Dynamics of Strongly Interacting Fermi Gases with Time-Dependent Interactions: Consequence of Conformal Symmetry

Jeff Maki,[1] Shizhong Zhang,[1,*] and Fei Zhou[2]

[1]*Department of Physics and HKU-UCAS Joint Institute for Theoretical and Computational Physics at Hong Kong, Guangdong-Hong Kong Joint Laboratory of Quantum Matter, The University of Hong Kong, Hong Kong, China*

[2]*Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia V6T 1Z1, Canada*

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In this Letter, we investigate the effects of a time-dependent, short-ranged interaction on the long-time expansion dynamics of Fermi gases. We show that the effects of the interaction on the dynamics is dictated by how it changes under a conformal transformation, and derive an explicit criterion for the relevancy of time-dependent interactions near both the strongly and noninteracting scale invariant limits. In addition, we show that it is possible to engineer interactions that give rise to nonexponential thermalization dynamics in trapped Fermi gases. To supplement the symmetry analysis, we perform hydrodynamic simulations to show that the moment of inertia of the trapped gas indeed follows a universal time dependence that is determined jointly by the conformal symmetry and time-dependent scattering length $a(t)$. Our results should also be relevant to the dynamics of other systems that are nearly scale invariant and that are governed by a nonrelativistic conformal symmetry.

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Introduction.—Conformal symmetry [1–3] imposes severe constraints on the dynamics of nonrelativistic scale invariant quantum systems due to an overarching SO(2,1) symmetry that can occur for quantum systems with dynamical critical exponent $z = 2$ [4–9]. The most prominent example is the unitary Fermi gas in three dimensions, for which the quantum critical point occurs when the scattering length tends to infinity [10]. At the critical point, the dynamics is severely constrained by a dynamical conformal symmetry, independent of the microscopic details of the system. For example, it leads to phenomena such as vanishing bulk viscosity [11], Efimovian expansion [12], eliptic flow in unitary Fermi gases [13] and the oscillations of the momentum distribution in a Tonks-Girardeau gas [14,15].

In this Letter, we show that for two-component Fermi gas away from its critical points, the dynamical conformal symmetry could still impose severe constraints on its dynamics. In particular, we show that in the long-time limit, its dynamics can be either fixed entirely by the conformal symmetry, or in the case when it is not, can be shown to depend on $a(t)$ in a universal fashion. In this case, both the strongly interacting quantum critical point and the noninteracting point represent scale invariant fixed points (SIFP) in terms of the renormalization group flow [16]. We derive explicit expressions describing the breaking of conformal symmetry based on how $a(t)$ transforms under the conformal transformation, and investigate its effects on its expansion dynamics and elliptic flow.

In the long-time limit, conformal symmetry also imposes constraints on the thermalization process of the system [17]. By tuning the time dependence of $a(t)$, it turns out that one can not only change the thermalization rate, but also engineer power-law thermalization. This provides a unique route for experimentalists to access the implication conformal symmetry in simple experimental settings, such as the damping of monopole oscillations.

Breaking of conformal symmetry with time-dependent interactions.—We begin by considering the expansion dynamics of a two-component Fermi gas near the strongly interacting SIFP in d dimension. The total Hamiltonian can be expanded as [9,18]

$$H(t) \approx H_s + \frac{1}{a^{d-2}(t)} C_a, \tag{1}$$

where H_s is the scale invariant Hamiltonian at the strongly interacting SIFP, and C_a is the contact operator [19–23]. For $d = 3$, $a(t)$ is simply the s-wave scattering length. The strongly interacting SIFP is defined when $1/a^{d-2}(t) = 0$. Equation (1) is valid to $O[a^{2-d}(t)]$, and is a good approximation provided that the rate of change of $a(t)$ is much slower than that set by the range r_0 of the two-body potential, $\hbar/mr_0^2$, which is usually very well satisfied in actual experiments.

