

## Testing Real Quantum Theory in an Optical Quantum Network

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Quantum theory is commonly formulated in complex Hilbert spaces. However, the question of whether complex numbers need to be given a fundamental role in the theory has been debated since its pioneering days. Recently it has been shown that tests in the spirit of a Bell inequality can reveal quantum predictions in entanglement swapping scenarios that cannot be modeled by the natural real-number analog of standard quantum theory. Here, we tailor such tests for implementation in state-of-the-art photonic systems. We experimentally demonstrate quantum correlations in a network of three parties and two independent EPR sources that violate the constraints of real quantum theory by over 4.5 standard deviations, hence disproving real quantum theory as a universal physical theory.

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The original formulation of quantum theory postulates states and measurements as operators in complex Hilbert spaces and uses tensor products to model system composition [1,2]. However, already some pioneers of the theory favored a real quantum theory over a complex quantum theory, i.e., using only real numbers in its mathematical formulation [3]. The general debate on the role of complex numbers in quantum theory has continued into the present [4–11].

Another long-standing debate in the foundations of quantum theory, nowadays settled, concerned the existence of local hidden variable theories to describe our world. Bell pioneered the idea of studying correlations in the outcome statistics of experiments to infer fundamental properties of their underlying physics [12]. In recent years, experimental implementations of such Bell tests have successfully ruled out local hidden variable theories [13–17]. Surprisingly, it was further shown that a natural generalization of Bell's test in a network can, contrary to their traditional counterparts [18,19], distinguish complex quantum theory from real quantum theory [20]. In a network in which parties are connected through several independent sources [21–23],

real quantum theory does not agree with all predictions of complex quantum theory [20]. This paves the way for experimentally distinguishing between the two theories in a quantum network based on independent sources. Here and in the rest of this paper, *real quantum theory* refers to a theory in which the real Hilbert spaces of independent systems are combined by the tensor product.

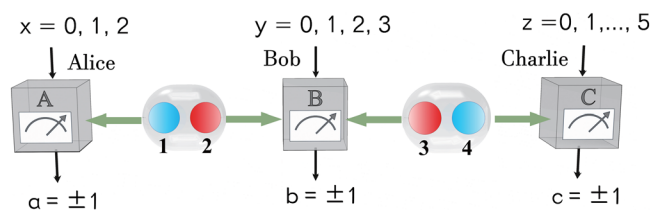


FIG. 1. The entanglement swapping scenario. Two sources, that may at most be classically correlated, distribute an entangled pair between Alice and Bob, and Bob and Charlie, respectively. Alice, Bob, and Charlie each select one of three, four, and six inputs, respectively, and perform corresponding quantum measurements on their respective shares. Each measurement gives one of two possible outcomes.

For this purpose, the simplest quantum network, describing the entanglement swapping scenario, suffices. As shown in Fig. 1, two independent sources each emit an EPR pair, the first one shared between Alice and Bob, and the second shared between Bob and Charlie. If Bob projects both of his particles onto an entangled basis, then Alice and Charlie are left in an entangled state when conditioned on Bob's outcome [24]. By appropriately combining the input-output probabilities of the network we arrive at a Bell-type correlation function, whose maximum value in real quantum theory can be upper bounded. Any experimental violation of this upper bound would disprove the universal validity of real quantum theory [20].

Photons emerge as a natural platform for realizing quantum networks of scalable distance as they are suitable carriers of quantum information. Therefore it is natural to use a photonic quantum network to test real quantum theory. Moreover, the pivotal requirement of independent sources can be stringently met in photonic implementations, whereas a recent similar experiment based on superconducting qubits implemented both sources on the same chip [25]. However, photonic implementations come with the difficulty that the protocol proposed in Ref. [20] requires a complete Bell state measurement. Even though such a measurement can be implemented using superconducting qubits [25], it is not amenable to the tools of standard linear optics [26,27], unless additional degrees of freedom are introduced and controlled [28]. Here, we resolve this issue by developing a new protocol that uses a partial Bell state measurement [29,30].

