Ground-State g Factor of Highly Charged ^{229}Th Ions: An Access to the M1 Transition Probability between the Isomeric and Ground Nuclear States

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A method is proposed to determine the $M1$ nuclear transition amplitude and hence the lifetime of the "nuclear clock transition" between the low-lying (~8 eV) first isomeric state and the ground state of ^{229}Th from a measurement of the ground-state g factor of few-electron ^{229}Th ions. As a tool, the effect of nuclear hyperfine mixing in highly charged ^{229}Th ions such as $^{229}\mathrm{Th}^{89+}$ or $^{229}\mathrm{Th}^{87+}$ is used. The ground-state-only g-factor measurement would also provide first experimental evidence of nuclear hyperfine mixing in atomic ions. Combining the measurements for H-, Li-, and B-like ^{229}Th ions has a potential to improve the initial result for a single charge state and to determine the nuclear magnetic moment to a higher accuracy than that of the currently accepted value. The calculations include relativistic, interelectronic-interaction, QED, and nuclear effects.

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The exceptionally low-energy (about 8 eV) isomeric state in ^{229}Th, which is connected to the ground state by a magnetic dipole (M1) transition, generates great interest among metrology institutes as well as atomic and nuclear physics communities [1–8] worldwide. Among others, ^{229}Th is considered as an ideal test bed of temporal variations of fundamental constants, as a nuclear γ-ray laser [9], or as an ideal candidate for a nuclear transition–based optical clock that eventually could serve as a new metrological frequency standard with unrivaled properties [10–12]. The practical realization of these applications requires the precise knowledge of the excitation energy as well as other fundamental nuclear properties such as nuclear magnetic moments of the ground state (g.s.) and the isomeric state (i.s.), and, as a key property of a clock, the lifetime of the isomer. The excitation energy was measured to an accuracy of about 2% (8.28(17) eV in Ref. [2] and 8.10(17) eV in Ref. [6]) and the magnetic moments of g.s. and i.s. were derived from experiments to precision of about 2% and 16%, respectively [13–15]. The lifetime of the neutral ^{229}Th is dominated by internal conversion that is more than 9 orders of magnitude stronger than the gamma decay [16,17]. Up to now, the internal conversion is also the only approach for direct detection of the isomer transition since neither the direct excitation nor the decay photons could be observed yet. Thus, to date there is no accurate experimental data on the $M1$ transition probability between these states (in accordance with Ref. [17], we neglect higher-order multipole contributions to the isomeric decay rate). For $^{229}\mathrm{Th}^{2+}$ in which internal conversion is energetically not possible, an experimental lower bound for the lifetime of the gamma decay of 1 min was reported [18]. Calculations of the reduced transition probability $B(M1)$ span the range from 0.005 to 0.048 Weisskopf units (W.u.) [17,19–23]. A very recent indirect estimation of $B(M1)$ from half-life measurements of other nuclear excited states in ^{229}Th yields 0.008(2) W.u. [24]. This estimation agrees with the most elaborated theoretical predictions, 0.006–0.008 W.u., of Refs. [22,23]. Yet, a precise experimental determination of this important value is still pending.

In this Letter, we propose a method for a highly sensitive experimental determination of the ^{229}Th transition probability that is deduced from a measurement of the g factor of highly charged $^{229}\mathrm{Th}^{q+}$ in its g.s. In few-electron ^{229}Th, the most tightly bound unpaired electron produces a strong magnetic field at the site of the nucleus and leads to a nuclear hyperfine mixing (NHM) of the states. The mixing coefficient b enters the g factor of the ion and contains the information of the $M1$-transition probability. Hence, the decay property of the i.s. can be experimentally deduced from an ion that is in the nuclear g.s. To date, measurements of the g factor of H- and Li-like ions with the nuclear charge number $Z = 6$–20 [25–33] have reached an accuracy of about 3×10^{-10} or better. It is expected that the same accuracy will be achieved in g-factor experiments with very heavy few-electron ions at the highly charged ion trap facility HITRAP at the accelerator complex of GSI/FAIR in

Darmstadt, Germany [34,35]. Alternatively, such high charge states can be produced at electron beam ion traps [36–38]. We show that the experimental determination of the ground-state g factor of H-like ^{229}Th ion to the precision of about 10^{-7} allows one to get the NHM mixing coefficient b to an accuracy of about 10^{-3}. Using this value of b and the excitation energy $\Delta E_{\rm nuc}$ known from the experiments [2,6], one can get $B(M1)$ with a few-percent accuracy. Furthermore, a comparison of the measurements of the ground-state g factors of H-, Li-, and B-like ^{229}Th ions improves the b value by about 1 order of magnitude and allows precise determination of the nuclear magnetic moment.