The dynamics of the density matrix $\rho(t)$ can be split into a matrix governed by the scale invariant Hamiltonian $\rho_s(t)$, and a matrix governed by the initial conditions and the breaking of conformal symmetry $\Gamma(t)$: $\rho(t) \approx \rho_s(t)\Gamma(t)$ [9,24]. The trivial dynamics of $\rho_s(t)$ are solely controlled by the so-called conformal tower states, which are defined as the eigenstates of the scale invariant gas inside an isotropic harmonic potential,

$$[H_s + \omega_0^2 C]|n,\ell\rangle = E_n^\ell |n,\ell\rangle, \tag{2}$$

where $C = \frac{1}{2} m \sum_i \mathbf{r}_i^2$ is the moment of inertia. In Eq. (2), ω_0 is an arbitrary trapping frequency, and $|n,\ell\rangle$ is the nth state in the ℓth conformal tower with energy E_n^ℓ. The trivial dynamics associated with the conformal towers have previously been used to study a variety of dynamical phenomena [5,9,12,14,17,18,25–33]. On the other hand, the nontrivial conformal symmetry breaking dynamics contained in $\Gamma(t)$ satisfy the following differential equation:

$$\partial_t \Gamma(t) = \frac{i}{a^{d-2}(t)} [e^{iH_s t} C_a e^{-iH_s t}, \Gamma(t)], \tag{3}$$

where $\Gamma(0) = \rho_0$ is the initial density matrix.

The effect of the scale symmetry breaking interactions, or, equivalently $\Gamma(t)$, can be understood by examining the matrix elements of $\Gamma(t)$ with respect to the conformal tower states to first order in perturbation theory:

$$\begin{aligned} &\langle n',\ell'|\Gamma(t \gg \omega_0^{-1})|n,\ell\rangle \\ &\quad \approx \langle n',\ell'|\Gamma(0)|n,\ell\rangle \\ &\qquad + i\left[\int_{\omega_0^{-1}}^{t} \frac{dt'}{\lambda^2(t')} \left(\frac{\lambda(t')}{a(t')}\right)^{d-2}\right] \langle n',\ell'|[\tilde{C}_a, \Gamma(0)]|n,\ell\rangle, \end{aligned} \tag{4}$$

where $\lambda(t) = \sqrt{1 + (\omega_0 t)^2}$ for expansion dynamics in free space and $\langle n',\ell'|\tilde{C}_a|n,\ell\rangle = \exp[i\pi(E_{n'}^{\ell'} - E_n^\ell)/(2\omega_0)]$ $\langle n',\ell'|C_a|n,\ell\rangle$. Equation (4) states that all the nontrivial dynamics contained in $\Gamma(t)$ must be a function of the effective coupling constant

$$g(t) \equiv \int_{\omega_0^{-1}}^{t} \frac{dt'}{\lambda^2(t')} \left(\frac{\lambda(t')}{a(t')}\right)^{d-2} \tag{5}$$

to first order in perturbation theory. In fact, in the long-time limit, one can show that all the leading dynamical effects due to the scale symmetry breaking can be written as a function of $g(t)$ to all orders of perturbation theory [18,24]. In the case of expansion inside a harmonic trap with time-dependent trap frequency $\omega(t)$, we need to replace $\lambda(t)$ in $g(t)$ by the solution to the following differential equation: $\omega_0^2 = \ddot{\lambda}(t)\lambda^3(t) + \omega^2(t)\lambda^4(t)$ with $\lambda(0) = 1$ and $\dot{\lambda}(0) = 0$, and $\omega(t)$ the time-dependent harmonic trap frequency with $\omega(t=0) = \omega_0$ [24].

Generally, irrespective of the detailed form of $a(t)$, if $g(t)$ saturates in the long-time limit, then $\Gamma(t \gg \omega_0^{-1})$ tends to a constant and the dynamics are entirely controlled by $\rho_s(t)$, which in turn is expressed by the time-dependent rescaling of the conformal tower states. In other words, there is an emergent conformal symmetry constraining the dynamics in the long-time limit. Alternatively, if $g(t)$ has nontrivial dynamics in the long-time limit, the effects of the conformal symmetry breaking perturbation will be important and there will be no emergent conformal symmetry.

