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Demonstration of Complete Information Trade-Off in Quantum Measurement

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While an information-disturbance trade-off in quantum measurement has been at the core of foundational quantum physics and constitutes a basis of secure quantum information processing, recently verified reversibility of a quantum measurement requires to refine it toward a complete version of information trade-off in quantum measurement. Here we experimentally demonstrate a trade-off relation among all information contents, i.e., information gain, disturbance, and reversibility in quantum measurement. By exploring quantum measurements applied on a photonic qutrit, we observe that the information of a quantum state is split into three distinct parts accounting for the extracted, disturbed, and reversible information. We verify that such different parts of information are in trade-off relations not only pairwise but also triplewise all at once, and find that the triplewise relation is tighter than any of the pairwise relations. Finally, we realize optimal quantum measurements that inherently preserve quantum information without loss of information, which offer wider applications in measurement-based quantum information processing.

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Quantum measurement is at the heart of foundational quantum physics [1] and plays a major role to readout information in quantum technologies [2–6]. However, since a quantum measurement inevitably disturbs the measured system, the amount of extracted information from a quantum state has a certain limit against the state disturbance [7–10]. A long-standing wisdom of this has been "the more information of a quantum state is extracted by a quantum measurement the more the state is disturbed" [11–18]. Such a trade-off relation has both fundamental and practical importance in establishing a basis of secure quantum information processing [19,20].

However, a series of recent works observed that a quantum state disturbed by a quantum measurement weakly interacting with the measured system can be faithfully recovered by a postmeasurement operation [21–38]. This implicates that a part of total information remains and allows us to recover the original quantum state after the measurement. Accounting for such a reversible information, the reversibility of quantum measurement has been quantified and analyzed as an additional information content in quantum measurement [24–27,39]. In this line, reversing or undoing quantum measurement has been realized in various physical qubits [32–39], and applied for quantum error correction [28], gate operation [29,30], and decoherence suppressions [31–34]. Therefore, verifying trade-off relations encompassing all information contents, i.e., information gain, disturbance, and reversibility in quantum measurement, has become crucial for developing reliable quantum technologies.

In this Letter, we experimentally verify a complete information trade-off relation in quantum measurement. To that end, we use a scheme for varying the type and strength of quantum measurement on path-encoded photonic qutrits and explore the amounts of extracted, disturbed, and reversible information by performing different types of quantum measurements. We show that three information contents are quantitatively linked by a triplewise trade-off relation obeying the fundamental upper bounds derived in Ref. [40]. To our knowledge, this is the first demonstration of the complete trade-off relation among information gain, disturbance, and reversibility of quantum measurement all at once. It experimentally proves that the triplewise trade-off relation is tighter than any of the pairwise relations [12,27,40] and reveals the emergence of the reversibility in multidimensional quantum measurements. Finally, we establish optimal quantum measurements inherently preserving quantum information, in the sense that all information is changed into another form by measurement without any missing part, which would find wider applications in measurement-based quantum information processing.

Assume that an observer performs a quantum measurement to obtain information from a quantum state $|\psi\rangle$ [see Fig. 1(a)]. A quantum measurement can be described by operators $\hat{M}_r$ satisfying $\sum_r \hat{M}_r^\dagger \hat{M}_r = \hat{I}$. For each outcome r, the state is altered to $|\psi_r\rangle = \hat{M}_r|\psi\rangle/\sqrt{p(r,\psi)}$, where $p(r,\psi) = \langle\psi|\hat{M}_r^\dagger \hat{M}_r|\psi\rangle$, and the observer estimates the state as $|\tilde{\psi}_r\rangle$. The amount of *information gain* by the