The approach is based on the NHM effect in highly charged ^{229}Th ions [39–46]. NHM is most pronounced in one-electron $^{229}\mathrm{Th}^{89+}$, three-electron $^{229}\mathrm{Th}^{87+}$, or five-electron $^{229}\mathrm{Th}^{85+}$ with an unpaired valence $j = 1/2$ electron. In these charge states, in addition to the ordinary hyperfine structure, the very strong magnetic field of up to $\sim$28 MT ($^{229}\mathrm{Th}^{89+}$) of the unpaired electron mediates a mixing of the $F = 2$ levels of the g.s. and i.s., i.e., a mixing of nuclear ground and isomeric levels with the same electronic state. In contrast to NHM, hyperfine mixing of electronic states (often termed hyperfine quenching) has been studied theoretically as well as experimentally for a large number of atomic metastable ions with charge states ranging from neutral atoms or singly charged ions [47–49] up to extreme cases such as two-electron $^{155}\mathrm{Gd}^{62+}$ and $^{157}\mathrm{Gd}^{62+}$ [50] or $^{197}\mathrm{Au}^{77+}$ [51].

NHM results in an additional small energy shift, but more notably, the lifetime of the i.s. decreases drastically for $^{229}\mathrm{Th}^{89+}$ by up to 6 orders of magnitude, from a few hours down to a few tens of ms. It is noted that, due to this vast increase of the transition rates, NHM might also become a key asset for laser spectroscopy of the nuclear transition. In fact, the experimental parameters become similar to the ones of successful storage-ring laser experiments of ordinary hyperfine transitions in $^{209}\mathrm{Bi}^{82+}$ and $^{209}\mathrm{Bi}^{80+}$ [52,53].

The mixing coefficient b is a function of the nuclear excitation energy $\Delta E_{\rm nuc}$ and the transition probability $B(M1)$. In the case of small mixing, it can be approximated as $b \sim \sqrt{B(M1)}/\Delta E_{\rm nuc}$, where the proportionality coefficient can be calculated to a good accuracy for a given ion. NHM is well known for muonic atoms (see, e.g., Refs. [54–56] and references therein) but has not been measured in conventional atoms or ions yet. Thus, the proposed g-factor measurements of few-electron ^{229}Th would also provide experimental evidence of the electronic NHM effect.

For a $^{229}\mathrm{Th}^{q+}$ g.s. ion ($I^\pi = 5/2^+$) with a single $j = 1/2$ valence electron, the hyperfine interaction splits the g.s. of the ion into two sublevels with the total angular momentum $F = 2$ and $F = 3$. Similarly, the i.s. ($I^\pi = 3/2^+$) splits into sublevels with $F = 1$ and $F = 2$. Because of the NHM, the $F = 2$ states can be represented as

$$\overline{|5/2^+, F=2\rangle} = \sqrt{1-b^2}|5/2^+, F=2\rangle - b|3/2^+, F=2\rangle, \quad (1)$$

$$\overline{|3/2^+, F=2\rangle} = \sqrt{1-b^2}|3/2^+, F=2\rangle + b|5/2^+, F=2\rangle. \quad (2)$$