To exemplify this difference, we consider for simplicity a power-law time dependence for the scattering length

$$a(t) = a_0(\eta t)^\gamma, \tag{6}$$

where γ is a real number, while a_0 and η have units of length, and frequency, respectively. For this perturbation, one can evaluate Eq. (5) analytically:

$$g(t \gg \omega_0^{-1}) = \frac{1}{\omega_0^2}\left(\frac{\omega_0}{a_0\eta^\gamma}\right)^{d-2} \frac{t^{d-3-\gamma(d-2)}}{d-3-\gamma(d-2)} + g_0, \tag{7}$$

for some constant g_0 that describes the short-time physics. $g(t)$ has a nontrivial time dependence in the long-time limit if

$$d - 3 \geq (d-2)\gamma \qquad \text{strong interactions.} \tag{8}$$

In three dimensions, Eq. (8) reduces to $\gamma \leq 0$. Therefore, any static perturbation, $\gamma = 0$ is marginal, while any perturbation that moves away from the strongly interacting SIFP, $\gamma < 0$, is relevant, consistent with previous studies [9,18]. Similarly, in one dimension we find that a perturbation near the strongly interacting SIFP becomes relevant when $\gamma \geq 2$. In this case, a static perturbation is irrelevant, and one needs to have a scattering length that increases quadratically, $\gamma = 2$, in order to break the conformal symmetry. Therefore, the one-dimensional strongly interacting SIFP is much more resilient against perturbations.

Similar arguments apply around the weakly interacting gases. Expanding the Hamiltonian around the noninteracting SIFP, $a^{d-2}(t) = 0$, one finds the following definition of the effective coupling constant:

$$g(t) = \int_{\omega_0^{-1}}^{t} \frac{dt'}{\lambda^2(t')} \left(\frac{a(t')}{\lambda(t')}\right)^{d-2}, \tag{9}$$

and the following criterion for the breaking of conformal symmetry for time dependence of $a(t)$ given by Eq. (6):

$$d - 1 \leq (d-2)\gamma \qquad \text{weak interactions.} \tag{10}$$

The relevancy of $a(t)$ in the weak coupling limit reads $\gamma \geq 2$ in three dimensions while for one dimension: $\gamma \leq 0$. The three- (one-) dimensional criterion near the strongly interacting SIFP is equivalent to the one- (three-) dimensional criterion for weak interactions. In Fig. 1, we show a schematic for how the effective coupling constant $g(t)$ changes with time, and the associated time dependence of the scattering length $a(t)$ near both the strongly interacting

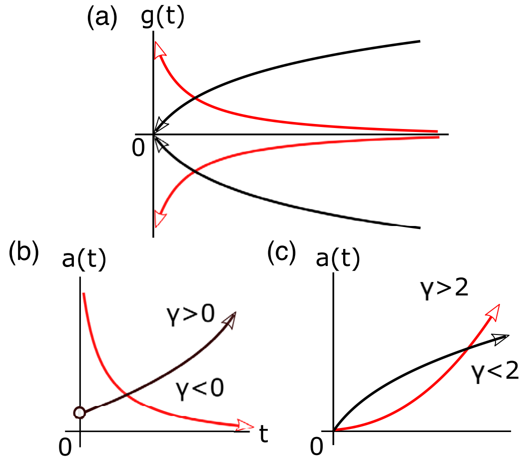

FIG. 1. (a) Schematic of the effective coupling constant $g(t)$ near a SIFP $g(t) = 0$. The definition of $g(t)$ is given by Eq. (5) near the strongly interacting SIFP, and by Eq. (9) for weak interactions. The black line represents irrelevant symmetry breaking interactions, while the red line denotes relevant ones. The relevancy condition for γ is given by Eq. (8) near the strongly interacting SIFP, and Eq. (10) for weak interactions. (b) and (c) The physical time dependence of $a(t)$ for relevant and irrelevant interactions near the strongly and weakly interacting SIFPs in $d = 3$, respectively.

and noninteracting SIFPs in three dimensions. A relevant perturbation would lead to an increase of the effective coupling constant $g(t)$ as a function of time.