Consider the scenario illustrated in Fig. 1. We focus on a quantum protocol in which each source emits the EPR state  $|\Phi^+\rangle = [(|00\rangle + |11\rangle)/\sqrt{2}]$ . Alice, Bob, and Charlie independently perform measurements with random inputs  $x \in \{0, 1, 2\}$ ,  $y \in \{0, 1, 2, 3\}$ , and  $z \in \{0, 1, 2, 3, 4, 5\}$ , respectively. Their measurement outcomes are labeled  $a, b, c \in \{+1, -1\}$ . Alice's three measurements are chosen as  $\{\sigma_X, \sigma_Y, \sigma_Z\}$ , and Charlie's six measurements are chosen as  $\{[(\sigma_X + \sigma_Y)/\sqrt{2}], [(\sigma_X - \sigma_Y)/\sqrt{2}], [(\sigma_Y + \sigma_Z)/\sqrt{2}], [(\sigma_Y - \sigma_Z)/\sqrt{2}], [(\sigma_X + \sigma_Z)/\sqrt{2}], [(\sigma_X - \sigma_Z)/\sqrt{2}]\}$ , where  $\sigma_X, \sigma_Y, \sigma_Z$  are Pauli observables. Bob's four measurements each correspond to discriminating one of the four Bell states. Specifically, the outcome  $b = 1$  corresponds respectively to a projection onto the Bell state  $|\Phi^+\rangle$ ,  $|\Phi^-\rangle = [(|00\rangle - |11\rangle)/\sqrt{2}]$ ,  $|\Psi^-\rangle = [(|01\rangle - |10\rangle)/\sqrt{2}]$  and  $|\Psi^+\rangle = [(|01\rangle + |10\rangle)/\sqrt{2}]$ . As in any Bell test, by suitably combining the probabilities  $p(a, b, c|x, y, z)$ , we define a Bell-type correlation function,

$$W = \frac{1}{5} \sum_{y=0}^3 T_y - \frac{4}{5} \sum_{y=0}^3 p(b = 1|y), \quad (1)$$

where  $y = y_2 y_1 \in \{00, 01, 10, 11\}$  in binary notation and  $T_y = (-1)^{y_1 + y_2} (S_{11y} + S_{12y}) - (-1)^{y_1} (S_{21y} - S_{22y}) +$

$(-1)^{y_1 + y_2} (S_{15y} + S_{16y}) + (-1)^{y_2} (S_{35y} - S_{36y}) - (-1)^{y_1} (S_{23y} + S_{24y}) + (-1)^{y_2} (S_{33y} - S_{34y})$ , with  $S_{xyz} = \sum_{a,c=\pm 1} acp(a, b = 1, c|x, y, z)$ . Using the tools developed in Ref. [20], the value of Eq. (1) in real quantum theory is upper bounded by (see Ref. [31] for the derivation)

$$W_{\text{RQT}} \lesssim 0.7486. \quad (2)$$

This bound holds even if Alice, Bob, and Charlie are allowed global shared classical randomness, i.e., the two sources may be classically, but not quantumly, correlated. In turn, complex quantum theory predicts the value

$$W_{\text{CQT}} = \frac{6\sqrt{2} - 4}{5} \approx 0.8971. \quad (3)$$

Hence an experimental observation of  $W_{\text{EXP}} > 0.7486$  is sufficient to rule out real quantum theory.

A schematic of our optical quantum network implementing the states and measurements above is depicted in Fig. 2. By driving a type-0 spontaneous parametric down-conversion (SPDC) process in a periodically poled MgO doped lithium niobate (PPLN) crystal with the pump laser at  $\lambda_p = 779$  nm [38], each EPR source probabilistically emits a pair of photons in state  $|\Phi^+\rangle = [(|HH\rangle + |VV\rangle)/\sqrt{2}]$  at the phase-matched wavelengths 1560 nm (signal) and 1556 nm (idler), where  $|H\rangle$  and  $|V\rangle$  represent respectively the horizontal and vertical polarization states. The two EPR sources each deliver their signal photon to Bob and their idler photon to Alice and Charlie, respectively. To realize the protocol measurements, Bob lets the two signal photons sequentially pass polarization beamsplitters (PBS) and directs the four outputs to single-photon detectors ( $D_3, D_4, D_5, D_6$ ) via optical fibers. The half-wave plates (HWP) before and after the first PBS are adjusted to the proper orientations upon receiving  $y$ . The resulting two-photon coincidence detection between  $D_3$  and  $D_5$  or  $D_4$  and  $D_6$  assigns  $b = +1$  and prepares Alice and Charlie to be in the Bell state  $|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^-\rangle$ , or  $|\Psi^+\rangle$  according to the random input  $y \in \{0, 1, 2, 3\}$ , respectively. Similarly, upon receiving the random inputs  $x$  and  $z$ , Alice and Charlie each respectively adjust the relevant HWP and quarter-wave plate (QWP) to perform the single-qubit measurements outlined above. The single-photon detection events at each detector are time tagged to produce the correlation analysis (more detail can be found in Ref. [31]).