The NHM coefficient b can be determined from

$$b^2 = \frac{1}{2} - \frac{1}{2}\frac{|E_1 - E_2|}{\sqrt{(E_1-E_2)^2 + 4|V_{21}|^2}}, \quad (3)$$

where $E_1 = E_1^0 + V_{11}$ and $E_2 = E_2^0 + V_{22}$ are the energies of the $F = 2$ g.s. and i.s. ions neglecting the mixing effect, $E_{1,2}^0$ are the energies in the absence of the hyperfine splitting (we choose $E_1^0 = 0$ and hence $E_2^0 = \Delta E_{\rm nuc}$), V_{11} and V_{22} are the corresponding expectation values of the hyperfine interaction, and V_{21} is the nondiagonal matrix element of the hyperfine interaction. The energies including the NHM effect are given by

$$\overline{E}_{1,2} = \frac{E_1 + E_2}{2} \mp \frac{1}{2}\sqrt{(E_1-E_2)^2 + 4|V_{21}|^2}. \quad (4)$$

In the case of small mixing ($b \ll 1$), expanding Eqs. (3) and (4) in the parameter $V_{21}/(E_1 - E_2)$, we find

$$b \approx b_0 \equiv -\frac{V_{21}}{E_1 - E_2}, \quad (5)$$

$$\overline{E}_{1,2} \approx E_{1,2} \pm \frac{|V_{21}|^2}{E_1 - E_2}. \quad (6)$$

Theoretical results for the hyperfine splitting (HFS) in H-, Li-, and B-like ^{229}Th are presented in Table I in terms of the matrix elements V_{ik}. These values are obtained using the presently available experimental values of the nuclear magnetic moments, $\mu^{(1)} = 0.360(7)\mu_N$ for the g.s. and $\mu^{(2)} = -0.37(6)\mu_N$ for the i.s. [13–15], where μ_N is the nuclear magneton. The presented results have been calculated using in part Refs. [57–65]. The details of the calculations are considered in the Supplemental Material [66]. Figure 1 shows the hyperfine structure of the g.s. and i.s. of $^{229}\mathrm{Th}^{89+}$ in the absence of (center) and including NHM (right). As a reference, on the left side the levels for the bare nucleus are displayed. For $B(M1) = 0.008$ W.u., the NHM effect yields a matrix element of $V_{21} = -0.24$ eV and shifts the $(3/2^+, F = 2)$ and $(5/2^+, F = 2)$ sublevels by 0.007 eV up and down, respectively. For the considered range of $B(M1)$ values from 0.005 to 0.048 W.u., the shift varies from 0.004 to 0.042 eV. We note that the direct determination of $B(M1)$ from the g.s. HFS measurement is rather problematic because of the large theoretical uncertainty originating from the nuclear magnetization distribution correction (the

TABLE I. The theoretical values of the hyperfine-interaction matrix elements V_{ik}, their ratios η_{ik} defined by Eq. (19), electronic g factor g_e, the HFS correction to the g factor, and the coefficients A, B, and C for H-, Li-, and B-like ^{229}Th ions. The values of $\mu^{(1)} = 0.360(7)\mu_N$ and $\mu^{(2)} = -0.37(6)\mu_N$ [13–15] are used.

Contribution	$^{229}\mathrm{Th}^{89+}$	$^{229}\mathrm{Th}^{87+}$	$\eta_{ik}^{(2s/1s)}$	$^{229}\mathrm{Th}^{85+}$	$\eta_{ik}^{(2p/1s)}$
$V_{11}/(\mu^{(1)}/\mu_N)$ [eV]	−1.109 (16)	−0.1833 (27)		−0.062 01 (31)	
$V_{22}/(\mu^{(2)}/\mu_N)$ [eV]	0.783 (14)	0.1293 (25)		0.044 12 (26)	
V_{11} [eV]	−0.399 (10)	−0.0660 (16)	0.1653 (2)	−0.0223 (4)	0.0559 (5)
V_{22} [eV]	−0.290 (47)	−0.0478 (77)	0.1651 (2)	−0.0163 (26)	0.0564 (6)
V_{21}/d [eV]	−0.498 (11)	−0.0823 (20)	0.1652 (3)	−0.027 96 (22)	0.0562 (8)
g_e	1.676 202 (3)	1.920 397 (3)		0.585 842 (3)	
δg_{HFS}	0.000 000 185 (11)	0.000 000 053 6 (21)		−0.000 005 11 (5)	
A	0.698 610 (16)	0.800 358 (16)		0.244 293 (16)	
B	−0.000 057 41	−0.000 057 41		−0.000 057 41	
C	−0.279 458 (2)	−0.320 158 (2)		−0.097 732 (2)	

Bohr-Weisskopf effect). The determination via the g factor considered below is much less sensitive to this effect.