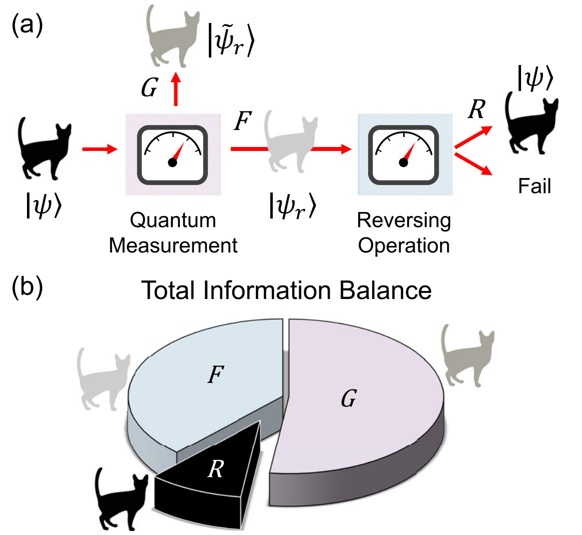


FIG. 1. (a) Quantum measurement is performed on a quantum state $|\psi\rangle$. Information can be extracted by estimating the state based on the measurement outcomes, i.e., $|\tilde{\psi}_r\rangle$, the amount of which is the information gain G. The input state is altered to $|\psi_r\rangle$ and the transmitted (undisturbed) information can be evaluated as the operation fidelity F. Then, a reversing operation can recover $|\psi\rangle$ probabilistically with the reversibility R. (b) The information of a quantum state is split into three parts, G, F, and R, by a quantum measurement.

observer can then be quantified by the closeness between $|\psi\rangle$ and $|\tilde{\psi}_r\rangle$ as $G = \int d\psi \sum_r p(r,\psi)|\langle\tilde{\psi}_r|\psi\rangle|^2$ [12]. On the other hand, the *operation fidelity* defined by the closeness between $|\psi\rangle$ and $|\psi_r\rangle$ as $F = \int d\psi \sum_r p(r,\psi)|\langle\psi_r|\psi\rangle|^2$ accounts for the amount of transmitted or undisturbed information. This in turn represents the amount of disturbance by $1-F$. The observer then applies a reversing operation attempting to restore the initial state $|\psi\rangle$. The *reversibility* R can then be defined as the maximum success probability of the faithful recovery of $|\psi\rangle$. A reversing operator $\hat{R}^r$ satisfies $\hat{R}^r\hat{M}_r|\psi\rangle \propto |\psi\rangle, \ \forall \ |\psi\rangle$. A quantum measurement can thus split the information of a quantum state into three parts, i.e., the extracted G, transmitted (undisturbed) F, and reversible R information [see Fig. 1(b)]. These quantities characterize a quantum measurement.

In what follows, we shall explore information contents by applying quantum measurements given in the form of

$$\hat{M}_r \equiv \hat{V}_r(\lambda_0^r|0\rangle\langle 0| + \lambda_1^r|1\rangle\langle 1| + \lambda_2^r|2\rangle\langle 2|)\hat{U}_r, \qquad (1)$$

satisfying $\sum_r \hat{M}_r^\dagger \hat{M}_r = I$ (set $\hat{V}_r = \hat{U}_r = I$ for simplicity), where different inputs of $\lambda_{i=0,1,2}^r$ determine the type of measurement. By exploring G, F, and R by changing the type and strength of measurements, we aim to demonstrate a complete trade-off relation. To implement quantum measurements in Eq. (1), we employ a heralded single-photon qutrit state. A signal photon can have three possible path modes, lower $|0\rangle$, middle $|1\rangle$, and upper $|2\rangle$ path modes after a set of half-wave plates (HWPs) and polarizing beam displacer as shown in Fig. 2. Polarization of photons in all path modes is arranged to be a horizontal polarization state. The relative phases ϕ among three path modes can be controlled by a set of two quarter-wave plates (QWPs) fixed at 45° and HWP with an angle of α without changing the polarization state of the signal photons with $\phi = 2\alpha$. Then, we can prepare an arbitrary qutrit state $|\psi\rangle = a_0|0\rangle + e^{i\phi_1}a_1|1\rangle + e^{i\phi_2}a_2|2\rangle$, where $\sum_i a_i^2 = 1$.