In two dimensions, there is a single noninteracting SIFP. Equation (10) then states that any scattering length with a power-law time dependence is irrelevant to the expansion dynamics. This conclusion is consistent with the fact that two-body interaction depending on the scattering length logarithmically, i.e., the quantum anomaly [34–38].

Consequences of broken conformal symmetry on eliptic flow.—The conclusions of the previous section are independent of the initial state of the system, and depend only on the conformal SO(2,1) symmetry. As an example, let us consider the effects of a time-dependent perturbation on the expansion dynamics of three-dimensional Fermi gases in an anisotropic trap, i.e., elliptic flow [13,39]. Let us define the moments of inertia as

$$\langle r_i^2\rangle(t) = \frac{1}{N}\int d\mathbf{r} r_i^2 n(\mathbf{r}, t), \qquad i = x, y, z, \tag{11}$$

where $n(\mathbf{r}, t)$ is the density of the gas. As shown in Ref. [9] and detailed in the Supplemental Material [24], near a SIFP, $\langle r_i^2\rangle(t)$ takes the following form in the long-time limit:

$$\langle r_i^2\rangle(t \gg \omega_i^{-1}) \approx \left[A_i + \frac{B_i}{\omega_i t} + \frac{D_i}{(\omega_i t)^2} + E_i g(t)\right] t^2, \tag{12}$$

where terms involving A_i, B_i, and D_i are guaranteed by conformal symmetry, and their numerical value depends on the initial conditions [40] and the short-time dynamics. On the other hand, the scale breaking term depends linearly on the effective coupling constant $g(t)$ defined in Eq. (5), with a proportionality constant E_i.

Depending on the time dependence of $g(t)$, one can change the asymptotic behavior of the moment of inertia. If the perturbation is relevant, the scale breaking term proportional to $g(t)$ becomes parametrically larger than the conformal symmetric terms in the long-time limit. A similar situation can also occur for $0 < \gamma < 1$. In this case the perturbation is irrelevant to the long-time dynamics, but the leading correction to the conformal physics is now given by the scale breaking term: $g(t) \propto t^{-\gamma}$.

From Eq. (12), one can show that the aspect ratio takes the perturbative form in the long-time limit:

$$\lim_{t\to\infty} \frac{\langle r_x^2\rangle(t)}{\langle r_y^2\rangle(t)} \approx \frac{A_x}{A_y}\left[1 + \left(\frac{E_x}{A_x} - \frac{E_y}{A_y}\right) g(t)\right]. \tag{13}$$

Conformal symmetry requires that the aspect ratio saturates to a constant with a correction of $O[(\omega_0 t)^{-1}]$. However, for irrelevant perturbations with $0 < \gamma < 1$, the aspect ratio still approaches a constant in the long time limit but with a different time dependence. If the interaction is relevant though, $\gamma \le 0$, the aspect ratio will not saturate due to the nontrivial time dependence induced by the conformal breaking interactions encoded in $g(t)$.

Equations (12), (13) constitute one of the main results of this Letter. It is to be emphasized that the simple dependence on $g(t)$ reflects not only the time dependence of $a(t)$, but also the conformal symmetry that is present at the SIFP, as is evident in the derivation of Eq. (5). Thus an experimental confirmation of the Eqs. (12), (13) would constitute an indirect experimental verification of the existence of conformal tower states.

