can be used to put constraints on the UV behavior of the gravitational scattering amplitudes.

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Introduction.—In recent years *S*-matrix positivity bounds and bootstrap methods have matured into a powerful tool to constrain low-energy effective field theories (EFTs). The original nonlinear *S*-matrix positivity bounds of the 1970s were largely concerned with constraints on individual partial wave coefficients where often experimental data was forthcoming, or constraints on the amplitude within the Mandelstam triangle which is only well defined in gapped theories [1]. The modern use of positivity bounds was reinvigorated in Ref. [2], based on earlier work [3,4], where it was emphasized that these act as constraints on EFTs, including on massless ones once mild assumptions are made. These constraints are interconnected with causality considerations and for Lorentz invariant nongravitational theories where causality and locality are precisely defined, there are now a large number of robust bounds on Wilson coefficients from EFTs. The linear forward limit bounds of Ref. [2] were extended away from the forward limit in Ref. [5], and for particles of arbitrary spins in Refs. [6,7]. These generalized bounds have been used to provide powerful constraints on low-energy EFTs, for instance, in Refs. [8–27]. Subsequently, the linear positivity bounds were generalized to a set of nonlinear constraints in Refs. [28–30] using the same methodology as Stieltjes moment positivity bounds derived in Refs. [31–34]. More recently, the full use of crossing symmetry has tightened these nonlinear statements [35–40], which strongly overlap with *S*-matrix bootstrap bounds [41–44].

The application of these methods to theories with gravity is, however, less developed since the precise rules for causality are far less established [45–49]. In Refs. [49,50] it was argued that based on causality considerations, the usual amplitude combinations which are demanded to be positive could admit a small Planck scale suppressed negativity in the presence of gravity, and this was confirmed by the novel impact-parameter bounds of Ref. [51] as well as in Ref. [52]. This apparent gravitational weakening of positivity bounds is intimately connected with the fact that perturbative corrections to the sound speed in gravitational EFTs can appear to be superluminal [53–62] even though causality is never violated as these effects are not resolvable [47–49].

There are two central problems with extending the usual positivity methods to 2-2 scattering amplitudes when including gravity:

The first is that massless graviton loops give rise to a branch cut which extends to $t = 0$, preventing the continuation of the partial wave expansion from $t < 0$ to $t \geq 0$, where the positivity bounds ought to be satisfied (in terms of the standard s, t, and u Mandelstam variables). However, since the start of the branch cut is associated to light loops, progress can be made by either directly removing the low-energy loops in the manner of the improved positivity bounds [6,10], or applying the bounds to the tree level of the massless and light states, while loops of the heavy fields can be included or integrated out (see, e.g., Refs. [27,51]). Since loops of massless modes are not included, this also removes any issue with IR divergences that are pertinent in four dimensions.

The second is the presence of a massless t-channel pole associated with gravitational exchange. Since this pole grows as s^2, it is not possible to subtract it and to continue to use a dispersion relation with two subtractions as is allowed for nongravitational theories. However, there is no difficulty in working with a dispersion relation with more than two subtractions and hence many nontrivial positivity bounds and *S*-matrix bootstrap constraints have been applied by focusing on the higher order EFT operators (see, for example, Ref. [27] for an excellent recent discussion).

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The impact-parameter bounds of Ref. [51] evade both problems by working at $t<0$ and looking for a new set of positive integrals not immediately related to the standard bounds. The resulting bounds are then consistent with those conjectured in Refs. [50,63] but controlled by the lightest massive state integrated out.

One approach to tighten this bound is to assume a Regge behavior for the UV completion [52,64–67] as given below in Eq. (8). This behavior arises in weakly coupled string theory, but it can also be argued for universally, and folding this information into the bounds of the graviton-photon scattering either leads to the presence of a new tower of higher spin states starting at least at the TeV scale, or a violation of the Froissart bound, an IR/UV mixing, or a constraint on the slope of the residue of the Regge pole.

