

## SU(2)-in-SU(1,1) Nested Interferometer for High Sensitivity, Loss-Tolerant Quantum Metrology

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We present experimental and theoretical results on a new interferometer topology that nests a SU(2) interferometer, e.g., a Mach-Zehnder or Michelson interferometer, inside a SU(1,1) interferometer, i.e., a Mach-Zehnder interferometer with parametric amplifiers in place of beam splitters. This SU(2)-in-SU(1,1) nested interferometer (SISNI) simultaneously achieves a high signal-to-noise ratio (SNR), sensitivity beyond the standard quantum limit (SQL) and tolerance to photon losses external to the interferometer, e.g., in detectors. We implement a SISNI using parametric amplification by four-wave mixing (FWM) in Rb vapor and a laser-fed Mach-Zehnder SU(2) interferometer. We observe path-length sensitivity with SNR 2.2 dB beyond the SQL at power levels (and thus SNR) 2 orders of magnitude beyond those of previous loss-tolerant interferometers. We find experimentally the optimal FWM gains and find agreement with a minimal quantum noise model for the FWM process. The results suggest ways to boost the in-practice sensitivity of high-power interferometers, e.g., gravitational wave interferometers, and may enable high-sensitivity, quantum-enhanced interferometry at wavelengths for which efficient detectors are not available.

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The use of squeezing [1,2] and entanglement [3,4] allows advanced interferometers to detect signals that would otherwise be buried in quantum mechanical noise. The signal-to-noise ratio (SNR) is a central figure of merit in any such sensing application. The signal strength can be increased by using a larger flux of photons, while quantum noise can be reduced below the shot-noise level using nonclassical states of light [1]. Approaches that use single squeezed beams and SU(2) interferometers, e.g., Michelson or Mach-Zehnder interferometers [5], have been successful in producing more than 10 dB of noise suppression [6,7] and have been applied in interferometers with very high photon flux [8,9] to achieve both large and quantum-enhanced signal-to-noise ratios. In gravitational wave detectors, a variety of effects limit the useful power such that both high flux and nonclassical input are required to achieve maximum sensitivity [10].

A current limitation of this approach is the loss of squeezing that accompanies optical losses, which break the photon-photon correlations of a strongly squeezed state. In Aasi *et al.* [9], for example, 2.2 dB of extra sensitivity was achieved, even though 10.3 dB of squeezing was available, due in large part to the 44% system efficiency. Losses external to the interferometer itself, including photodetection inefficiency, significantly impact the achievable quantum advantage [9–11].

An alternative approach is to use in-principle-noiseless quantum optical methods to amplify the interferometer signal, rather than aiming to suppress quantum noise. For example, the SU(1,1) interferometer [12] (SUI) has the topology of the Mach-Zehnder interferometer (MZI), but the splitting and recombination elements are parametric amplifiers (PAs), rather than passive beam splitters [13]. The SNR of the SUI can be insensitive to external loss,