Hydrodynamics with time-dependent interactions.—The above arguments are based on the analysis of the effects of symmetry breaking interactions on the density matrix of the system and one would expect that in the regime where hydrodynamics applies, the results obtained above, e.g., Eq. (12), should also be reproduced by a hydrodynamic theory [13,41,42]. Here we show that this is indeed the case. To this end we examine the expansion dynamics of a trapped Fermi gas near the strongly interacting SIFP with time-dependent interactions, when subject to a quench of the harmonic trapping potential: $\omega_0 \to \omega_f$, where $\omega_f \ll \omega_0$. For simplicity, we will assume that both the initial and final trapping potential are isotropic. Thus the only source of broken conformal symmetry is due to the changing interaction. To describe the expansion, we make the following scaling ansatz for the moment of inertia:

$$\langle r^2\rangle(t) = \sum_{i=x,y,z} \langle r_i^2\rangle(t) = \lambda_a^2(t)\langle r^2\rangle(0). \tag{14}$$

From standard hydrodynamic arguments [13,41,42] and by performing an expansion near the strongly interacting SIFP, one can obtain the following differential equation (see Supplemental Material [24]):

$$\begin{aligned}\frac{d^2\lambda_a^2(t)}{dt^2} &= 2(\omega_0^2+\omega_f^2) - 4\omega_f^2\lambda_a^2(t)\\ &\quad + \tilde{C}_a\left[\frac{1}{\lambda_a^{4-d}(t)}\left(\frac{1}{\tilde{a}(t)}\right)^{d-2} - 1\right]\\ &\quad + 2\tilde{C}_a\int_0^t dt'\left(\frac{1}{\tilde{a}(t')}\right)^{d-1}\frac{1}{\lambda_a^{4-d}(t)}\frac{d\tilde{a}(t')}{dt'}\\ &\quad - \tilde{\zeta}\left(\frac{\lambda_a^2(t)}{\tilde{a}^2(t)}\right)^{d-2}\left(\frac{1}{\lambda_a^2(t)}\frac{d\lambda_a^2(t)}{dt} - \frac{2}{\tilde{a}(t)}\frac{d\tilde{a}(t)}{dt}\right),\end{aligned} \tag{15}$$

where we have defined

$$\tilde{a}(t) = \frac{a(t)}{a(0)}, \quad \tilde{C}_a = \frac{2\langle C_a\rangle(0)}{\langle r^2\rangle(0)a^{d-2}(0)}, \quad \tilde{\zeta} = d^2\int\frac{d\mathbf{r}}{N}\frac{\zeta(\mathbf{r},0)}{\langle r^2\rangle(0)}, \tag{16}$$

and $\zeta(\mathbf{r},t)$ is the local bulk viscosity that depends quadratically on the inverse scattering length, $1/a(t)^2$ near the SIFP [43–46]. The initial conditions are given by $\lambda_a(0)=1$ and $\dot{\lambda}_a(0)=0$. First, let us consider the expansion into free space ($\omega_f=0$). At the strongly interacting SIFP, $a(t)=\infty$, the solution to Eq. (15) is $\lambda^2(t)=1+(\omega_0 t)^2$, the same as that based on conformal symmetry. For finite $a(t)$, the solution of Eq. (15) is given by $\lambda_a^2(t)$ that differ from $\lambda^2(t)$: $\delta\lambda^2(t)\equiv[\lambda_a^2(t)-\lambda^2(t)]/\lambda^2(t)$. In the long time limit and for weak conformal symmetry breaking, one expects that $\delta\lambda^2(t)$ to be simply proportional to $g(t)$ [see Eq. (12)].

In Fig. 2, we present $\delta\lambda^2(t)$ in both $d=3$ [Fig. 2(a)] and $d=1$ [Fig. 2(b)], and for various values of γ in Eq. (6). As one can see in both cases the differences become substantially larger when γ satisfies the relevancy condition, Eqs. (8). On the other hand, if the symmetry breaking interaction is irrelevant, then the long-time dynamics closely track the conformal invariant solution. In addition, as we show in the inset, $\delta\lambda^2(t)$ is also proportional to $g(t)$ [see Eq. (12)] for a substantial time window that extends to a time t about $\omega_0 t\approx 30$, which would allow enough time for observation experimentally. The deviation from linearity at even longer times is due to the fact that the bulk viscosity ζ is no longer proportional to $1/a^{2(d-2)}(t)$ as $a(t)$ deviates substantially from its critical value in the long-time limit.