In order to falsify real quantum theory, it is essential to stringently meet the requirement that the two EPR sources are independent up to classical synchronization [20]. In our network experiment, the time reference of all events is set to the 15 GHz internal clock of a pulse pattern generator (PPG) in the EPR source  $S_1$ , which triggers the PPG in the EPR source  $S_2$ . In each EPR source, the PPG sends triggers at a rate of 250 MHz to enable the distributed feedback (DFB) laser to switch on the electric current. Switching

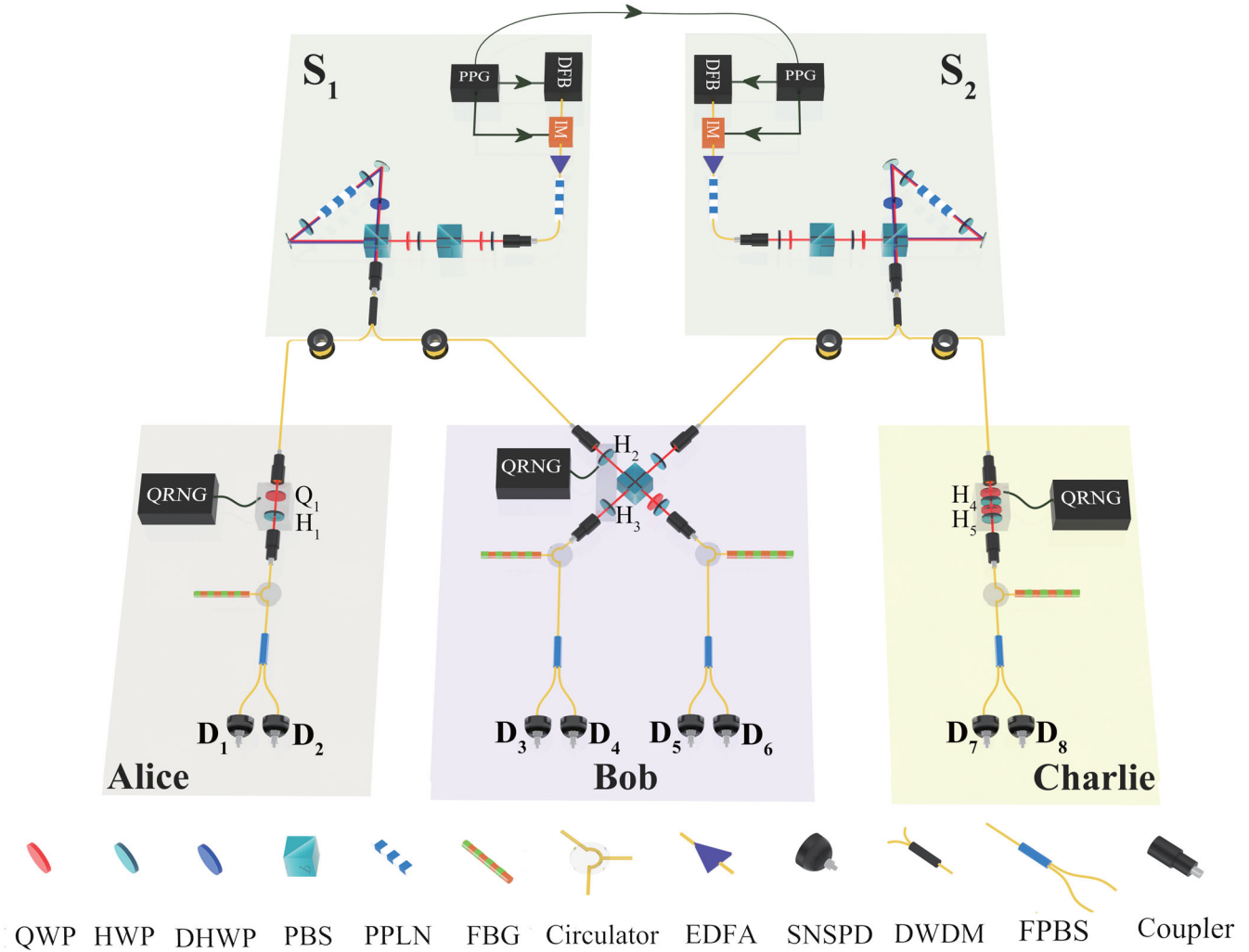


FIG. 2. Schematic of the experiment. The setup consists of two EPR sources,  $S_1$  and  $S_2$ , and three measurement nodes, Alice, Bob, and Charlie. In each EPR source, the laser pulse is injected into a Sagnac loop interferometer containing a PPLN crystal to produce a pair of photons in Bell state  $|\Phi^+\rangle$  via the SPDC process [38]. The EPR source delivers the signal photon to Bob and the idler photon to Alice (Charlie). Bob performs measurement with the two signal photons, which prepares Alice and Charlie in a Bell state. Alice and Charlie measure the idler photons according to the inputs from the quantum random number generator. PPG in EPR source  $S_1$  triggers the PPG in EPR source  $S_2$ ; DHWP: HWP for dual wavelength; SPBS: spatial PBS;  $D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8$  are superconducting nanowire single-photon detectors (SNSPDs); FPBS: fibre PBS; DWDM: dense wavelength division multiplexer.