Gravitational positivity bounds.—In what follows, we shall consider scattering amplitudes that can be consistently computed while including tree and loop level contributions from all massive states, but only trees from massless ones (i.e., no massless loops). At low energies, these amplitudes admit an expansion which is determined by the tree amplitudes of the low-energy EFT obtained from integrating out all massive states. In the specific context of graviton scattering with other light states $Xh\to Xh$, the dangerous graviton loops may be removed by taking a scaling limit of the exact scattering amplitude $\mathcal{A}(s,t)$ as

$$\tilde{\mathcal{A}}_{Xh\to Xh}(s,t)\equiv\lim_{M_{\rm Pl}\to\infty}M_{\rm Pl}^2\mathcal{A}_{Xh\to Xh}(s,t). \tag{1}$$

The central point is that graviton loops enter the amplitude $\mathcal{A}$ only at order $1/M_{\rm Pl}^4$, and so by taking this scaling, graviton loops are automatically projected out in an RG independent manner [68]. This reduced amplitude satisfies unitarity in the sense [69]

$$\begin{aligned}{\rm Disc}\tilde{\mathcal{A}}_{Xh\to Xh}=&\frac{1}{2}\sum_Y[(2\pi)^4\delta^4(k_h+k_X-k_Y)\\&\times\tilde{\mathcal{A}}_{Xh\to Y}\tilde{\mathcal{A}}^*_{Xh\to Y}],\end{aligned} \tag{2}$$

where Y is a complete set of states in the UV completion not including gravitons and

$$\tilde{\mathcal{A}}_{Xh\to Y}\equiv\lim_{M_{\rm Pl}\to\infty}M_{\rm Pl}\mathcal{A}_{Xh\to Y}, \tag{3}$$

which is enough to ensure positivity for elastic scattering processes. With the graviton loops removed, the only remaining dangerous singularities are the graviton t-channel pole and the loops of any other massless states such as the photon. For the process we shall be considering, the latter are largely harmless and will not contribute at the order we shall be interested in.

We now make the standard assumption that the amplitude $\tilde{\mathcal{A}}_{Xh\to Xh}(s,t)$ admits a dispersion relation with two subtractions in the physical region $t<0$. Although the Froissart bound [70–72] does not strictly apply to massless states, reasonable causality considerations applied to the scattering amplitude in impact parameter space in the physical region imply the bound

$$\lim_{|s|\to\infty}s^{-2}\tilde{\mathcal{A}}_{Xh\to Xh}(s,t)=0\quad\text{for } t<0, \tag{4}$$

throughout the complex s plane. In particular in $D\geq 5$ this bound is expected for low-energy EFTs that descend from string theory, and so a violation of the $<|s|^2$ growth would hence violate predictions from perturbative string theory. In $D=4$ the situation is more subtle because of IR divergences; however, by working in the scaling limit (1) we have removed any dangerous IR contributions from graviton loops. Hence assuming (4), and assuming that light loops do not spoil the standard analyticity conditions, the amplitude enjoys a twice-subtracted dispersion relation for $t<0$:

$$\begin{aligned}\tilde{\mathcal{A}}_s(s,t)=&a_s(t)+b_s(t)s+(s\text{-and }u\text{-channel poles})\\&+\frac{s^2}{\pi}\int_0^\infty d\mu\frac{{\rm Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^2(\mu-s)}+\frac{u^2}{\pi}\int_0^\infty d\mu\frac{{\rm Disc}\tilde{\mathcal{A}}_u(\mu,t)}{\mu^2(\mu-u)},\end{aligned} \tag{5}$$

where $\tilde{\mathcal{A}}_s$ denotes the s-channel process $Xh\to Xh$ and $\tilde{\mathcal{A}}_u$—the crossed process $X\bar{h}\to X\bar{h}$. The key observation is that while the amplitude on the left-hand side of Eq. (5) contains a spin-$2t$-channel pole, the pole does not explicitly appear in the dispersion relation valid for $t<0$, on the right-hand side of Eq. (5). Hence the pole is found within the dispersive integral. More concretely, as we approach $t=0$ from below,

$$\lim_{t\to 0^-}\left(\int_0^\infty d\mu\frac{{\rm Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^2(\mu-s)}+s\leftrightarrow u\right)\sim\frac{1}{t}. \tag{6}$$