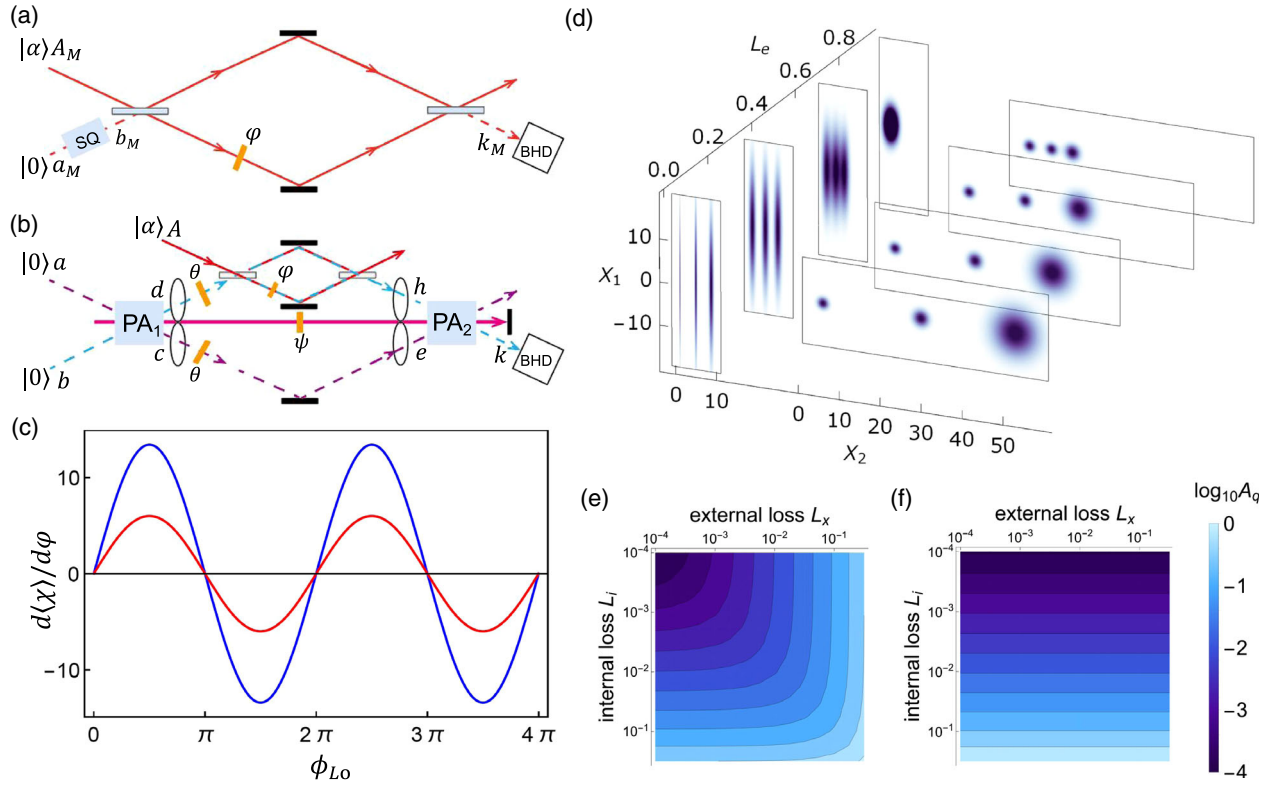


FIG. 1. Comparison of SQ-MZI and SISNI in theoretical simulations. (a),(b) Topology of SQ-MZI and SISNI, respectively. BHD, balanced homodyne detection. Insets show the various input states in the amplitude quadrature ( $X_1$ )—phase quadrature ( $X_2$ ) phase space. For both interferometer types, a coherent state  $|\alpha\rangle$  feeds the “bright input” port of the SU(2) interferometer, while a nonclassical state feeds the “dark input” port. (c) Signal gain  $d\mathcal{X}/d\varphi$  versus local oscillator phase  $\phi_{LO}$  for SQ-MZI (red) and SISNI (blue), for the lossless case. Both configurations have  $|\alpha|^2 = 36$ . (d) Shading indicates Wigner distributions for the output mode  $\hat{k}$  and  $\hat{k}_M$  versus signal phase  $\varphi$  (three equispaced  $\varphi$  values are shown in  $X_1, X_2$  phase space) and versus external loss  $L_e$ . For larger loss values, degradation of SNR, i.e., overlap of uncertainty areas, is evident in the SQ-MZI case, but not that of SISNI. (e),(f) Quantum sensitivity advantage  $A_q \equiv \langle \delta\varphi^2 \rangle / \langle \delta\varphi^2 \rangle_{\text{SQL}}$  in decibels, calculated from Eqs. (2) and (3), as a function of internal and external loss for (left) SISNI and (right) SQ-MZI.

i.e., to losses outside of the loop formed by the two paths and PAs [14]. Similar advantages can be obtained in a “truncated” SUI that replaces the second PA with correlated phase-sensitive detection [15].

The SUI approach has its own limitations. The PAs, as their name suggests, are used as amplifiers rather than light sources *per se*. To increase photon flux and signal strength, seed light is introduced into the upstream PA to stimulate the generation of bright two-mode squeezed beams. While the PA process can be noiseless in theory, the best implementations to date are based on atomic four-wave mixing (FWM), which inevitably introduces additional noise [16–22]. This FWM noise grows faster with seed power than does the signal, thereby setting an intrinsic limit to SNR. This drawback has to date limited the phase-sensing light power to tens of microwatts in SUI interferometers [13,23,24]. Similarly, nonlinear-optical effects limit the practical twin-beam power of FWM in optical fiber [25].