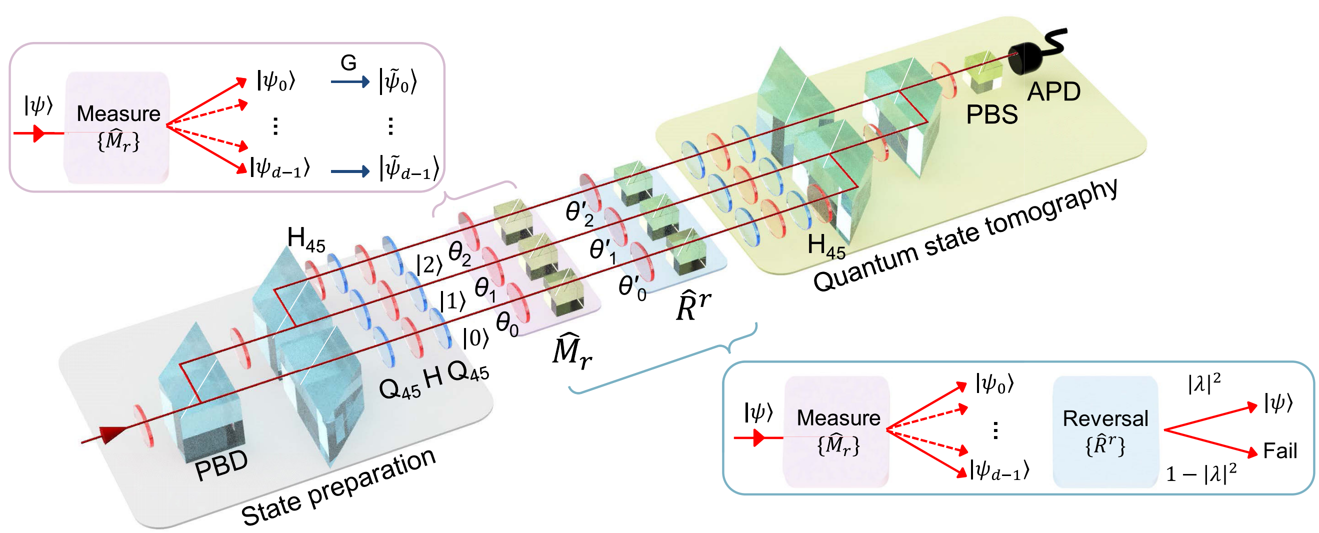


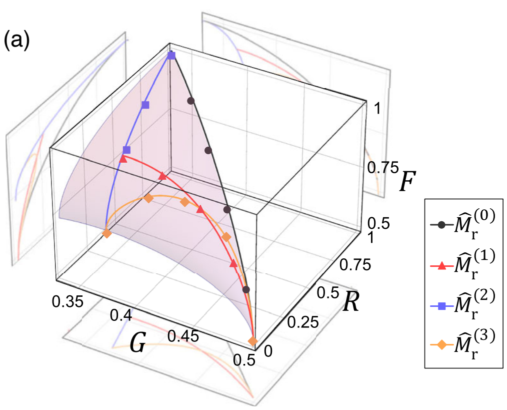
FIG. 2. Schematic of the experimental setup. Quantum measurement and reversing operators are realized using a set of HWPs and PBSs. By controlling the angles θ_i of HWPs, a generalized qutrit measurement operator can be implemented as $\hat{M}_r = \cos 2\theta_0|0\rangle\langle 0| + \cos 2\theta_1|1\rangle\langle 1| + \cos 2\theta_2|2\rangle\langle 2|$ and the reversal operator can be obtained by the same way as $\hat{R}^r = \cos 2\theta'_0|0\rangle\langle 0| + \cos 2\theta'_1|1\rangle\langle 1| + \cos 2\theta'_2|2\rangle\langle 2|$. (PBD, polarizing beam displacer; H_{45}, half-wave plate fixed at 45°; H, half-wave plate; Q, quarter-wave plate; PBS, polarizing beam splitter; APD, avalanche photodiode.).

The prepared states undergo $\hat{M}_r$ or both $\hat{M}_r$ and $\hat{R}^r$. See Supplemental Material [41] for experimental details.

To evaluate G and F for a qutrit system, we perform the symmetric and informationally complete positive-operator-valued measures (SIC POVM) with nine states [42]. POVMs for $\hat{M}_r$ and $\hat{R}^r$ are realized with a set of HWPs and polarizing beam splitters (PBSs). The transmission amplitude for each path is modified by using a set of HWP and PBS. Different types of quantum measurement operator in Eq. (1) can be realized by adjusting the angle θ_i of HWP as $\lambda_i^r = \cos 2\theta_i$ with $i = 0, 1, 2$ as shown in Fig. 2. The reversing operator $\hat{R}^r$ can be implemented by the same way as $\hat{M}_r$ using a set of HWPs of θ_i' and PBSs.