The radiative $M1$ transition probability w_0 between i.s and g.s. in the bare ^{229}Th nucleus is ($\hbar = c = 1$, $\alpha = e^2/(4\pi), e < 0$):

$$w_0 = \frac{1}{4\pi}\frac{\omega^3}{3} d^2 \mu_N^2 = \alpha \frac{\omega^3}{12}\frac{d^2}{m_p^2}. \qquad (7)$$

Here, ω is the transition frequency, m_e and m_p are the electron and proton masses, respectively, $d = \langle 3/2^+ \| \boldsymbol{\mu}^{(\mathrm{n})} \| 5/2^+ \rangle / \mu_N$ is the reduced matrix element of the nuclear magnetic moment operator $\boldsymbol{\mu}^{(\mathrm{n})}$ between the i.s. and g.s., expressed in the nuclear magnetons. d is directly related to the reduced transition probability $B(M1)$ in the Weisskopf units: $B_{\mathrm{W.u.}} = d^2/30$.

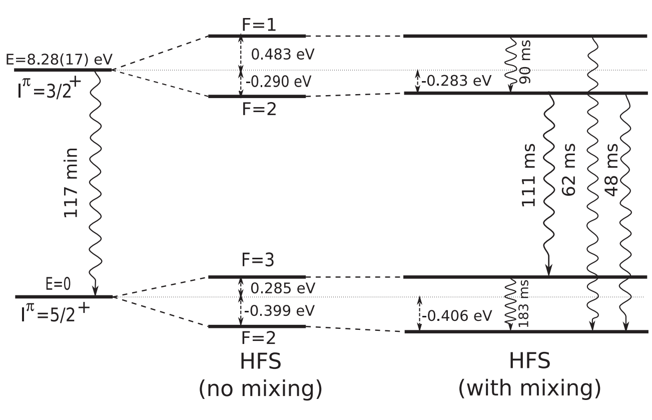

FIG. 1. Energy levels of $^{229}\mathrm{Th}^{89+}$ g.s. and i.s. Left: the g.s. and i.s. of the bare thorium nucleus. Center: including ordinary hyperfine structure but neglecting NHM. Right: including hyperfine structure and NHM. For the displayed values $B(M1) = 0.008$ W.u. was used. The wavy lines accompanied by numbers indicate the radiative transitions and the associated lifetimes. The scale is not maintained.

Taking into account that the transition wavelength is much larger than the size of the ion, the mixed $M1$ transition probability between the F and F' states is given by

$$w_{F\to F'} = \frac{1}{4\pi}\frac{4}{3}\omega^3 \frac{1}{(2F+1)} \times \sum_{M_F, M_{F'}} |\langle \overline{F' M_{F'} j I'} | [\boldsymbol{\mu}^{(\mathrm{e})} + \boldsymbol{\mu}^{(\mathrm{n})}] | \overline{F M_F j I} \rangle|^2, \qquad (8)$$

where $\boldsymbol{\mu}^{(\mathrm{e})}$ is the magnetic moment operator of the electronic subsystem and ω is the transition frequency. In the case of the $(F = 2) \to (F' = 2)$ transition, the calculation using the Eckart-Wigner theorem yields

$$w_{(F=2,\mathrm{up})\to(F'=2,\mathrm{low})} = \frac{1}{4\pi}\omega^3 \frac{25}{18}\left[b\sqrt{1-b^2}\left(g_e \mu_B + \frac{14}{5} g_I^{(1)} \mu_N - \frac{9}{5} g_I^{(2)} \mu_N \right) - (1 - 2b^2)\frac{\sqrt{2}}{5\sqrt{5}} d\mu_N \right]^2, \qquad (9)$$

where g_e is the electronic g factor, including the relativistic, QED, interelectronic-interaction, and nuclear effects, $g_I^{(1)}$ and $g_I^{(2)}$ are the nuclear g factors ($\mu = g_I \mu_N I$) of the g.s. and i.s., respectively, and μ_B is the Bohr magneton. For the other transitions, we find