Here we propose and demonstrate a new SUI-based approach, the SU(2)-in-SU(1,1) nested interferometer (SISNI), which combines advantages of SU(1,1) and

SU(2) interferometry. As illustrated in Fig. 1(b), a SU(2) interferometer is nested inside a SU(1,1) interferometer: the “signal” beam from the upstream PA (PA<sub>1</sub>) is fed into the dark input port of a SU(2) interferometer, while a bright coherent state is fed into the bright input port. The light emerging from the SU(2) dark output port is then recombined with the idler beam in the downstream PA (PA<sub>2</sub>). The advantage of the SISNI can be explained as follows: in the *dark fringe* condition, the SU(2) dark output is a noiseless copy of the dark input, displaced by an amplitude  $\propto |\alpha|\varphi$ . The displacement indicates the phase  $\varphi$ , boosted by the strong coherent state magnitude  $|\alpha|$ . The SU(1,1) interferometer, meanwhile, is a loss-tolerant detector of translations [14]. In this way, the SISNI achieves the large signal strength of SU(2) interferometry and the loss-tolerant quantum noise reduction of the SUI approach.

In principle, the same advantages can be achieved with single-mode squeezers at the input (for sub-shot-noise operation) and at the output (for loss tolerance) [1]. The SU(1,1) provides both functions in a simple implementation and also gives one the possibility to sense with one

wavelength and detect at another [26]. Another modified SUI was recently proposed, the so-called ‘‘pumped-up SUI’’ [27], which achieves large photon flux by detecting interference of the signal, idler, and also pump. This approach appears attractive for atomic interferometry, with possible implementation in spinor Bose-Einstein condensates [28] or hybrid atom-light systems [29].

The quantum optical performance of each strategy can be analyzed by considering cascaded linear input-output relations [30,31]. For a SU(2) interferometer, e.g., MZI, the input and output have the relation

$$\hat{k}_M = \hat{N}_M + \sqrt{\eta} \left[ \cos\left(\frac{\varphi}{2}\right) \hat{b}_M - i \sin\left(\frac{\varphi}{2}\right) \hat{A}_M \right], \quad (1)$$

where  $\hat{N}_M = \sqrt{L_e} \hat{D}_M + \sqrt{L_i(1-L_e)/2} (\hat{B}_M - \hat{C}_M)$  and  $\eta = (1-L_i)(1-L_e)$ . A strong coherent state  $|\alpha\rangle$  is injected into the MZI bright input  $\hat{A}_M$ , vacuum enters by modes  $\hat{B}_M$ ,  $\hat{C}_M$ , and  $\hat{D}_M$ , and  $L_i$  and  $L_e$  indicate internal (affecting both beams of MZI) and external (after the second MZI beam splitter) photon loss probabilities, respectively. The dark fringe condition is  $\varphi = \varphi_0 + \varphi_{\text{sig}}$ , where  $\varphi_0 = 0$  is the operating point and  $\varphi_{\text{sig}} \ll 1$  is the signal phase. The term in  $\sin(\varphi_{\text{sig}}/2) \hat{A}$  contributes the signal, which can be large if mode  $\hat{A}$  contains a bright coherent state. In low-loss conditions, the term in  $\cos(\varphi_{\text{sig}}/2) \hat{b}_M$  contributes nearly all the noise; mode  $\hat{A}$ , being a coherent state, contributes unit noise scaled down by the small prefactor  $\sin(\varphi_{\text{sig}}/2)$ .

A squeezed-light MZI (SQ-MZI), injects squeezed light into the MZI dark input, as shown in Fig. 1(a). This places the mode  $\hat{b}_M$  in Eq. (1) in a vacuum squeezed state; that is,  $\hat{b}_M = G \hat{a}_M + g \hat{a}_M^\dagger$ , where  $G = \sqrt{1+g^2}$  is the amplification gain of the single-mode squeezer and  $\hat{a}_M$  is a vacuum state. This suppresses noise in one quadrature of mode  $\hat{b}_M$  and increases the SNR. The SNR, after optimization of the squeezing and interferometer phases, is

$$\zeta_{\text{SQ-MZI}} = \frac{\eta |\alpha|^2 \varphi_{\text{sig}}^2}{1 + \eta [(G-g)^2 - 1]}, \quad (2)$$

where  $|\alpha|^2$  is the input power and  $(G+g)^{-2}$  is the degree of squeezing at the dark port input. When  $(G+g)^{-2} = 1$ , the above equation describes a conventional MZI with standard quantum limit (SQL) sensitivity. Following Eq. (2), we show how a phase coded squeezed state evolves with loss in the left part of Fig. 1(d). We note the noise is extremely sensitive to loss since its noise level is lower than that of a vacuum state.