Since the pole does not arise in the discontinuity, it must arise from the failure of the integral to converge as $t\to 0$. At the same time we know that a dispersion relation with three subtractions is well behaved even for $t>0$ (again assuming massless loops do not contribute). This implies that $\int_0^\infty d\mu[{\rm Disc}\tilde{\mathcal{A}}_{s,u}(\mu,t)/\mu^3(\mu-s)]$ is a convergent integral for small $t>0$. We thus conclude that as $\mu\to\infty$ the discontinuity behaves as

$$\begin{cases}{\rm Disc}\tilde{\mathcal{A}}_{s,u}(\mu,t)<\mu^2, & \text{for } t<0,\\ \mu^2<{\rm Disc}\tilde{\mathcal{A}}_{s,u}(\mu,t)<\mu^3, & \text{for small } t>0.\end{cases} \tag{7}$$

Assuming the mildest analytic behavior for the t dependence of the discontinuity in either of the s and u channels, we are necessarily led to the Regge assumption for fixed t near $t=0$ [52,64–67],

$$\lim_{\mu\to+\infty}\mathrm{Disc}\tilde{\mathcal{A}}_{s,u}(\mu,t)=r_{s,u}(t)\Lambda_r^4\left(\frac{\mu}{\Lambda_r^2}\right)^{\alpha_{s,u}(t)},\qquad(8)$$

where $\alpha(t)$ is the Regge trajectory, and $r(t)$ is related to the residue of the associated Regge pole. The scale Λ_r is freely chosen as it can be absorbed into $r(t)$ and is introduced for later convenience. The Regge slopes satisfy $\alpha(t)<2$ for $t<0$ and $\alpha(t)>2$ for $t>0$ [73]. Given the absence of massless loops we expect $\alpha(t)$ to be analytic at $t=0$. We stress that we are led to this Regge assumption without any input from string theory, although the latter is certainly consistent with it (see, e.g., Refs. [74,75]). Defining for each channel the difference

$$R(\mu,t)\equiv\mathrm{Disc}\tilde{\mathcal{A}}(\mu,t)-r(t)\Lambda_r^4\left(\frac{\mu}{\Lambda_r^2}\right)^{\alpha(t)}\qquad(9)$$

(where we omit the s, u subscripts unless needed), the dispersion relation may be reorganized into

$$\begin{aligned}\tilde{\mathcal{A}}(s,t)&=a_s(t)+b_s(t)s+(s\text{-channel poles})\\&\quad+\frac{s^2r_s(t)}{\pi[2-\alpha_s(t)]}+\frac{s^2}{\pi}\int_0^{\Lambda_r^2}d\mu\frac{\mathrm{Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^3}\\&\quad+\frac{s^2}{\pi}\int_{\Lambda_r^2}^{\infty}d\mu\frac{R_s(\mu,t)}{\mu^3}+\frac{s^3}{\pi}\int_0^{\infty}d\mu\frac{\mathrm{Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^3(\mu-s)}\\&\quad+s\leftrightarrow u.\end{aligned}\qquad(10)$$

The crucial difference is that the dispersion relation (10) is now also valid for $t>0$, unlike Eq. (5), since all of the dispersive integrals are convergent. The t-channel pole is now explicit in the Regge slope contribution

$$\lim_{t\to0}\frac{s^2r(t)}{\pi[2-\alpha(t)]}\sim-\frac{s^2r}{\pi\alpha' t},\qquad(11)$$

given that $\alpha(t)$ is analytic in t at $t=0$, so that $\alpha(t)=2+\alpha' t+\mathcal{O}(t^2)$. Thus defining the amplitude with poles removed in all three channels in a crossing symmetric way

$$\hat{\mathcal{A}}(s,t)\equiv\tilde{\mathcal{A}}(s,t)-(s\text{-}, u\text{-}, \text{and } t\text{-channel poles}),\qquad(12)$$

we infer the forward limit positivity bound

$$\begin{aligned}\partial_s^2\hat{\mathcal{A}}(0,0)>&-\frac{r_s}{\pi\alpha'_s}\left(2\frac{r'_s}{r_s}-\frac{\alpha''_s}{\alpha'_s}\right)\\&+\frac{2}{\pi}\int_{\Lambda_r^2}^{\infty}d\mu\frac{R_s(\mu,0)}{\mu^3}+s\leftrightarrow u.\end{aligned}\qquad(13)$$