Next, let us consider the case of a quenched harmonic potential: $\omega_0\to\omega_f$. At the SIFP, both $\tilde{C}_a$ and $\tilde{\zeta}$ vanish. Equation (15) then predicts undamped oscillations at exactly $2\omega_f$, identical to the scale invariant solution [47]. When the scale invariance is broken, however, damping sets in and leads to the thermalization of the gas. In Fig. 3, we show the numerical solutions of Eq. (15) near the strongly interacting SIFP in three dimensions [Fig. 3(a)] and one dimension [Fig. 3(b)], as one moves away from the SIFP. For an equivalent change in the Hamiltonian, Eq. (1), the one-dimensional Fermi gas is more stable against conformal symmetry breaking than its three-dimensional

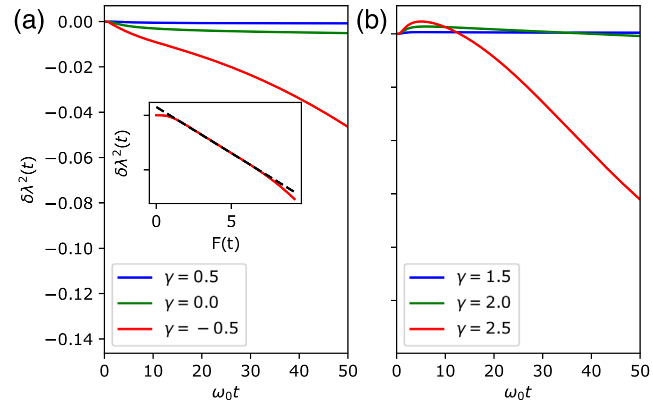

FIG. 2. Solutions to the hydrodynamic equations [Eq. (15)] in free space with a time-dependent scattering length [Eq. (6)] near the strongly interacting SIFP for (a) 3D and (b) 1D. Here we set $\eta=1.5\omega_0$. For relevant symmetry breaking perturbations, i.e., $\gamma=-0.5$ in 3D and $\gamma=2.5$ in 1D, the deviation from the conformal solution becomes significant and is accurately described by Eq. (12). The inset in (a) shows the linear relation between $\delta\lambda^2(t)$ and $g(t)$, which holds for approximately $0<\omega_0 t<30$. For irrelevant symmetry breaking interactions, $\gamma=0.5$ in 3D and $\gamma=2$ in 1D, the hydrodynamic simulation closely tracks the conformal solution in the long-time limit.

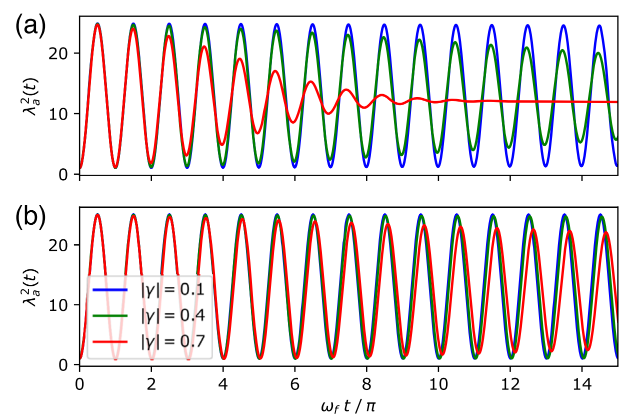

FIG. 3. Numerical solution to the hydrodynamic equations [Eq. (15)] when one moves away from the strongly interacting SIFP for (a) $d=3$ and (b) $d=1$. To compare these two situations, the symmetry breaking interactions in the Hamiltonian, Eq. (1), are equivalent in $d=3$ and $d=1$. That is, we use the same values of the coupling constants, $\eta=\omega_0$, $|\tilde{C}_a|=0.01$, $\tilde{\zeta}=\tilde{C}_a^2$ for both $d=1,3$ (For repulsive interactions $\tilde{C}_a>0$ for $d=3$ and $\tilde{C}_a<0$ for $d=1$), and the same value of $|\gamma|$ ($\gamma<0$ for $d=3$, and $\gamma>0$ for $d=1$).

counterpart. This is exactly what we expect based on the relevancy criterion given in Eq. (8).