We realize different $\hat{M}_r^{(t)}$ with $t = 0, 1, 2, 3$ (the explicit forms are given in the table in Fig. 3) by varying the measurement strength parametrized by p. We evaluate G and F for $\hat{M}_r^{(t)}$ with specific p by analyzing the quantum state tomography for nine pure states of SIC POVM. R is evaluated by analyzing the final state after both $\hat{M}_r$ and $\hat{R}^r$ are performed. See Supplemental Material for the detailed information on how to extract G, F, and R quantities from the experimental data [41]. To verify that the initial state is retrieved after reversing operation, we perform quantum process tomography (QPT) for analyzing the realized operation. Detailed information on QPT results are provided in Supplemental Material [41].

Figure 3 presents the main results. The obtained G, R, and F for each $\hat{M}_r^{(t)}$ and p are plotted as a marker in Fig. 3(a). We observe that the amount of G, F, and R tend to vary in a trade-off manner and global exchanges occur among them as p changes. The amount of disturbance $1 - F$ and information gain G by $\hat{M}_r^{(0)}$, $\hat{M}_r^{(1)}$, $\hat{M}_r^{(2)}$ exhibit monotonic increases as increasing p, while the reversibility R decreases. $\hat{M}_r^{(3)}$ draws a nontrivial quadratic curve of information exchanges as $\lambda_{i=1,2}^r$ is given as a quadratic function of p. The amount of G, F, and R depending on p are plotted in Supplemental Material for each quantum measurement $\hat{M}_r^{(t)}$ [41]. While these demonstrate triplewise quantitative links among G, F, and R for different types of quantum measurements, all the obtained results hold a certain upper limit represented as a shaded surface in Fig. 3(a), which meets the bound [40]:



	$\lambda_{i=r}^r$	$\lambda_{i=r+1}^r$	$\lambda_{i=r+2}^r$	p range
$\hat{M}_r^{(0)}$	$\sqrt{p}$	$\sqrt{\frac{1-p}{2}}$	$\sqrt{\frac{1-p}{2}}$	$1/3 \le p \le 1$
$\hat{M}_r^{(1)}$	$\sqrt{p}$	$\sqrt{\frac{(1-p)(3-p)}{3}}$	$\sqrt{\frac{p(1-p)}{3}}$	$0.4586 \le p \le 1$
$\hat{M}_r^{(2)}$	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{1-2p}$	$1/3 \le p \le 1/2$
$\hat{M}_r^{(3)}$	$\sqrt{p}$	$\sqrt{p^2 - \frac{5}{2}p + \frac{3}{2}}$	$\sqrt{-p^2 + \frac{3}{2}p - \frac{1}{2}}$	$1/2 \le p \le 1$

Table I. Different types of quantum measurements with $r = 0, 1, 2$ ($r \equiv r$ mod 3).