Now crucially the above formula is valid for any value of Λ_r, and in particular we are free to choose Λ_r to be some scale much larger than the actual scale at which the Regge behavior kicks in. Since $R(s,t)$ is the subleading term in the Regge behavior, we would expect it to be suppressed by more than $\ln s$ relative to the leading part [76]. Hence we may scale $\Lambda_r\to\infty$ to ensure that the bound is effectively

$$\partial_s^2\hat{\mathcal{A}}(0,0)>-\beta_s[2(\ln r_s)'-(\ln\alpha'_s)']+s\leftrightarrow u,\qquad(14)$$

where we have defined $\beta_{s,u}=r_{s,u}/(\pi\alpha'_{s,u})$. The β's combined as $\beta=\beta_s+\beta_u$ which can be matched against the actual low energy t-channel pole

$$\tilde{\mathcal{A}}(s,t)=-\beta\frac{s^2}{t}+\hat{\mathcal{A}}(s,t)+(s\text{ and }u\text{ poles}).\qquad(15)$$

In practice for the scattering of only massless states, β is either of order one or zero depending on whether the given process allows t-channel graviton exchange. Moreover, the slope of the Regge residue is always positive by virtue of unitarity and the partial wave expansion

$$\partial_t[\mathrm{Disc}\tilde{\mathcal{A}}_{s,u}(\Lambda_r^2,0)]>0\Rightarrow r'_{s,u}>0.\qquad(16)$$

In the rest of this Letter, we will focus on the photon-graviton scattering process $Ah\to Ah$ accounting for standard model (SM) effects and inferring the implications of the bound (14). Remarkably, we shall see that this provides us a bound on UV rather than on IR physics.

AhAh positivity.—Because of the universal nature of the graviton coupling, all electroweak and QCD sector particles contribute to the $Ah\to Ah$ scattering process. We may start by considering all the SM particles to be minimally coupled to gravity in a covariant way, although the implications of this Letter are insensitive to that assumption,

$$\mathcal{L}=-\frac{M_{\mathrm{Pl}}^2}{2}R+\mathcal{L}_{\mathrm{SM}}(g_{\mu\nu},A_\mu,\psi,W^\pm,Z,\mathrm{QCD},\ldots).\qquad(17)$$

The graviton enters the metric as $g_{\mu\nu}=\eta_{\mu\nu}+2h_{\mu\nu}/M_{\mathrm{Pl}}$, and A_μ designates the photon. Every charged lepton ψ, the W bosons and the QCD sector enter the photon-graviton scattering and the relevant diagrams are schematically shown in Fig. 1 of the Supplemental Material [77]. In practice, however, up to order s^2 in the amplitude, the effects of all contributions from the SM to the $Ah\to Ah$ amplitude can be captured by the following operators:

$$\mathcal{L}=-\frac{M_{\mathrm{Pl}}^2}{2}R-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+b_3F_{\mu\nu}F_{\rho\sigma}R^{\mu\nu\rho\sigma}+\mathcal{O}_{\mathrm{dim}\geq8},\qquad(18)$$

where we have ignored operators that are either topological or removable by field redefinitions (and hence do not contribute independently to the amplitude) as well as dim-8 or higher operators that are irrelevant to this discussion. In practice the value of b_3 is dominated by the effects of the electron loops [53],

$$b_3 = -\frac{\alpha}{360\pi m_e^2}\left[1+\mathcal{O}\left(\frac{m_e^2}{m_W^2},\frac{m_e^2}{m_{\rm meson}^2}\right)\right], \tag{19}$$