from below to above the lasing threshold, the DFB laser emits a 2 ns laser pulse at the wavelength of 1558 nm with a randomized phase per trigger [38]. We further shorten the pulse width to 90 ps with an intensity modulator (IM). After passing through an erbium-doped-fiber-amplifier, a PPLN waveguide to double the frequency, and a wavelength-division multiplex filter, the produced laser pulses at  $\lambda_p = 779$  nm drive the SPDC process as described in above, for which we keep the photon-pair production rate at about 0.0025 per trigger to strongly mitigate the multiphoton effect. We pass photons through fiber Bragg gratings (FBGs) with bandwidths of 3.3 GHz before entering single-photon detectors to suppress the spectral distinguishability between photons from different EPR sources. Quantum tomography measurements indicate that

the state fidelity is greater than 0.99 for the two-photon states produced at EPR sources  $S_1$  and  $S_2$  with respect to the targeted Bell state  $|\Phi^+\rangle$  and greater than 0.96 for the two-photon state of Alice and Charlie with respect to the Bell states  $|\Phi^+\rangle$ ,  $|\Phi^-\rangle$ ,  $|\Psi^+\rangle$ , and  $|\Psi^-\rangle$  after entanglement swapping, respectively. The lack of coherence between pulses of the same laser and of different lasers ensures that the independence of the sources is stringently enforced in the network experiment.

We switch the measurement settings every 30 seconds, reserving the first 10 seconds to reset the measurement settings, including quantum random number generation and wave plate rotation, and the remaining 20 seconds for data collection. The average four-photon coincident detection event rate is 1.24 per switching cycle. We collect 26 954