The SISNI method can help us to overcome the loss sensitivity, as shown in Fig. 1(b). A nondegenerate optical parametric amplifier is used to generate twin beams  $\hat{c}$  and  $\hat{d}$ , then the signal mode  $\hat{d}$  is injected into the dark input port

of the MZI. When the MZI is locked at a dark fringe, the noise at the dark output will be still correlated to the ‘‘idler’’ mode  $\hat{e}$ . Finally, mode  $\hat{h}$  and  $\hat{e}$  are injected into the second PA, where destructive interference partially cancels quantum noise at the output. Here the noise term  $\hat{b}_M$  in Eq. (1) is replaced by one of the twin beams, i.e.,  $\hat{d}_M = (G_1 \hat{a} + g_1 \hat{b}^\dagger) e^{i\theta/2}$ , where  $\theta$  is the common-mode phase of the SU(1,1) interferometer paths. The relative phase is irrelevant to the signal (see Supplemental Material [30]). The optimized SNR of SISNI is

$$\zeta_{\text{SISNI}} = \frac{\eta_s G_2^2 |\alpha|^2 \varphi_{\text{sig}}^2}{L + (\eta_s G_2^2 + \eta_i g_2^2)(G_1^2 + g_1^2) - 4\sqrt{\eta_s \eta_i} G_1 G_2 g_1 g_2}, \quad (3)$$

where  $L = L_e + g_2^2(1-L_e)L_{ii} + G_2^2(1-L_e)L_{is}$  and  $L_\beta$  with  $\beta \in \{is, ii, e\}$  indicate, respectively, internal loss of the signal mode [including loss in the SU(2) interferometer] and idler mode of the SU(1,1) interferometer and external loss after PA<sub>2</sub>.  $G_1 = \sqrt{1+g_1^2}$  and  $G_2 = \sqrt{1+g_2^2}$  are the amplification gains of PA<sub>1</sub> and PA<sub>2</sub>, respectively, where  $1/(G_1+g_1)^2$  indicates the degree of two-mode squeezing generated by PA<sub>1</sub>. The right part of Fig. 1(d) represents Eq. (3), which shows the same noise performance as SUI. We note that the SISNI improves the SNR relative to the conventional MZI with the same  $|\alpha|^2$ .

To demonstrate the above-described quantum advantage at large signal strength, we implemented a SISNI as shown in Fig. 2(a). PA<sub>1</sub> and PA<sub>2</sub> are implemented as FWM processes [32] in <sup>85</sup>Rb, with amplification gains  $G_1$  and  $G_2$ . A MZI, formed by two linear beam splitters and mirrors with piezoelectric transducers (PZTs), is nested in one arm of the SUI. The two input ports of PA<sub>1</sub> are fed with vacuum to avoid excess noise from the FWM process. Laser light is injected into the bright input port of the MZI. Local oscillator (LO) beams are generated by FWM process to implement balanced homodyne detection. We note that the interferometer becomes a simple MZI if the PAs’ pump light is blocked. The relative phase  $\phi$  of the two PA pumps is locked to minimum net amplification by a quantum noise locking technique [33], and a coherent modulated locking technique is used to maintain the MZI at the dark fringe condition [34]. To lock the phase of the LO on the phase quadrature, we use a method described by Liu *et al.* [25], in which the input coherent state is amplitude modulated and the envelope of modulations seen at the homodyne detector is fed back to the LO phase.