The damping of the oscillation amplitude $\lambda_a^2(t)$ can be described by the following phenomenological equation: $\lambda_a^2(t) = \lambda_a^2(0)\exp[-\Gamma_d(t)t]$, where the effective damping rate $\Gamma_d(t)$ is given by

$$\frac{1}{\Gamma_d(t)} \approx \frac{2B}{\omega_0}\frac{\tilde{a}^{2(d-2)}(t)}{\tilde{\zeta}}, \tag{17}$$

for some constant B that depends on the details of the time dependence of $a(t)$. Equation (17) suggests that equilibrationj or thermalization can be sped up or slowed down by changing the time dependence of the scattering length. Consider the following time-dependent scattering length:

$$\tilde{a}^{d-2}(t) = \left[\frac{1}{\sqrt{1+(\eta t)^2}}\right]^{-1/2}. \tag{18}$$

In the long-time limit, $\tilde{a}^{d-2}(t) \propto (\eta t)^{1/2}$. In this case, the damping of the local maxima of the moment of inertia follows a power-law behavior:

$$\lambda_a^2(t) \propto \frac{\omega_0^2+\omega_f^2}{2\omega_0^2}\left(1+\frac{A}{(\eta t)^{\alpha}}\right), \qquad \alpha = \frac{\tilde{\zeta}}{2\eta}, \tag{19}$$

for some constant A. In the limit $t \to \infty$, the system thermalizes to a universal value: $\lambda_a^2(t\to\infty) = \lambda_{\rm th}^2 = (\omega_0^2+\omega_f^2)/(2\omega_0^2)$. To test this hypothesis, we fit the local maxima of the oscillations to a power-law decay, and find results consistent with Eq. (19). The power-law decay is shown in Fig. 4, and as one can see, is very accurate at describing the damping physics.

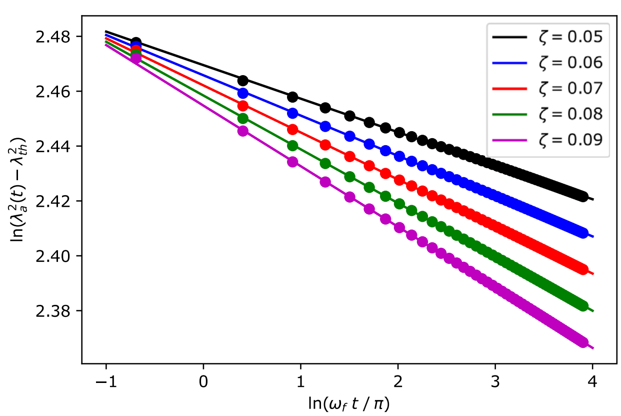

FIG. 4. Power-law damping of the peak height with respect to the final thermalized value, $\lambda_{\rm th}^2 = (\omega_0^2+\omega_f^2)/(2\omega_0^2)$, in the oscillations of a Fermi gas near the strongly interacting SIFP, as a function of the bulk-viscosity $\tilde{\zeta}$, when $d=3$. Here we set $\omega_f = \omega_0/5$, $\tilde{C}_a = 0.05$, and $\eta = 2\omega_0$. The time-dependent scattering length is given by Eq. (18).

Conclusions.—In this Letter, we provided a useful criterion for understanding whether a time-dependent scale breaking perturbation is relevant to the long-time dynamics. The time dependence of the perturbation can be used to enhance or diminish the effects of broken conformal invariance, which can be implemented in experiments. One unique way of using the time-dependent scattering length is to engineer power-law thermalization in trapped Fermi gases. Although we focused on the application to atomic gases, we stress that these results extend to other quantum systems with dynamical exponent $z=2$, such as the Lifshitz transition in solid state materials [48].

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*shizhong@hku.hk

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