where m_e is the electron mass, $\alpha = q_e^2/(4\pi)$ is the fine-structure constant, and q_e is the electric charge. Following the conventions laid out in the Supplemental Material [77], the nonzero definite helicity $Ah \to Ah$ amplitudes are given by

$$\begin{aligned}
M_{\rm Pl}^2\mathcal{A}_{++\to++} &= M_{\rm Pl}^2\mathcal{A}_{--\to--} = -\frac{s^2}{t} + \mathcal{O}(s^3),\\
M_{\rm Pl}^2\mathcal{A}_{++\to--} &= M_{\rm Pl}^2\mathcal{A}_{--\to++} = 2b_3 t^2 + \mathcal{O}(s^3),\\
M_{\rm Pl}^2\mathcal{A}_{++\to-+} &= M_{\rm Pl}^2\mathcal{A}_{--\to+-} = 2b_3 su,
\end{aligned} \tag{20}$$

wherein the amplitude $\mathcal{A}_{h_1,h_2\to h_3,h_4}$, h_1, h_3 are the photon polarizations, and h_2, h_4 the graviton ones. These amplitudes are consistent with those derived in Ref. [78] when $b_3 = 0$. The remaining amplitudes can be expressed in terms of the amplitudes given above by using the fact that the amplitudes are symmetric under parity and the s, u crossing symmetry, i.e.,

$$\mathcal{A}_{h_1,h_2\to h_3,h_4}(s,t,u) = \mathcal{A}_{h_1,\bar{h}_4\to h_3,\bar{h}_2}(u,t,s). \tag{21}$$

In particular, this implies that $\mathcal{A}_{+-\to+-}(s,t,u) = \mathcal{A}_{++\to++}(u,t,s)$. As one can see from the above results, there is no contribution to the positivity bounds on elastic definite-helicity amplitudes coming from the b_3 term.

We can further consider initial and final photon and graviton states with indefinite polarizations,

$$|A\rangle = a_+|+\rangle + a_-|-\rangle \quad \text{and} \quad |h\rangle = h_+|+\rangle + h_-|-\rangle,$$

where $a_\pm$, $h_\pm$ are complex numbers normalized so that $|a_+|^2 + |a_-|^2 = 1$, etc. Subtracting the poles in all three channels, the result is sensitive to the indefinite state of the photon (see Supplemental Material [77] for details),

$$\partial_s^2\hat{\mathcal{A}}(0,0) = -8b_3\mathrm{Re}(a_+a_-^*), \tag{22}$$

which is sign indefinite, regardless of the sign of b_3. In particular, we may make the reasonable choice $a_+ = 1/\sqrt{2}$ and $a_- = \mathrm{Sign}(b_3)/\sqrt{2}$ for which

$$\partial_s^2\hat{\mathcal{A}}(0,0) = -4|b_3|. \tag{23}$$

This is where the application of the positivity bounds (14) is particularly insightful as it leads to

$$2\beta_s(\ln r_s)' + 2\beta_u(\ln r_u)' > 4|b_3| + \beta_s(\ln\alpha_s')' + \beta_u(\ln\alpha_u')'.$$

The gravitational positivity bounds are thus violated unless either $(\ln r)'$ or the Regge slope $(\ln\alpha')'$ are bounded by the ratio of the mass of the electron to its charge. In this sense, the positivity bound can only be viewed as an IR constraint on UV physics.