The output performance of SISNI and MZI under the same operating conditions are shown in Fig. 2. The black trace in Fig. 2(b) is the output noise level of MZI at the dark fringe, which is also the vacuum noise level. The blue trace in Fig. 2(b) shows the noise level of PA<sub>2</sub>, 6 dB above vacuum noise. To have an experimentally accessible

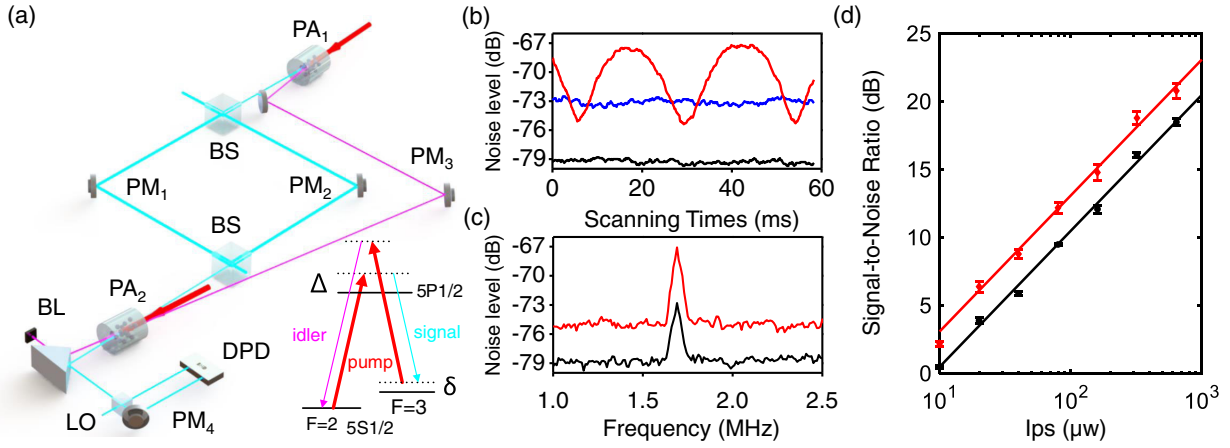


FIG. 2. Experimental demonstration of quantum enhancement of the SISNI. (a) Schematic of experiment. BS, 50/50 beam splitter; PM, mirror mounted with piezoelectric transducer as phase modulator; DPD, differential photodetector; BL, beam block. Inset shows the double- $\Lambda$  level structure of the PA process, which uses the D1 line of  $^{85}\text{Rb}$ . The two pump beams (red arrows) are frequency degenerate,  $\Delta = 1$  GHz and  $\delta = 2$  MHz.  $\text{PA}_1$  and  $\text{PA}_2$  act as a source of two-mode squeezing and amplifier, respectively. (b) Noise of the MZI and SISNI. Graph shows measured noise of the output phase quadrature in a bandwidth of 100 kHz about 1.7 MHz, versus time as  $\phi$ , the relative phase of the PAs, is scanned using  $\text{PM}_3$ . MZI phase is locked to dark fringe. Traces show MZI SQL (black), implemented by blocking both PA pumps, MZI output amplified by  $\text{PA}_2$  (blue), implemented by blocking only  $\text{PA}_1$  pump, and SISNI when MZI locked at dark fringe as the  $\text{SU}(1,1)$  phase is scanned (red). (c) Measured noise spectra of MZI (black) and SISNI (red) outputs, acquired as in (b), but with PAs relative phase locked to noise minimum and a sinusoidal phase signal at 1.7 MHz applied via  $\text{PM}_1$ . (d) Measured SNR, defined as spectral peak over white background level from spectra as in (c), versus phase-sensing light intensity ( $I_{\text{ps}}$ ) for MZI (black) and SISNI (red). Error bars show  $\pm 1\sigma$  statistical variation. Lines show fits with  $\text{SNR} = A|\alpha|^2$ . The QNG of  $\text{PA}_1$  and  $\text{PA}_2$  are  $G_{q1} = 4$  dB and  $G_{q2} = 6$  dB, respectively.

measure of PA gain, we define the quantum noise gain (QNG) as  $G_q \equiv \langle \delta^2 \hat{X}(\theta) \rangle$  resulting from vacuum inputs. For an ideal PA,  $G_q = G^2 + g^2$ . The red trace in Fig. 2(b) shows noise reduction (minimum) and antireduction (maximum) by scanning the phase of twin beams with the MZI locked at the dark fringe. It shows 2.4 dB of noise reduction below the noise level of  $\text{PA}_2$ , while the QNG of  $\text{PA}_1$  is set as 4 dB. The inset in Fig. 2(c) shows the power spectrum of MZI (black trace) and SISNI (red trace) when a signal at 1.7 MHz is introduced by modulating the PZT. As above, both the MZI and SUI are locked at their respective dark conditions. The measured SNRs [35] of the MZI and SISNI are 4.8(2) and 7.0(3) dB, respectively, which indicates a 2.2(5) dB SNR enhancement.