$$w_{(F=3,\mathrm{low})\to(F'=2,\mathrm{low})} = \frac{1}{4\pi}\omega^3 \frac{5}{9}\left[\sqrt{1-b^2}(g_e \mu_B + g_I^{(1)} \mu_N) - b\frac{\sqrt{2}}{\sqrt{5}} d\mu_N \right]^2, \qquad (10)$$

$$w_{(F=1,\mathrm{up})\to(F'=2,\mathrm{up})} = \frac{1}{4\pi}\omega^3 \frac{5}{6}\left[\sqrt{1-b^2}(g_e \mu_B + g_I^{(2)} \mu_N) + b\frac{\sqrt{2}}{\sqrt{5}} d\mu_N \right]^2, \qquad (11)$$

$$w_{(F=2,\mathrm{up})\to(F'=3,\mathrm{low})} = \frac{1}{4\pi}\omega^3\frac{7}{9}\left[b(g_e\mu_B+g_I^{(1)}\mu_N)+\sqrt{1-b^2}\frac{\sqrt{2}}{\sqrt{5}}d\mu_N\right]^2, \quad (12)$$

$$w_{(F=1,\mathrm{up})\to(F'=2,\mathrm{low})} = \frac{1}{4\pi}\omega^3\frac{5}{6}\left[b(g_e\mu_B+g_I^{(2)}\mu_N)-\sqrt{1-b^2}\frac{\sqrt{2}}{\sqrt{5}}d\mu_N\right]^2. \quad (13)$$

Except for Eq. (12), these equations are in agreement with those in Ref. [42] if we replace g_e with its one-electron Dirac value and neglect the contributions containing μ_N. As to the $(F=2,\mathrm{up})\to(F'=3,\mathrm{low})$ transition, in Ref. [42] the corresponding expression contains a coefficient of 7/6 instead of 7/9 as obtained here. For $b=0$, Eq. (10) agrees with that from Ref. [67].

The M1 transition probabilities and the related lifetimes ($\tau=1/w$) evaluated by Eqs. (9)–(13) for $B(M1)$ in the range from 0.005 to 0.048 W.u. are listed in the Supplemental Material [66]. For $B(M1)=0.008$ W.u. the values of the lifetimes are presented in Fig. 1. We note that the M1 transition probabilities between the "up" and "low" states [Eqs. (9), (12), and (13)] are approximately linearly proportional to $B(M1)$.

Let us consider an ion of ^{229}Th with one valence electron exposed to a homogeneous magnetic field **B** directed along the z axis. Assuming that the Zeeman splitting is much smaller than the hyperfine splitting, $\Delta E_{\mathrm{magn}} \ll \Delta E_{\mathrm{HFS}}$, the linear (in B) part of the energy shift can be written as

$$\Delta E_{\mathrm{magn}} = g\mu_B B M_F, \quad (14)$$

where M_F is the z projection of the total atomic angular momentum F. We refer to the Supplemental Material [66] for the details regarding the g-factor theory. The g factor of the ground $F=2$ state can be conveniently written as

$$g = Ab^2 + Bdb\sqrt{1-b^2} + C, \quad (15)$$

where the coefficients A, B, and C do not depend on b and d,

$$A = \frac{5}{12}g_e + \frac{m_e}{m_p}\left[\frac{7}{6}g_I^{(1)} - \frac{3}{4}g_I^{(2)}\right], \quad (16)$$

$$B = -\frac{1}{3\sqrt{10}}\frac{m_e}{m_p}, \quad (17)$$

$$C = -\frac{1}{6}g_e - \frac{7}{6}\frac{m_e}{m_p}g_I^{(1)} + \delta g_{\mathrm{HFS}}. \quad (18)$$

The total theoretical values of the electronic g factor g_e for H-, Li-, and B-like thorium are presented in Table I. They have been obtained using in part the results from Refs. [60,61,68–74]. The last term in Eq. (18) describes the HFS correction to the g factor [75–78]. Since this term is rather small, it can be evaluated at $b=0$. The results of this evaluation (see the Supplemental Material [66]) are presented in Table I.