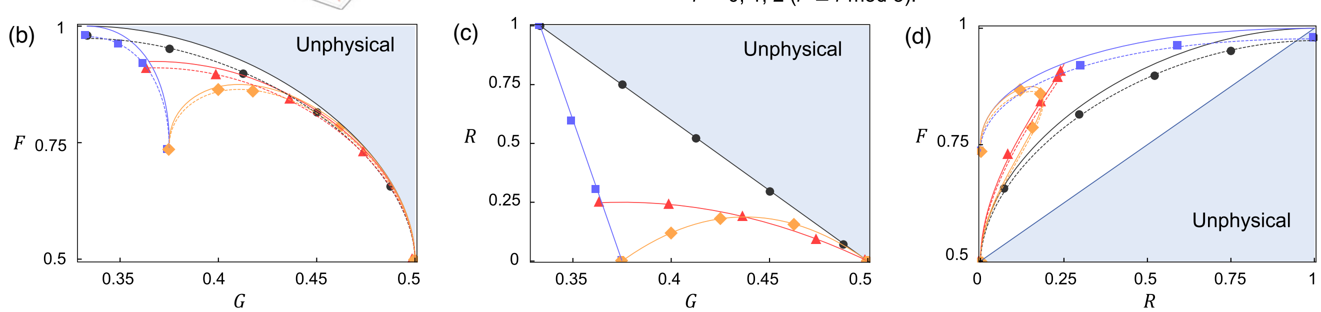


FIG. 3. Complete trade-off relations in quantum measurement. (a) Experimentally obtained G, F, R are plotted for different quantum measurements listed in Table I shown herein in blue shading. Black, red, blue, and yellow markers refer to experimental results for $\hat{M}_r^{(t)}$ with $t = 0, 1, 2, 3$ and different p, respectively, and solid lines represent the expectation value evaluated by continuously changing p for an ideal $\hat{M}_r^{(t)}$. The upper bound of the triplewise trade-off relation G-F-R is represented as a surface. Each pairwise relation is obtained by projecting onto (b) G-F, (c) G-R, and (d) F-R planes, respectively. The shaded region indicates unphysical regions due to the bound of each pairwise relations. The dashed lines represent the fitted trade-off relations assuming a nonideal input state $\hat{\rho}(e) = e|\psi\rangle\langle\psi| + (1-e)I/3$. The error parameters e are obtained to be $e = 0.960$, $e = 0.977$, $e = 0.966$, and $e = 0.980$ for $\hat{M}_r^{(0)}$, $\hat{M}_r^{(1)}$, $\hat{M}_r^{(2)}$, and $\hat{M}_r^{(3)}$, respectively. The error bars represent one standard deviation obtained by performing 100 Monte Carlo simulation runs by taking into account the Poissonian photon counting statistics. The error bars are too small to be visible.

$$\sqrt{F-\frac{1}{d+1}} \leq \sqrt{G-\frac{1}{d+1}}+\sqrt{\frac{R}{d(d+1)}} + \sqrt{(d-2)\left(\frac{2}{d+1}-G-\frac{R}{d(d+1)}\right)}. \qquad (2)$$

Notably, G, F, and R for $\hat{M}_r^{(t)}$ with $t=0$, 1, 2, and 3 are always on the surface irrespective of p, i.e., saturate Eq. (2).

To investigate the pairwise trade-off relations G-F, G-R, and F-R, we project the 3D plot in Fig. 3(a) onto each corresponding plane. The obtained results are plotted in Figs. 3(b)–3(d), where the border to the shaded region indicates the theoretical bounds of G-F, G-R, and F-R relation, respectively (see Supplemental Material [41]). We can observe the quantitative links between two selected pairs of information contents as varying p. At two extremal points in each plot, i.e., when representing a von Neumann projection (e.g., $\hat{M}_r^{(0)}$ with $p=1$) or a unitary operation (e.g., $\hat{M}_r^{(2)}$ with $p=1/3$), all measurements reach the upper bounds. Otherwise, each measurement $\hat{M}_r^{(t)}$ shows different tendencies. For example, $\hat{M}_r^{(0)}$ enables us to reach the upper bound of G-F and G-R trade-off relations for any value of p. On the other hand, none of $\hat{M}_r^{(1)}$, $\hat{M}_r^{(2)}$, $\hat{M}_r^{(3)}$ allows us to attain G, F, R saturating the bounds of the pairwise relations. Remarkably, our result shows that a certain class of quantum measurements that saturates neither of the pairwise trade-off bounds can saturate the triplewise trade-off bound. This in turn indicates that the triplewise trade-off relation is tighter than any of the pairwise trade-off relations.

We note that optimizing quantum measurement is generally aimed at extracting information without any loss of information. Previously, an optimal measurement has been required to saturate either the information-disturbance G-F [43] or the information-reversibility G-R trade-off bound [39]. Within this condition, $\hat{M}_r^{(0)}$ is an optimal measurement extracting information G maximally for a fixed amount of F, but $\hat{M}_r^{(1)}$ is nonoptimal as shown in Fig. 3(b). However, our result is a clear evidence that, when G, F, and R are simultaneously taken into account, there exist optimal quantum measurements beyond the previously identified ones. In fact, we can reverse $\hat{M}_r^{(1)}$ to faithfully recover the original quantum state with probability R, which accounts for the gap between F and G regarded as a missing part of information previously. Therefore, we can generally define an *optimal quantum measurement* as a measurement that transfers the total information of a quantum state into G, F, and R without an unaccounted part of information as illustrated in Fig. 1(b), i.e., a quantum measurement inherently preserving quantum information. An optimal quantum measurement thus saturates at least one of the trade-off bounds. As a result, all the measurements $\hat{M}_r^{(t)}$ with $t=0$, 1, 2, and 3 realized in our experiment are optimal by saturating the triplewise tight trade-off bound G-F-R.