The above results were obtained for optimal gains  $G_{q1} = 4$  dB and  $G_{q2} = 6$  dB. The existence of an optimum is an important characteristic of the SISNI with atomic FWM, not present in models with ideal PAs. As shown in Fig. 3, the SNR enhancement, measured as described above, increases with increasing  $\text{PA}_2$  gain until saturation, and in the range studied decreases with increasing  $\text{PA}_1$  gain. This behavior can be understood as the result of atomic dephasing: this introduces excess noise in the  $\text{PA}_1$  output, which is further amplified by  $\text{PA}_2$ . A detailed model including atomic dephasing and internal losses of 16% and 10% in the signal and idler arms, respectively, agrees well with all experimental results (see Supplemental Material [30]).

To verify that our scheme is able to maintain the SNR enhancement at high photon flux, we measure the SNR of MZI and SISNI while increasing the laser input power. As can be seen in Fig. 2(d), both MZI and SISNI performance are well fit by  $\text{SNR} = A|\alpha|^2$ , which confirms their photon

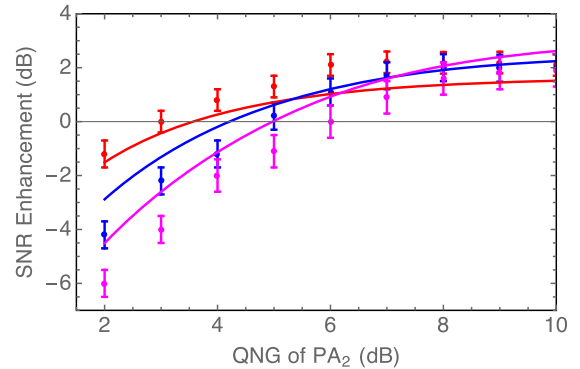


FIG. 3. Characterization of the trade-off between gain and FWM noise in the SISNI. Horizontal axis shows QNG of  $\text{PA}_2$ , adjusted by control of the pump power. Vertical axis shows the SNR advantage of the SISNI over the MZI at equal coherent state input power, with positive values indicating advantage for the SISNI. Points and error bars indicate mean and  $\pm 1$  standard deviation from repeated measurements. Red, blue, and magenta data points and curves correspond to  $\text{PA}_1$  QNG levels 4, 6, and 8 dB, respectively. Curves show fits based on a minimal PA noise model [30,36].

shot-noise limited performance. By the fit, the SISNI SNR is 2.2 dB above that of the MZI, comparable to that seen in other SUI experiments [25,34,37]. The advantage is observed up to nearly 1 mW, which is hundreds of times higher than the  $\mu\text{W}$  levels of previous SUIs [13,23,38]. The interferometer, built from dielectric mirrors and beam splitters, could withstand much higher powers. In a SISNI, the PAs need not support high powers.

The SISNI is tolerant of loss in the detection process, because the PA process boosts the signal above the vacuum noise level. PA processes, e.g., FWM, have been demonstrated from radio [39] to XUV [40] wavelengths, whereas high-quantum-efficiency detectors exist for a much more limited range. Provided these PA processes can be quantum noise limited, the SISNI will enable sensitive, quantum-enhanced measurements in previously inaccessible spectral regions. The SISNI, like the SUI, can use one wavelength to sense and another to detect, as in [41], and can make simultaneous measurement of phase and amplitude, one at each mode [25]. Possible use cases include reduced-damage probing of materials [42] and nanoimaging [43].

In conclusion, we have experimentally demonstrated a new interferometer topology, the SU(2)-in-SU(1,1) nested interferometer. A detailed analysis shows that this topology combines the loss tolerance of SU(1,1) interferometry with the large signal strength of SU(2) interferometry. Experimentally, we have demonstrated 2.2 dB of signal-to-noise ratio improvement beyond the standard quantum limit, with optical power levels, and thus signal-to-noise ratios, beyond the reach of traditional SU(1,1) interferometry.

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