The values of the coefficients A, B, and C, including their uncertainties, which are mainly limited by the experimental input data, are given in Table I. B is presently known to a relative accuracy of 10^{-10}. The largest uncertainty of the coefficient C is due to the second term in Eq. (18). It amounts to about 2×10^{-6} and stems from the g.s. nuclear magnetic moment. The relative uncertainty of the coefficient A does not exceed 7×10^{-5} and is mainly determined by the i.s. nuclear magnetic moment.

In Table II, the individual terms contributing to the g factor [Eq. (15)] are given for several $B(M1)$ values in the range from 0.005 to 0.048 W.u. Assuming a state-of-art g-factor measurement, we find the relative uncertainty δb_{exp} to which the NHM coefficient b can be derived from the experiment. The obtained value of b, together with the experimental value of the excitation energy ΔE_{nuc} and the theoretical values of the g.s. and i.s. HFS, yields the matrix element V_{21} using Eq. (3). Employing the relations between d, V_{21}, and $B(M1)$ (see the Supplemental Material [66]) one can deduce $B(M1)$ with a few-percent accuracy, which depends equally on the accuracy of ΔE_{nuc} and the accuracy of the ratio V_{21}/d. The theoretical uncertainty of V_{21}/d is mainly due to the Bohr-Weisskopf correction to the HFS.

In the case of B-like thorium, the accuracy of C is not high enough to determine b, since the contribution of Ab^2 becomes comparable to the uncertainty of C. However, let us consider the ratios of the HFS matrix elements $V_{ik}^{(2s)}$ of Li-like ions and $V_{ik}^{(2p)}$ of B-like ions to the ones $V_{ik}^{(1s)}$ of H-like ions:

$$\eta_{ik}^{(2s/1s)} = V_{ik}^{(2s)}/V_{ik}^{(1s)}, \qquad \eta_{ik}^{(2p/1s)} = V_{ik}^{(2p)}/V_{ik}^{(1s)}. \quad (19)$$

These ratios can be calculated to a higher accuracy than the individual matrix elements (see the Supplemental Material [66]). This offers the opportunity in combined measurement for different charge states to determine improved values for b and $\mu^{(1)}$. The numerical values of $\eta_{ik}^{(2s/1s)}$ and $\eta_{ik}^{(2p/1s)}$ are given in Table I. The uncertainties are mainly due to the Bohr-Weisskopf effect. Let us now rewrite Eq. (15) in the form

$$g^{\mathrm{exp}} + \frac{1}{6}g_e^{\mathrm{th}} - \delta g_{\mathrm{HFS}} = Ab^2 + Bdb\sqrt{1-b^2} - \frac{7}{6}\frac{m_e}{m_p}g_I^{(1)}, \quad (20)$$

where g^{exp} is the experimental value of the total ground-state g factor and g_e^{th} is the theoretical value of the electronic g factor. Considering the difference of these equations for different charge states, we eliminate the last term on the

TABLE II. The individual contributions to the right-hand side of Eq. (15) for H-, Li-, and B-like ions of ^{229}Th. The uncertainty of C [Eq. (18)] is defined by the ground-state nuclear magnetic moment, while the uncertainty of A [Eq. (16)] is due to the isomeric-state nuclear magnetic moment. The NHM coefficient b is evaluated for the given values of $B(M1)$ and $\Delta E_{\rm nuc} = 8.28(17)$ eV [2] using the approximate (b_0) and the exact (b) equations. Its uncertainty caused by the uncertainties of $\Delta E_{\rm nuc}$ and V_{21} as well as the related uncertainties of the contributions Ab^2 and $Bdb\sqrt{1-b^2}$ are omitted. $\delta b_{\rm exp}$ indicates the relative uncertainty of b, to which it can be determined from Eq. (15), provided the experimental value of g is measured to an accuracy higher than that of C.