A loss of information may arise in quantum measurement due to the effect of noise, ignorance in estimating quantum states, or inherent nonoptimality. (i) In our experiment, we consider the effect of noise to the input states as $\hat{\rho}(e)=e|\psi\rangle\langle\psi|+(1-e)I/3$, resulting in dashed lines in Fig. 3. We can also take into account noise that makes the final output state in Fig. 1(a) deviated from the original input, i.e., $\hat{R}^r\hat{M}_r|\psi\rangle \propto |\psi'\rangle \neq |\psi\rangle$, which reduces R [40]. So, the effect of noise generally brings about a degradation of the amount of either G, F, or R so that none of the trade-off relations can be saturated. (ii) Ignorance and inefficiency when estimating a quantum state from measurement data may directly reduce the amount of information gain G [35,39]. (iii) Interestingly, it turns out that the form of quantum measurement itself can also induce information loss. For example, consider a measurement defined by $\hat{M}_0^{(4)}=|0\rangle\langle 0|+\sqrt{1-p}|1\rangle\langle 1|+|2\rangle\langle 2|$ and $\hat{M}_1^{(4)}=\sqrt{p}|1\rangle\langle 1|$ for $0\leq p\leq 1$. We experimentally obtain G, F, and R by changing p and find that none of the trade-off bounds can be saturated (see Fig. 4) (except when $p=0$). $\hat{M}_r^{(4)}$ may be inherently nonoptimal as there exists a part of information that is not changed into any of G, F, R, even if the measurement is performed perfectly without noise and ignorance (see Supplemental Material [41]).

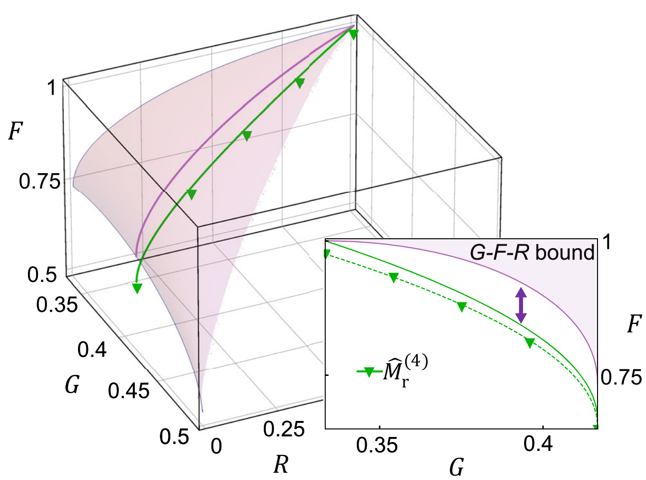


FIG. 4. Information loss by a weak quantum measurement. Experimentally obtained G, F, and R for $\hat{M}_r^{(4)}$ are plotted by markers. The green solid line represents the changes of G, F, and R obtained assuming ideally performed $\hat{M}_r^{(4)}$ in the region $0\leq p\leq 1$, while the purple solid line corresponds to the bound of the triplewise trade-off relation. $\hat{M}_r^{(4)}$ cannot reach the upper bound of the triplewise trade-off relation except when it represents a unitary operator $p=0$. The green dashed line refers to results obtained assuming a nonideal input state with $e=0.963$. Experimental results for pairwise trade-off relations for $\hat{M}_r^{(4)}$ are provided in Supplemental Material [41].

From a fundamental point of view, our result is a first experimental proof of the complete trade-off relation among the extracted, disturbed, and reversible information. We observed that the triplewise trade-off relation is tighter than any pairwise trade-off relations. It implicates that the conventional wisdom, on the information-disturbance trade-off, should be rephrased more rigorously into "the more information is obtained by quantum measurement, the more the state is disturbed or less recoverable." We also realized quantum measurement that preserves quantum information in the sense that all the information of a quantum state is transferred to G, F, and R without any missing part. In addition, our Letter raises a fundamental question on the information loss by an inherently non-optimal quantum measurement, which may provide a deep insight on the quantum to classical transition and loss of quantumness as increasing the dimensionality of quantum measurement [44,45].