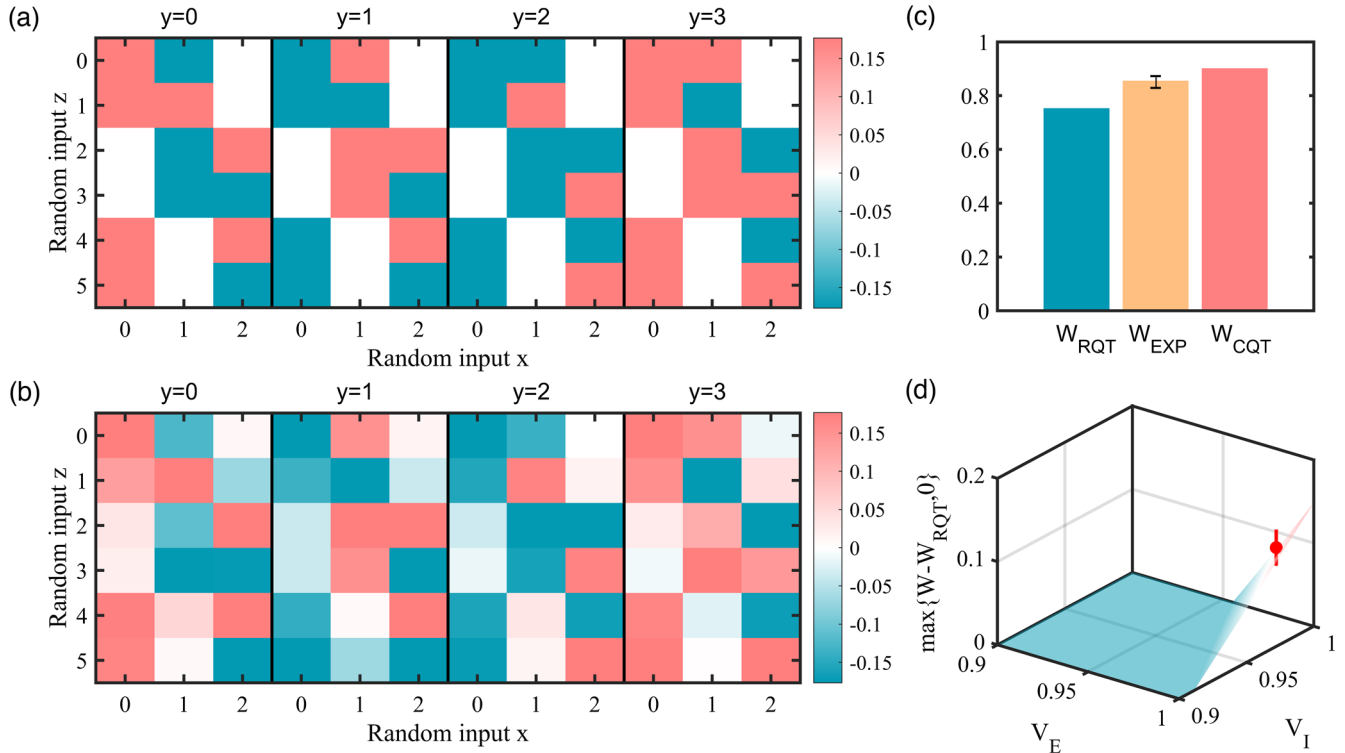


FIG. 3. Experimental results. The ideal (a) and experimentally measured (b) values  $S_{xzy} = \sum_{a,c=\pm 1} ac p(a, b = 1, c | x, y, z)$ . (c) The values of  $W$  in different scenarios.  $W_{\text{RQT}}$ : real quantum theory,  $W_{\text{CQT}}$ : complex quantum theory,  $W_{\text{EXP}}$ : experimental result in our optical quantum network. (d) An emulation of the experiment based on complex quantum theory. The value of the red dot is given by  $W_{\text{EXP}} - W_{\text{RQT}}$ . Error bars shown in (c) and (d) represent 1 standard deviation in the experiment.

four-photon coincidence events in 21 742 switching cycles. Since we only record Bob's outcome  $b = +1$ , we use our characterization of the efficiency of the detection device to estimate the associated probability. For each cycle, we estimate  $p(b = +1 | y)$  through the quantity  $[(N_{b=+1|y} N_A N_C) / (N_{AC} N_{AB} N_{BC})]$ , where  $N_{AC}$ ,  $N_{AB}$ ,  $N_{BC}$  are the two-party photon coincidence numbers,  $N_A$  and  $N_C$  are the one-party photon detection numbers, and  $N_{b=+1|y}$  is the number of four-photon coincidence events when Bob obtains outcome  $b = +1$  (derivation details can be found in Ref. [31]). In Figs. 3(a) and 3(b), we compare the  $3 \times 4 \times 6 = 72$  different experimentally measured values of  $S_{xzy}$ , averaged with respect to different cycles, with their corresponding theoretical values. The results uphold a good agreement. Putting it all together, we then obtain  $W_{\text{EXP}} = 0.8508 \pm 0.0218$ , which exceeds the upper bound  $W_{\text{RQT}} \approx 0.7486$  set by real quantum theory by 4.70 standard deviations [Fig. 3(c)]. We also perform an emulation of the experiment based on complex quantum theory. Consider that the EPR source emits a nonideal state,  $\rho_{\text{EPR}} = v_E |\Phi^+\rangle\langle\Phi^+| + (1 - v_E)I/4$ , and the photons from the two EPR sources interfere with a nonideal visibility  $v_I$ . With  $v_E = 0.9909$  and  $v_I = 0.9844$  determined in the experiment, complex quantum theory predicts  $W = 0.8404$ , which is consistent with the experimental result  $W_{\text{EXP}}$  within a standard deviation [Fig. 3(d)].

We have experimentally demonstrated that real quantum theory is incompatible with the observed data in our optical quantum network experiment. The independent sources in the network guarantee that the observed correlations cannot be simulated by real quantum theory [20]. The rapid development in photonic or hybrid quantum technologies, in particular with regard to more efficient detectors, faster switching, higher-quality entanglement sources, and longer-distance entanglement distribution, leaves us optimistic of even more sophisticated future experiments. Research in quantum networks has enjoyed a rapid growth thanks to the role they are likely to play in quantum communications, distributed quantum computing, and the future quantum Internet. In addition to these potential applications, our work highlights that they are also powerful frameworks for devising and implementing tests of fundamental aspects of quantum theory.

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