$B(M1)$	b_0	b	Ab^2	$Bdb\sqrt{1-b^2}$	C	$\delta b_{\rm exp}$
			^{229}Th^{89+}			
0.005	−0.0230	−0.0230	0.000 368	0.000 001	−0.279 458 (2)	3×10^{-3}
0.008	−0.0291	−0.0290	0.000 589	0.000 001	−0.279 458 (2)	2×10^{-3}
0.015	−0.0398	−0.0397	0.001 102	0.000 002	−0.279 458 (2)	1×10^{-3}
0.030	−0.0563	−0.0560	0.002 193	0.000 003	−0.279 458 (2)	5×10^{-4}
0.048	−0.0712	−0.0707	0.003 490	0.000 005	−0.279 458 (2)	3×10^{-4}
			^{229}Th^{87+}			
0.005	−0.003 84	−0.003 84	0.000 011 8	0.000 000 1	−0.320 158 (2)	8×10^{-2}
0.008	−0.004 86	−0.004 86	0.000 018 9	0.000 000 1	−0.320 158 (2)	5×10^{-2}
0.015	−0.006 65	−0.006 65	0.000 035 4	0.000 000 3	−0.320 158 (2)	3×10^{-2}
0.030	−0.009 41	−0.009 41	0.000 070 8	0.000 000 5	−0.320 158 (2)	14×10^{-3}
0.048	−0.011 90	−0.011 90	0.000 113 3	0.000 000 8	−0.320 158 (2)	9×10^{-3}
			^{229}Th^{85+}			
0.005	−0.001 31	−0.001 31	0.000 000 42	0.000 000 03	−0.097 732 (2)	
0.008	−0.001 65	−0.001 65	0.000 000 67	0.000 000 05	−0.097 732 (2)	
0.015	−0.002 26	−0.002 26	0.000 001 25	0.000 000 09	−0.097 732 (2)	
0.030	−0.003 20	−0.003 20	0.000 002 50	0.000 000 17	−0.097 732 (2)	
0.048	−0.004 05	−0.004 05	0.000 004 00	0.000 000 28	−0.097 732 (2)	

right-hand side. The derived equation without $g_I^{(1)}$ can be used for a more precise determination of b employing Eq. (19) and the following relations between b and V_{ik}:

$$b^2 = b_0^2 \frac{2}{1+4b_0^2+\sqrt{1+4b_0^2}} \approx b_0^2(1-3b_0^2), \quad (21)$$

$$b_0 = \frac{V_{21}}{\Delta E_{\rm nuc}} \frac{1}{1+(V_{22}-V_{11})/\Delta E_{\rm nuc}}. \quad (22)$$

Using the obtained value of b and Eq. (20) for one of the ions, we can determine $g_I^{(1)}$ [and hence $\mu^{(1)}$] to a higher accuracy. Assuming the reasonable experimental uncertainty of $< 10^{-7}$ for g-factor measurements of both H-like ^{229}Th^{89+} and Li-like ^{229}Th^{87+}, the accuracy of the coefficient b can be improved by a factor of 10 compared to the one for ^{229}Th^{89+} only. Likewise, the experimental value for the magnetic moment $\mu^{(1)}$ can be improved tenfold compared to the currently accepted value. Similarly, one can use the g-factor experiments on Li- and B-like ions to refine the value of $\mu^{(1)}$.

Concluding, by a precise measurement of the g factor in H- or Li-like ion of ^{229}Th, the much-sought-after lifetime of the nuclear clock transition can be determined experimentally on a few-percent level. Remarkably, to achieve this goal a measurement of the ion in its ground state can be used, meaning that the nuclear lifetime is determined completely without the nuclear decay. The approach uses NHM, which is very pronounced in the considered charge states. The experimental accuracy of typical g-factor experiments today is orders of magnitude higher than required by the proposed method. The complete formulas for the transition probabilities have been derived including relativistic, electron-electron correlation, QED, and nuclear contributions. Further substantial improvements can be achieved if several charge states are compared. As a byproduct, the precise measurement of the ground-state nuclear magnetic moment is deduced. In addition, in the course of such a measurement, evidence for NHM in atomic ions can be obtained. NHM is a fascinating research topic on its own since it allows the manipulation of nuclear lifetimes by orders of magnitude simply by attachment or removal of a single electron. For example, in the He-like ^{229}Th^{88+} ion with paired electrons, the effect is absent, and the lifetime corresponds to the one of the bare nucleus, i.e., about 2 hours. Removal of one electron shortens the lifetime to a few tens of ms (H-like ^{229}Th^{89+} ion), while the attachment of an electron increases the lifetime to several seconds (Li-like $^{229}Th^{87+}$ ion).

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