While our demonstration is executed based on photonic qutrits, the results are valid for arbitrary dimensional systems. We note that the role of the reversibility R emerges in multidimensional Hilbert space, while the triplewise trade-off relation reduces to the information-disturbance relation when $d = 2$ [40]. Our results may thus be useful in high-dimensional quantum information processing [46]. We have demonstrated different types of optimal quantum measurements. Such measurements can be classified into different sets according to which trade-off relation they saturate. Different optimal quantum measurements may suit different applications. In order to estimate or discriminate quantum states, an optimal measurement that leads to maximum information gain G with minimal disturbance (i.e., maximum F) may be required [39]. To transfer or stabilize qubits, e.g., in quantum teleportation [47] or quantum error correction [28,48], maximum reversibility R and minimal information gain G may be desirable as a reversing operation plays an important role to recover the input information. Implementing optimal quantum measurements in specific quantum information protocols may be a next step of research.

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[1] W. Heisenberg, Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik, Z. Phys. **43**, 172 (1927).

[2] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2010).

[3] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, England, 2009).

[4] K. Jacobs, *Quantum Measurement Theory* (Cambridge University Press, Cambridge, England, 2014).

[5] C. H. Bennett, G. Brassard, C. Crpeau, R. Jozsa, A. Peres, and W. K. Wootters, Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels, Phys. Rev. Lett. **70**, 1895 (1993).

[6] H. J. Briegel, D. E. Browne, W. Dr, R. Raussendorf, and M. Van den Nest, Measurement-based quantum computation, Nat. Phys. **5**, 19 (2009).

[7] H. J. Groenewold, A problem of information gain by quantal measurements, Int. J. Theor. Phys. **4**, 327 (1971).

[8] G. Lindblad, An entropy inequality for quantum measurements, Commun. Math. Phys. **28**, 245 (1972).

[9] M. Ozawa, On information gain by quantum measurements of continuous observables, J. Math. Phys. (N.Y.) **27**, 759 (1986).

[10] C. A. Fuchs and A. Peres, Quantum-state disturbance versus information gain: Uncertainty relations for quantum information, Phys. Rev. A **53**, 2038 (1996).

[11] C. A. Fuchs and K. Jacobs, Information-tradeoff relations for finite-strength quantum measurements, Phys. Rev. A **63**, 062305 (2001).

[12] K. Banaszek, Fidelity Balance in Quantum Operations, Phys. Rev. Lett. **86**, 1366 (2001).

[13] K. Banaszek and I. Devetak, Fidelity trade-off for finite ensembles of identically prepared qubits, Phys. Rev. A **64**, 052307 (2001).

[14] G. M. D'Ariano, On the heisenberg principle, namely on the information-disturbance trade-off in a quantum measurement, Fortschr. Phys. **51**, 318 (2003).

[15] M. F. Sacchi, Information-Disturbance Tradeoff in Estimating a Maximally Entangled State, Phys. Rev. Lett. **96**, 220502 (2006).

[16] F. Buscemi, M. Hayashi, and M. Horodecki, Global Information Balance in Quantum Measurements, Phys. Rev. Lett. **100**, 210504 (2008).

[17] S. Luo, Information conservation and entropy change in quantum measurements, Phys. Rev. A **82**, 052103 (2010).

[18] M. Berta, J. M. Renes, and M. M. Wilde, Identifying the information gain of a quantum measurement, IEEE Trans. Inf. Theory **60**, 7987 (2014).

[19] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Quantum cryptography, Rev. Mod. Phys. **74**, 145 (2002).

[20] C. H. Bennett and G. Brassard, Quantum cryptography: Public key distribution and coin tossing, in *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing* (IEEE Press, New York, 1984), Vol. 175.

[21] M. Ueda and M. Kitagawa, Reversibility in Quantum Measurement Processes, Phys. Rev. Lett. **68**, 3424 (1992).

[22] A. Royer, Reversible Quantum Measurements on a Spin 1/2 and Measuring the State of a Single System, Phys. Rev. Lett. **73**, 913 (1994).

[23] M. Ueda, N. Imoto, and H. Nagaoka, Logical reversibility in quantum measurement: General theory and specific examples, Phys. Rev. A **53**, 3808 (1996).

[24] A. N. Jordan and A. N. Korotkov, Uncollapsing the wavefunction by undoing quantum measurements, Contemp. Phys. **51**, 125 (2010).

[25] H. Terashima, Information, fidelity, and reversibility in single-qubit measurements, Phys. Rev. A **83**, 032114 (2011).

[26] H. Terashima, Information, fidelity, and reversibility in general quantum measurements, Phys. Rev. A **93**, 022104 (2016).

[27] Y. W. Cheong and S.-W. Lee, Balance between Information Gain and Reversibility in Weak Measurement, Phys. Rev. Lett. **109**, 150402 (2012).