Reverse bootstrapping.—The main observation is that the expected positivity bounds informed by a UV Regge behavior (14) appear to be violated by the amount (23) which is sensitive to the mass to charge ratio of the lightest charged particle in the SM, namely, the electron. We will go through a list of potential implications and emphasize that irrespectively to how nature resolves this tension, the SM does provide a remarkable constraint on UV physics: 1. *Regge residue:* The first possible resolution is that the residue of the Regge pole associated with the scattering of gravitons and photons varies at a scale related to the electron mass to charge ratio, $(\ln r)' \geq (m_e/q_e)^{-2} \sim (10^{-3}\ \mathrm{GeV})^{-2}$ [79]. This is a remarkable outcome as the Regge behavior as indicated in Eq. (8) is typically only related to the behavior of UV physics and one would not expect it to be set by the electron mass scale. 2. *Regge slope:* Another way out could be to set the scale of the Regge slope to be of order of the electron mass to charge ratio, $|\ln\alpha'|' \sim |b_3|^{-1} \sim m_e^2/q_e^2$. This would then imply the presence of a higher spin Regge pole already at the scale $\sqrt{m_e M_s/q_e}$ where $M_s = 1/\sqrt{\alpha'}$, leading to an infinite tower of higher spins starting at or below about 10^4 TeV [80]. 3. *Causality or locality:* In order to derive the positivity bounds, a certain level of causality or locality has been postulated when assuming the Froissart-like bound (4). While this bound is preserved for amplitudes derived from perturbative string theory in $D \geq 5$, it is possible that it is not technically applicable in the context of gravitational EFTs. In $D = 4$ the bound is known to be more subtle due to IR divergences, however by working with $\tilde{\mathcal{A}}$ defined in Eq. (1) we have removed the dangerous graviton loops. If failure of this bound were the reason why the amplitude (23) carries such a high level of negativity, the consequences for UV physics and string theory in particular would be significant. 4. *Light loops and gaplessness:* We have argued that to the order we are interested in, the amplitude is insensitive to graviton and photon loops. Graviton loops are Planck scale suppressed and do not enter $\tilde{\mathcal{A}}$ by construction. At low energies photon loops contribute at best as $s^2 t\log t$, so $\partial_s^2\hat{\mathcal{A}}$ is finite at $s = t = 0$ even if $\partial_s^3\hat{\mathcal{A}}$ and higher derivatives of the amplitude are not. More dangerous is the fact that photon loops may undermine the Froissart-like bound (4). While technically possible as a resolution, it would be indicative of a nontrivial UV/IR mixing and fall under the previous category as a weakening of locality.

The possibility of introducing other new physics is discussed in the Supplemental Material [77] and we argue that other than the inclusion of an infinite tower of higher spin at the TeV scale or lower, there is no new beyond standard model (BSM) physics nor nonminimal couplings that could ameliorate the situation.

For pragmatic reasons, we have focused our discussion on graviton-photon exchange as it shows a clear level of negativity within known SM physics. All the arguments presented here are however generic and apply to any U(1). In particular for any other dark sector U(1) or BSM physics, the pole-subtracted graviton-gauge field indefinite scattering amplitude will always acquire a negative contribution that scales as the mass of the lightest particle charged under this U(1). For instance, imagining a dark photon and charged dark matter particles under this dark U(1) as in Ref. [81], one would expect the residue of the Regge behavior to carry a scale as small as the lightest charged dark matter particle, a scale which could, in principle, be extremely low. In some of these models, the dark photon could also be massive hence avoiding any IR divergences issues. Whether we are dealing with the actual photon or with another gauge field, there are no other operators one could include into the EFT that would change our results and the positivity bounds cannot be read as a constraint on the cutoff of the EFT. Rather the constraint has to be imposed directly at the level of either the Regge behavior, the Froissart bound, the mixing with IR loops, or the presence of an infinite tower of higher spin states at the scale $\sqrt{mM_s/q}$, where m and q are the mass and charge of the lightest charged particle. Interestingly, the scale associated with this behavior is closely related to that entering the weak gravity conjecture [63,64,82–84].

Irrespective of which avenue is the most likely explanation, our findings show how SM physics has to be woven into UV physics.

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[77] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.051602 for more details. The supplementary material contains the conventions used and the technical details of the computation of the photon-graviton scattering amplitude. It also includes an extended discussion about the possibility of introducing new physics in order to avoid the conclusions reached in the main text and a figure of the relevant Feynman diagrams contributing to the photon-graviton scattering process within the electroweak theory.

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