[28] M. Koashi and M. Ueda, Reversing Measurement and Probabilistic Quantum Error Correction, Phys. Rev. Lett. **82**, 2598 (1999).

[29] A. N. Korotkov and A. N. Jordan, Undoing a Weak Quantum Measurement of a Solid-State Qubit, Phys. Rev. Lett. **97**, 166805 (2006).

[30] H. Terashima and M. Ueda, Nonunitary quantum circuit, Int. J. Quantum. Inform. **03**, 633 (2005).

[31] A. N. Korotkov and K. Keane, Decoherence suppression by quantum measurement reversal, Phys. Rev. A **81**, 040103 (R) (2010).

[32] Y.-S. Kim, Y.-W. Cho, Y.-S. Ra, and Y.-H. Kim, Reversing the weak quantum measurement for a photonic qubit, Opt. Express **17**, 11978 (2009).

[33] Y.-S. Kim, J.-C. Lee, O. Kwon, and Y.-H. Kim, Protecting entanglement from decoherence using weak measurement and quantum measurement reversal, Nat. Phys. **8**, 117 (2012).

[34] H.-T. Lim, J.-C. Lee, K.-H. Hong, and Y.-H. Kim, Avoiding entanglement sudden death using single-qubit quantum measurement reversal, Opt. Express **22**, 19055 (2014).

[35] G. Chen *et al.*, Experimental Test of the State Estimation-Reversal Tradeoff Relation in General Quantum Measurements, Phys. Rev. X **4**, 021043 (2014).

[36] N. Katz, M. Neeley, M. Ansmann, R. C. Bialczak, M. Hofheinz, E. Lucero, A. O'Connell, H. Wang, A. N. Cleland, J. M. Martinis, and A. N. Korotkov, Reversal of the Weak Measurement of a Quantum State in a Superconducting Phase Qubit, Phys. Rev. Lett. **101**, 200401 (2008).

[37] P. Schindler, T. Monz, D. Nigg, J. T. Barreiro, E. A. Martinez, M. F. Brandl, M. Chwalla, M. Hennrich, and R. Blatt, Undoing a Quantum Measurement, Phys. Rev. Lett. **110**, 070403 (2013).

[38] J.-C. Lee, Y.-C. Jeong, Y.-S. Kim, and Y.-H. Kim, Experimental demonstration of decoherence suppression via quantum measurement reversal, Opt. Express **19**, 16309 (2011).

[39] H.-T. Lim, Y.-S. Ra, K.-H. Hong, S.-W. Lee, and Y.-H. Kim, Fundamental Bounds in Measurements for Estimating Quantum States, Phys. Rev. Lett. **113**, 020504 (2014).

[40] S.-W. Lee, J. Kim, and H. Nha, Complete information balance in quantum measurement, Quantum **5**, 414 (2021).

[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.128.050401 for experimental details, theoretical bounds for each measurement, information contents depending on p, and QPT results.

[42] H.-T. Lim, Y.-S. Kim, Y.-S. Ra, J. Bae, and Y.-H. Kim, Experimental realization of an approximate transpose operation for qutrit systems using a structural physical approximation, Phys. Rev. A **86**, 042334 (2012).

[43] F. Sciarrino, M. Ricci, F. De Martini, R. Filip, and L. Mišta, Realization of a Minimal Disturbance Quantum Measurement, Phys. Rev. Lett. **96**, 020408 (2006).

[44] J. Kofler and Č. Brukner, Classical World Arising out of Quantum Physics under the Restriction of Coarse-Grained Measurements, Phys. Rev. Lett. **99**, 180403 (2007).

[45] H. Jeong, Y. Lim, and M. S. Kim, Coarsening Measurement References and the Quantum-to-Classical Transition, Phys. Rev. Lett. **112**, 010402 (2014).

[46] M. Erhard, M. Krenn, and A. Zeilinger, Advances in high-dimensional quantum entanglement, Nat. Rev. Phys. **2**, 365 (2020).

[47] S. Pirandola, J. Eisert, C. Weedbrook, A. Furusawa, and S. L. Braunstein, Advances in quantum teleportation, Nat. Photonics **9**, 641 (2015).

[48] D. A. Lidar and T. A. Brun, *Quantum Error Correction* (Cambridge University Press, Cambridge, England, 2013).